Lecture 09:

Friday, October 18, 2002

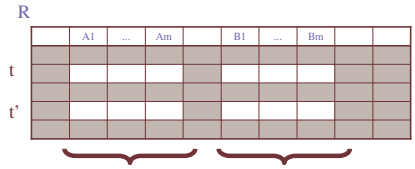
Outline

- Functional dependencies (3.4)
- Rules about FDs (3.5)
- Design of a Relational schema (3.6)

Functional Dependencies

Definition: $A_1, ..., A_m \rightarrow B_1, ..., B_n$ holds in R if:

$\forall t, t' \in R, (t.A_1=t'.A_1 \wedge ... \wedge t.A_m=t'.A_m \Rightarrow t.B_1=t'.B_1 \wedge ... \wedge t.B_m=t'.B_m)$



Formal definition of a key

- A **key** is a set of attributes $A_1, ..., A_n$ s.t. for any other attribute B, $A_1, ..., A_n \rightarrow B$
- A **minimal key** is a set of attributes which is a key and for which no subset is a key
- Note: book calls them **superkey** and **key**

Examples of Keys

- Product(name, price, category, color)
  name, category → price
  category → color

  Keys are: {name, category} and all supersets
- Enrollment(student, address, course, room, time)
  student → address
  room, time → course
  student, course → room, time

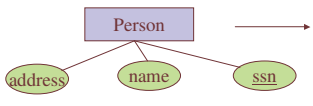
  Keys are: {in class}

Finding the Keys of a Relation

Given a relation constructed from an E/R diagram, what is its key?

Rules:

1. If the relation comes from an entity set, the key of the relation is the set of attributes which is the key of the entity set.

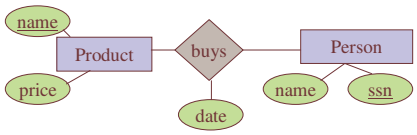


Person(address, name, ssn)

Finding the Keys

Rules:

2. If the relation comes from a many-many relationship, the key of the relation is the set of all attribute keys in the relations corresponding to the entity sets



buys(name, ssn, date)

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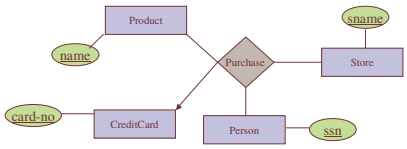
Finding the Keys

Except: if there is an arrow from the relationship to E, then we don't need the key of E as part of the relation key.



Purchase(name , sname, ssn, card-no)

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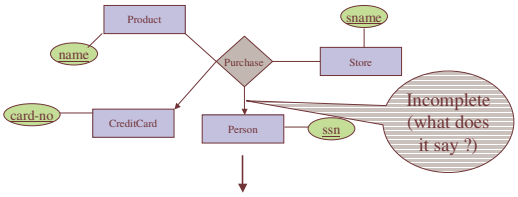
Expressing Dependencies

Say: "the CreditCard determines the Person"



Purchase(name , sname, ssn, card-no)

card-no → name

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Finding the Keys

More rules in the book – please read !

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Inference Rules for FD's

$A_1, A_2, \ldots A_n \longrightarrow B_1, B_2, \ldots B_m$

Is equivalent to

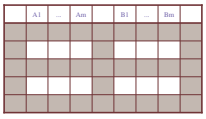
$A_1, A_2, \ldots A_n \longrightarrow B_1$

$A_1, A_2, \ldots A_n \longrightarrow B_2$

…

$A_1, A_2, \ldots A_n \longrightarrow B_m$

Splitting rule and Combing rule



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Inference Rules for FD's (continued)

$A_1, A_2, \ldots A_n \longrightarrow A_i$ **Trivial Rule**

where i = 1, 2, ..., n

Why ?

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Inference Rules for FD's (continued)

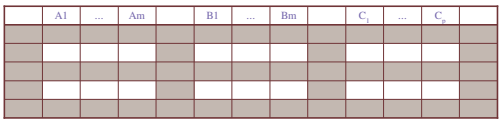
Transitive Closure Rule

If $A_1, A_2, \ldots A_n \longrightarrow B_1, B_2 \ldots, B_m$

and $B_1, B_2, \ldots B_m \longrightarrow C_1, C_2 \ldots, C_p$

then $A_1, A_2, \ldots A_n \longrightarrow C_1, C_2 \ldots, C_p$

Why ?



- Enrollment(student, major, course, room, time)
  - student → major
  - major, course → room
  - course → time

What else can we infer ? [in class]

Closure of a set of Attributes

Given a set of attributes *{A1, …, An}* and a set of dependencies S.
Problem: find all attributes *B* such that:
any relation which satisfies S also satisfies:
A1, …, An → B

The **closure** of *{A1, …, An}*, denoted *{A1, …, An}*$^+$,
is the set of all such attributes *B*

Closure Algorithm

Start with X={A1, …, An}.

Repeat until X doesn't change **do**:

if $B_1, B_2, \ldots B_n \longrightarrow C$ is in S, **and**
$B_i, B_2 \ldots B_n$ are all in X, **and**
C is not in X

then

add C to X.

Example

R(A,B,C,D,E,F)

A B → C
A D → E
B → D
A F → B

Closure of {A,B}: X = {A, B, }

Closure of {A, F}: X = {A, F, }

Why Is the Algorithm Correct ?

