Introduction to Database Systems
CSE 444

Lectures 6-7: Database Design
Outline

• Design theory: 3.1-3.4
  – [Old edition: 3.4-3.6]
Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = will study
- 3rd Normal Form = see book
First Normal Form (1NF)

- A database schema is in First Normal Form if all tables are flat

<table>
<thead>
<tr>
<th>Student</th>
<th>GPA</th>
<th>Courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>3.8</td>
<td>Math, DB, OS</td>
</tr>
<tr>
<td>Bob</td>
<td>3.7</td>
<td>DB, OS</td>
</tr>
<tr>
<td>Carol</td>
<td>3.9</td>
<td>Math, OS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>3.8</td>
</tr>
<tr>
<td>Bob</td>
<td>3.7</td>
</tr>
<tr>
<td>Carol</td>
<td>3.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
</tr>
<tr>
<td>DB</td>
</tr>
<tr>
<td>OS</td>
</tr>
</tbody>
</table>

May need to add keys.
Relational Schema Design

Conceptual Model:

Relational Model: plus FD’s

Normalization: Eliminates anomalies
Data Anomalies

When a database is poorly designed we get anomalies:

**Redundancy**: data is repeated

**Updated anomalies**: need to change in several places

**Delete anomalies**: may lose data when we don’t want
Relational Schema Design

Recall set attributes (persons with several phones):

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>PhoneNumber</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-1234</td>
<td>Seattle</td>
</tr>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-6543</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-2121</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

One person may have multiple phones, but lives in only one city.

Primary key is thus (SSN,PhoneNumber)

The above is in 1NF, but was is the problem with this schema?
Relational Schema Design

Recall set attributes (persons with several phones):

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>PhoneNumber</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-1234</td>
<td>Seattle</td>
</tr>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-6543</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-2121</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

Anomalies:
- Redundancy = repeat data
- Update anomalies = what if Fred moves to “Bellevue”?  
- Deletion anomalies = what if Joe deletes his phone number?  
  (what if Joe had only one phone #)
Relation Decomposition

Break the relation into two:

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>PhoneNumber</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-1234</td>
<td>Seattle</td>
</tr>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-6543</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-2121</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

Anomalies have gone:

- No more repeated data
- Easy to move Fred to “Bellevue” (how ?)
- Easy to delete all Joe’s phone numbers (how ?)
Relational Schema Design
(or Logical Design)

Main idea:
• Start with some relational schema
• Find out its \textit{functional dependencies} 
  – They come from the application domain knowledge!
• Use them to design a better relational schema
Functional Dependencies

- A form of constraint
  - Hence, part of the schema
- Finding them is part of the database design
- Use them to normalize the relations
Functional Dependencies (FDs)

Definition:

If two tuples agree on the attributes

\[ A_1, A_2, \ldots, A_n \]

then they must also agree on the attributes

\[ B_1, B_2, \ldots, B_m \]

Formally:

\[ A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m \]
When Does an FD Hold

Definition: \( A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n \) holds in \( R \) if:

\[
\forall t, t' \in R, \\
(t.A_1 = t'.A_1 \land \ldots \land t.A_m = t'.A_m \Rightarrow t.B_1 = t'.B_1 \land \ldots \land t.B_n = t'.B_n)
\]

<table>
<thead>
<tr>
<th>R</th>
<th>( A_1 )</th>
<th>( \ldots )</th>
<th>( A_m )</th>
<th>( B_1 )</th>
<th>( \ldots )</th>
<th>( B_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
</tr>
<tr>
<td>t'</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
<td>( \square )</td>
</tr>
</tbody>
</table>

\[\text{if } t, t' \text{ agree here} \quad \Rightarrow \quad \text{then } t, t' \text{ agree here}\]
Example

An FD holds, or does not hold on an instance:

<table>
<thead>
<tr>
<th>EmpID</th>
<th>Name</th>
<th>Phone</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>E0045</td>
<td>Smith</td>
<td>1234</td>
<td>Clerk</td>
</tr>
<tr>
<td>E3542</td>
<td>Mike</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E1111</td>
<td>Smith</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E9999</td>
<td>Mary</td>
<td>1234</td>
<td>Lawyer</td>
</tr>
</tbody>
</table>

EmpID → Name, Phone, Position
Position → Phone
but not Phone → Position
Example

<table>
<thead>
<tr>
<th>EmpID</th>
<th>Name</th>
<th>Phone</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>E0045</td>
<td>Smith</td>
<td>1234</td>
<td>Clerk</td>
</tr>
<tr>
<td>E3542</td>
<td>Mike</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E1111</td>
<td>Smith</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E9999</td>
<td>Mary</td>
<td>1234</td>
<td>Lawyer</td>
</tr>
</tbody>
</table>