- Show the following by induction:
  - For every B in X:
    - $A1, ..., An \rightarrow B$
- Initially $X = \{A1, ..., An\}$ -- holds
- Induction step: $B1, ..., Bm$ in X
  - Implies $A1, ..., An \rightarrow B1, ..., Bm$
  - We also have $B1, ..., Bm \rightarrow C$
  - By transitivity we have $A1, ..., An \rightarrow C$
- This shows that the algorithm is *sound*; need to show it is *complete*

Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its FD's
- Use them to design a better relational schema

Relational Schema Design (or Logical Design)

When a database is poorly designed we get anomalies:

- Redundancy: data is repeated
- Updated anomalies: need to change in several places
- Delete anomalies: may lose data when we don't want

Relational Schema Design

Recall set attributes (persons with several phones):

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name, City, but not SSN → PhoneNumber

Anomalies:

- Redundancy = repeat data
- Update anomalies = Fred moves to "Bellvue"
- Deletion anomalies = Fred drops all phone numbers: what is his city ?

Relation Decomposition

Break the relation into two:

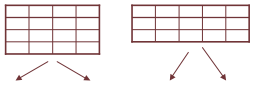
Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Relational Schema Design

Conceptual Model:

Relational Model: plus FD's



Normalization: Eliminates anomalies

Decompositions in General

$R(A_1, ..., A_n)$

Create two relations R1(B1, ..., Bm) and R2(C1, ..., Cp)

such that: B1, ..., Bm ∪ C1, ..., Cp = A1, ..., An

and:

R_1 = projection of R on $B_1, ..., B_m$
R_2 = projection of R on $C_1, ..., C_p$

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Incorrect Decomposition

- Sometimes it is incorrect:

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
DoubleClick	29.99	Camera

Decompose on : Name, Category and Price, Category

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Incorrect Decomposition

Name	Category
Gizmo	Gadget
OneClick	Camera
DoubleClick	Camera

Price	Category
19.99	Gadget
24.99	Camera
29.99	Camera

When we put it back:

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
OneClick	29.99	Camera
DoubleClick	24.99	Camera
DoubleClick	29.99	Camera

Cannot recover information

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Normal Forms

First Normal Form = all attributes are atomic

Second Normal Form (2NF) = old and obsolete

Third Normal Form (3NF) = this lecture

Boyce Codd Normal Form (BCNF) = this lecture

Others...

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Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if:

Whenever there is a nontrivial dependency $A_1, ..., A_n \rightarrow B$ in R, $\{A_1, ..., A_n\}$ is a key for R

In English (though a bit vague):

Whenever a set of attributes of *R* is determining another attribute, should determine ***all*** the attributes of *R*.

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Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

What are the dependencies?
SSN → Name, City
What are the keys?
{Name, SSN, PhoneNumber}
Is it in BCNF?

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Decompose it into BCNF

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN → Name, City

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

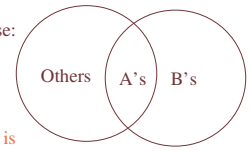
Summary of BCNF Decomposition

Find a dependency that violates the BCNF condition:

$A_1, A_2, \ldots A_n \longrightarrow B_1, B_2, \ldots B_m$

Heuristics: choose $B_1, B_2, \ldots B_m$ "as large as possible"

Decompose:



Is there a 2-attribute relation that is not in BCNF ?

Continue until there are no BCNF violations left.

Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN → name, age

age → hairColor

Decompose in BCNF (in class):

Step 1: find all keys

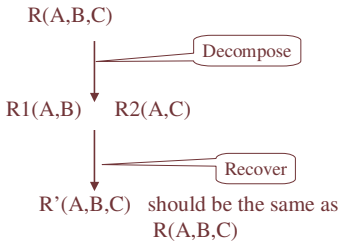
Step 2: now decompose

Other Example

- R(A,B,C,D) $A \longrightarrow B$, $B \longrightarrow C$
- Key: A, D
- Violations of BCNF: $A \rightarrow B$, $A \rightarrow C$, $A \rightarrow BC$
- Pick $A \rightarrow BC$: split into R1(A,BC) R2(A,D)
- What happens if we pick $A \rightarrow B$ first ?

Correct Decompositions

A decomposition is *lossless* if we can recover:



R' is in general larger than R. Must ensure R' = R

Correct Decompositions

- Given R(A,B,C) s.t. A→B, the decomposition into R1(A,B), R2(A,C) is lossless

3NF: A Problem with BCNF

Unit	Company	Product

FD's: Unit → Company; Company, Product → Unit
So, there is a BCNF violation, and we decompose.

Unit	Company

Unit → Company

Unit	Product

No FDs

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So What's the Problem?

Unit	Company
Galaga99	UW
Bingo	UW

Unit	Product
Galaga99	databases
Bingo	databases

No problem so far. All *local* FD's are satisfied.
Let's put all the data back into a single table again:

Unit	Company	Product
Galaga99	UW	databases
Bingo	UW	databases

Violates the dependency: company, product -> unit!

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Solution: 3rd Normal Form (3NF)

A simple condition for removing anomalies from relations:

A relation R is in 3rd normal form if :

Whenever there is a nontrivial dependency $A_1, A_2, ..., A_n \rightarrow B$ for R , then $\{A_1, A_2, ..., A_n\}$ a super-key for R,
or B is part of a key.

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