Position → Phone
Example

<table>
<thead>
<tr>
<th>EmpID</th>
<th>Name</th>
<th>Phone</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>E0045</td>
<td>Smith</td>
<td>1234</td>
<td>Clerk</td>
</tr>
<tr>
<td>E3542</td>
<td>Mike</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E1111</td>
<td>Smith</td>
<td>9876</td>
<td>Salesrep</td>
</tr>
<tr>
<td>E9999</td>
<td>Mary</td>
<td>1234</td>
<td>Lawyer</td>
</tr>
</tbody>
</table>

But not Phone → Position
Example

FD’s are constraints:
• On some instances they hold
• On others they don’t

<table>
<thead>
<tr>
<th>name</th>
<th>category</th>
<th>color</th>
<th>department</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gizmo</td>
<td>Gadget</td>
<td>Green</td>
<td>Toys</td>
<td>49</td>
</tr>
<tr>
<td>Tweaker</td>
<td>Gadget</td>
<td>Green</td>
<td>Toys</td>
<td>99</td>
</tr>
</tbody>
</table>

Does this instance satisfy all the FDs?
### Example

**name** $\rightarrow$ **color**

**category** $\rightarrow$ **department**

**color, category** $\rightarrow$ **price**

<table>
<thead>
<tr>
<th>name</th>
<th>category</th>
<th>color</th>
<th>department</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gizmo</td>
<td>Gadget</td>
<td>Green</td>
<td>Toys</td>
<td>49</td>
</tr>
<tr>
<td>Tweaker</td>
<td>Gadget</td>
<td>Black</td>
<td>Toys</td>
<td>99</td>
</tr>
<tr>
<td>Gizmo</td>
<td>Stationary</td>
<td>Green</td>
<td>Office-supp.</td>
<td>59</td>
</tr>
</tbody>
</table>

What about this one?
An Interesting Observation

If all these FDs are true:

- name → color
- category → department
color, category → price

Then this FD also holds:

- name, category → price

Why ??
Goal: Find ALL Functional Dependencies

• Anomalies occur when certain “bad” FDs hold

• We know some of the FDs

• Need to find all FDs
• Then look for the bad ones
Armstrong’s Rules (1/3)

\[ A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m \]

Is equivalent to

\[ A_1, A_2, \ldots, A_n \rightarrow B_1 \]
\[ A_1, A_2, \ldots, A_n \rightarrow B_2 \]
\[ \ldots \ldots \]
\[ A_1, A_2, \ldots, A_n \rightarrow B_m \]
Armstrong’s Rules (2/3)

\[ A_1, A_2, \ldots, A_n \rightarrow A_i \]

where \( i = 1, 2, \ldots, n \)

Why?

Trivial Rule
Transitive Rule

If \( A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m \)

and \( B_1, B_2, \ldots, B_m \rightarrow C_1, C_2, \ldots, C_p \)

then \( A_1, A_2, \ldots, A_n \rightarrow C_1, C_2, \ldots, C_p \)

Why?
Armstrong’s Rules (3/3)

Illustration

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>...</td>
<td>Aₘ</td>
<td>B₁</td>
<td>...</td>
<td>Bₘ</td>
<td>C₁</td>
</tr>
</tbody>
</table>
Example (continued)

Start from the following FDs:

1. name → color
2. category → department
3. color, category → price

Infer the following FDs:

<table>
<thead>
<tr>
<th>Inferred FD</th>
<th>Which Rule did we apply?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. name, category → name</td>
<td></td>
</tr>
<tr>
<td>5. name, category → color</td>
<td></td>
</tr>
<tr>
<td>6. name, category → category</td>
<td></td>
</tr>
<tr>
<td>7. name, category → color, category</td>
<td></td>
</tr>
<tr>
<td>8. name, category → price</td>
<td></td>
</tr>
</tbody>
</table>
### Example (continued)

**Answers:**

1. `name $\rightarrow$ color
2. `category $\rightarrow$ department
3. `color, category $\rightarrow$ price

<table>
<thead>
<tr>
<th>Inferred FD</th>
<th>Which Rule did we apply?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. <code>name, category $\rightarrow$ name</code></td>
<td>Trivial rule</td>
</tr>
<tr>
<td>5. <code>name, category $\rightarrow$ color</code></td>
<td>Transitivity on 4, 1</td>
</tr>
<tr>
<td>6. <code>name, category $\rightarrow$ category</code></td>
<td>Trivial rule</td>
</tr>
<tr>
<td>7. <code>name, category $\rightarrow$ color, category</code></td>
<td>Split/combine on 5, 6</td>
</tr>
<tr>
<td>8. <code>name, category $\rightarrow$ price</code></td>
<td>Transitivity on 3, 7</td>
</tr>
</tbody>
</table>

**THIS IS TOO HARD!** Let’s see an easier way.
Closure of a set of Attributes

Given a set of attributes $A_1, \ldots, A_n$

The closure, $\{A_1, \ldots, A_n\}^+ = \text{the set of attributes B}$

s.t. $A_1, \ldots, A_n \rightarrow B$

Example:

- name $\rightarrow$ color
- category $\rightarrow$ department
- color, category $\rightarrow$ price

Closures:

- $\text{name}^+ = \{\text{name, color}\}$
- $\{\text{name, category}\}^+ = \{\text{name, category, color, department, price}\}$
- $\text{color}^+ = \{\text{color}\}$
Closure Algorithm

$X = \{ A_1, \ldots, A_n \}$.

Repeat until $X$ doesn’t change do:

if $B_1, \ldots, B_n \rightarrow C$ is a FD and $B_1, \ldots, B_n$ are all in $X$

then add $C$ to $X$.

Example:

name $\rightarrow$ color
category $\rightarrow$ department
color, category $\rightarrow$ price

$\{\text{name, category}\}^+ = \{ \text{name, category, color, department, price} \}$

Hence: name, category $\rightarrow$ color, department, price
Example

In class:

\[ R(A, B, C, D, E, F) \]

\[
\begin{array}{ccc}
A, B & \rightarrow & C \\
A, D & \rightarrow & E \\
B & \rightarrow & D \\
A, F & \rightarrow & B \\
\end{array}
\]

Compute \( \{A, B\}^+ \) \( X = \{A, B, \} \)

Compute \( \{A, F\}^+ \) \( X = \{A, F, \} \)
Example

In class:

\[ R(A, B, C, D, E, F) \]

\[
\begin{array}{ccc}
A, B & \rightarrow & C \\
A, D & \rightarrow & E \\
B & \rightarrow & D \\
A, F & \rightarrow & B \\
\end{array}
\]

Compute \( \{A, B\}^+ \):
\[
X = \{A, B, C, D, E\}
\]

Compute \( \{A, F\}^+ \):
\[
X = \{A, F, \}
\]
Example

In class:

$$R(A,B,C,D,E,F)$$

\[
\begin{array}{ccc}
A, B & \rightarrow & C \\
A, D & \rightarrow & E \\
B & \rightarrow & D \\
A, F & \rightarrow & B \\
\end{array}
\]

Compute \(\{A,B\}^+\)  \(X = \{A, B, C, D, E\}\)

Compute \(\{A, F\}^+\)  \(X = \{A, F, B, C, D, E\}\)
Why Do We Need Closure

- With closure we can find all FD’s easily

- To check if $X \rightarrow A$
  - Compute $X^+$
  - Check if $A \in X^+$
Using Closure to Infer ALL FDs

Example:

\[
\begin{align*}
    & A, B \rightarrow C \\
    & A, D \rightarrow B \\
    & B \rightarrow D
\end{align*}
\]

Step 1: Compute \( X^+ \), for every \( X \):

\[
\begin{align*}
    A^+ &= A, \quad B^+ = BD, \quad C^+ = C, \quad D^+ = D \\
    AB^+ &= ABCD, \quad AC^+ = AC, \quad AD^+ = ABCD, \\
    & \quad BC^+ = BCD, \quad BD^+ = BD, \quad CD^+ = CD \\
    ABC^+ &= ABD+, \quad ACD^+ = ABCD \quad \text{(no need to compute— why ?)} \\
    BCD^+ &= BCD, \quad ABCD^+ = ABCD
\end{align*}
\]

Step 2: Enumerate all FD’s \( X \rightarrow Y \), s.t. \( Y \subseteq X^+ \) and \( X \cap Y = \emptyset \):

\[
\begin{align*}
    & AB \rightarrow CD, \quad AD \rightarrow BC, \quad BC \rightarrow D, \quad ABC \rightarrow D, \quad ABD \rightarrow C, \quad ACD \rightarrow B
\end{align*}
\]
Another Example

- Enrollment(student, major, course, room, time)
  - student $\rightarrow$ major
  - major, course $\rightarrow$ room
  - course $\rightarrow$ time

What else can we infer? [in class, or at home]
Keys

• A **superkey** is a set of attributes $A_1, ..., A_n$ s.t. for any other attribute $B$, we have $A_1, ..., A_n \rightarrow B$

• A **key** is a minimal superkey
  – I.e. set of attributes which is a superkey and for which no subset is a superkey
Computing (Super)Keys

• Compute $X^+$ for all sets $X$
• If $X^+$ = all attributes, then $X$ is a superkey
• List only the minimal $X$’s to get the keys
Example

Product(name, price, category, color)

name, category → price
category → color

What is the key?
Example

Product(name, price, category, color)

\[
\begin{array}{|l|}
\hline
\text{name, category } \rightarrow \text{ price} \\
\text{category } \rightarrow \text{ color} \\
\hline
\end{array}
\]

What is the key?

\((\text{name, category}) + = \{ \text{name, category, price, color} \}\)

Hence (name, category) is a key
Examples of Keys

Enrollment(student, address, course, room, time)

\[
\begin{align*}
\text{student} & \to \text{address} \\
\text{room, time} & \to \text{course} \\
\text{student, course} & \to \text{room, time}
\end{align*}
\]

(find keys at home)
Eliminating Anomalies

Main idea:

• $X \rightarrow A$ is OK if $X$ is a (super)key

• $X \rightarrow A$ is not OK otherwise
Example

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>PhoneNumber</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-1234</td>
<td>Seattle</td>
</tr>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-6543</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-2121</td>
<td>Westfield</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-1234</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

SSN $\rightarrow$ Name, City

What is the key?

$\{\text{SSN, PhoneNumber}\}$

Hence SSN $\rightarrow$ Name, City is a “bad” dependency

CSE 444 - Summer 2009
Key or Keys?

Can we have more than one key?

Given $R(A,B,C)$ define FD’s s.t. there are two or more keys
Key or Keys?

Can we have more than one key?

Given R(A,B,C) define FD’s s.t. there are two or more keys

<table>
<thead>
<tr>
<th>AB → C</th>
<th>A → BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC → A</td>
<td>B → AC</td>
</tr>
</tbody>
</table>

or

what are the keys here?

Can you design FDs such that there are three keys?
A simple condition for removing anomalies from relations:

A relation $R$ is in BCNF if:

If $A_1, ..., A_n \rightarrow B$ is a non-trivial dependency in $R$,
then $\{A_1, ..., A_n\}$ is a superkey for $R$

In other words: there are no “bad” FDs

Equivalently:
for all $X$, either $(X^+ = X)$ or $(X^+ = \text{all attributes})$
BCNF Decomposition Algorithm

repeat
  choose \( A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n \) that violates BCNF
  split \( R \) into \( R_1(A_1, \ldots, A_m, B_1, \ldots, B_n) \) and \( R_2(A_1, \ldots, A_m, [\text{others}] \)
  continue with both \( R_1 \) and \( R_2 \)
until no more violations

Is there a 2-attribute relation that is not in BCNF?

In practice, we have a better algorithm (coming up)
Example

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>PhoneNumber</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-1234</td>
<td>Seattle</td>
</tr>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>206-555-6543</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-2121</td>
<td>Westfield</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>908-555-1234</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

SSN $\rightarrow$ Name, City

What is the key?

\{SSN, PhoneNumber\} use SSN $\rightarrow$ Name, City to split
Example

<table>
<thead>
<tr>
<th>Name</th>
<th>SSN</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>123-45-6789</td>
<td>Seattle</td>
</tr>
<tr>
<td>Joe</td>
<td>987-65-4321</td>
<td>Westfield</td>
</tr>
</tbody>
</table>

SSN → Name, City

<table>
<thead>
<tr>
<th>SSN</th>
<th>PhoneNumber</th>
</tr>
</thead>
<tbody>
<tr>
<td>123-45-6789</td>
<td>206-555-1234</td>
</tr>
<tr>
<td>123-45-6789</td>
<td>206-555-6543</td>
</tr>
<tr>
<td>987-65-4321</td>
<td>908-555-2121</td>
</tr>
<tr>
<td>987-65-4321</td>
<td>908-555-1234</td>
</tr>
</tbody>
</table>

Let’s check anomalies:
• Redundancy?
• Update?
• Delete?
Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
  FD1: SSN \rightarrow name, age
  FD2: age \rightarrow hairColor

Decompose in BCNF (in class):
Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
   FD1: SSN $\rightarrow$ name, age
   FD2: age $\rightarrow$ hairColor

Decompose in BCNF (in class): What is the key?
   $\{SSN, phoneNumber\}$

But how to decompose?
Person(SSN, name, age)
Phone(SSN, hairColor, phoneNumber)
Or
Person(SSN, name, age, hairColor)
Phone(SSN, phoneNumber)
Or ....
BCNF Decomposition Algorithm

BCNF_Decompose(R)

find X s.t.: X ≠ X⁺ ≠ [all attributes]

if (not found) then “R is in BCNF”

let Y = X⁺ - X
let Z = [all attributes] - X⁺
decompose R into R1(X ∪ Y) and R2(X ∪ Z)
continue to decompose recursively R1 and R2
Find $X$ s.t.: $X \neq X^+ 
eq \text{[all attributes]}$

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
  SSN $\rightarrow$ name, age
  age $\rightarrow$ hairColor

Iteration 1: Person
SSN+ = SSN, name, age, hairColor
Decompose into: $P(\text{SSN, name, age, hairColor})$
  Phone(\text{SSN, phoneNumber})

Iteration 2: $P$
age+ = age, hairColor
Decompose: $\text{People(\text{SSN, name, age})}$
  $\text{Hair(\text{age, hairColor})}$
  $\text{Phone(\text{SSN, phoneNumber})}$

What are the keys?
Example

R(A,B,C,D)

A⁺ = ABC ≠ ABCD

R₁(A,B,C)
B⁺ = BC ≠ ABC

R₂(A,D)

What are the keys?

What happens if in R we first pick B⁺? Or A₂B⁺?
Decompositions in General

\[ R(A_1, \ldots, A_n, B_1, \ldots, B_m, C_1, \ldots, C_p) \]

\[ R_1(A_1, \ldots, A_n, B_1, \ldots, B_m) \]

\[ R_2(A_1, \ldots, A_n, C_1, \ldots, C_p) \]

\[ R_1 = \text{projection of } R \text{ on } A_1, \ldots, A_n, B_1, \ldots, B_m \]

\[ R_2 = \text{projection of } R \text{ on } A_1, \ldots, A_n, C_1, \ldots, C_p \]
Theory of Decomposition

• Sometimes it is correct:

<table>
<thead>
<tr>
<th>Name</th>
<th>Price</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gizmo</td>
<td>19.99</td>
<td>Gadget</td>
</tr>
<tr>
<td>OneClick</td>
<td>24.99</td>
<td>Camera</td>
</tr>
<tr>
<td>Gizmo</td>
<td>19.99</td>
<td>Camera</td>
</tr>
</tbody>
</table>

Lossless decomposition
Incorrect Decomposition

• Sometimes it is not:

<table>
<thead>
<tr>
<th>Name</th>
<th>Price</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gizmo</td>
<td>19.99</td>
<td>Gadget</td>
</tr>
<tr>
<td>OneClick</td>
<td>24.99</td>
<td>Camera</td>
</tr>
<tr>
<td>Gizmo</td>
<td>19.99</td>
<td>Camera</td>
</tr>
</tbody>
</table>

What’s incorrect??

Lossy decomposition
Decompositions in General

\[ R(A_1, ..., A_n, B_1, ..., B_m, C_1, ..., C_p) \]

\[ R_1(A_1, ..., A_n, B_1, ..., B_m) \]
\[ R_2(A_1, ..., A_n, C_1, ..., C_p) \]

If \( A_1, ..., A_n \rightarrow B_1, ..., B_m \)
Then the decomposition is lossless

Note: don’t need \( A_1, ..., A_n \rightarrow C_1, ..., C_p \)

BCNF decomposition is always lossless. WHY?
Optional

• The following four slides are optional
• The content will not be on any exam

• But please take a look because they motivate the need for 3NF

• It’s good to know at least why 3NF exists
General Decomposition Goals

1. Elimination of anomalies

2. Recoverability of information
   - Can we get the original relation back?

3. Preservation of dependencies
   - Want to enforce FDs without performing joins

Sometimes cannot decomposed into BCNF without losing ability to check some FDs
BCNF and Dependencies

<table>
<thead>
<tr>
<th>Unit</th>
<th>Company</th>
<th>Product</th>
</tr>
</thead>
</table>

FD’s: \text{Unit} \rightarrow \text{Company}; \quad \text{Company, Product} \rightarrow \text{Unit}

So, there is a BCNF violation, and we decompose.
BCNF and Dependencies

FD’s: Unit → Company; Company, Product → Unit
So, there is a BCNF violation, and we decompose.

In BCNF we lose the FD: Company, Product → Unit
3NF Motivation

A relation R is in 3rd normal form if:

Whenever there is a nontrivial dep. \( A_1, A_2, \ldots, A_n \rightarrow B \) for R, then \( \{A_1, A_2, \ldots, A_n\} \) is a super-key for R, or B is part of a key.

Tradeoffs

BCNF = no anomalies, but may lose some FDs
3NF = keeps all FDs, but may have some anomalies