## **CSE 442** - Data Visualization **Exploratory Data Analysis**



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## Deconstructions

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



## William Playfair, 1786



X-axis: year (Q) Y-axis: currency (Q) Color: imports/exports (N, O)



http://www.smartmoney.com/marketmap/

## Wattenberg's Map of the Market



Rectangle Area: market cap (Q) Rectangle Position: market sector (N), market cap (Q) Color Hue: loss vs. gain (N, O) Color Value: magnitude of loss or gain (Q)

#### Minard 1869: Napoleon's March



#### Single-Axis Composition









#### **Mark Composition**

Y-axis: temperature (Q)

**X-axis**: longitude (Q) / time (O)



Temp over space/time (Q x Q)

#### Mark Composition

Y-axis: longitude (Q)







Army position  $(Q \times Q)$  and army size (Q)



### Minard 1869: Napoleon's March



#### Depicts at least 5 quantitative variables. Any others?

## Formalizing Design

## **Choosing Visual Encodings**

Assume k visual encodings and n data attributes. We would like to pick the "best" encoding among a combinatorial set of possibilities of size  $(n+1)^k$ 

#### **Principle of Consistency**

The properties of the image (visual variables) should match the properties of the data.

#### **Principle of Importance Ordering**

Encode the most important information in the most effective way.

### Design Criteria [Mackinlay 86]

#### Expressiveness

A set of facts is *expressible* in a visual language if the sentences (i.e. the visualizations) in the language express all the facts in the set of data, and only the facts in the data.

#### Effectiveness

A visualization is more *effective* than another visualization if the information conveyed by one visualization is more readily perceived than the information in the other visualization.

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## Can not express the facts

A multivariate relation may be *inexpressive* in a single horizontal dot plot because multiple records are mapped to the same position.

•••	•••	•••				•••••			•••••						•• ••	•
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
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		0	10	20	30	40	50	 60	 70	 80		
		Value										

#### Expresses facts not in the data



Fig. 11. Incorrect use of a bar chart for the *Nation* relation. The lengths of the bars suggest an ordering on the vertical axis, as if the USA cars were longer or better than the other cars, which is not true for the *Nation* relation.

## A length is interpreted as a quantitative value.

## Design Criteria [Mackinlay 86]

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## **Design Criteria** [Mackinlay 86]

#### Expressiveness

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#### Design Criteria Translated

**Tell the truth and nothing but the truth** (don't lie, and don't lie by omission)

Use encodings that people decode better (where better = faster and/or more accurate)

## Mackinlay's Ranking



Conjectured effectiveness of encodings by data type

#### Mackinlay's Design Algorithm

APT - "A Presentation Tool", 1986

User formally specifies data model and type Input: ordered list of data variables to show

**APT searches over design space** Test expressiveness of each visual encoding Generate encodings that pass test Rank by perceptual effectiveness criteria

Output the "most effective" visualization

APT

Automatically generate chart for car data

Input variables:1. Price2. Mileage3. Repair4. Weight



#### Limitations of APT?

## Limitations of APT

**Does not cover many visualization techniques** Networks, hierarchies, maps, diagrams Also: 3D structure, animation, illustration, ...

**Does not consider interaction** 

Does not consider semantics / conventions

Assumes single visualization as output

## Summary: Data & Image Models

#### **Formal specification**

Data model: relational data; N,O,Q types Image model: visual encoding channels Encodings map data to visual variables

**Choose expressive and effective encodings** Rule-based tests of expressiveness Perceptual effectiveness rankings

**Question**: how do we establish effectiveness criteria? *Subject of perception lectures*...

## What was the **first** data visualization?



~6200 вс Town Map of Catal Hyük, Konya Plain, Turkey

0 BC



 $\bigcirc$ 

~950 AD Position of Sun, Moon and Planets



Sunspots over time, Scheiner 1626



Longitudinal distance between Toledo and Rome, van Langren 1644



The Rate of Water Evaporation, Lambert 1765



The Rate of Water Evaporation, Lambert 1765

# The **Golden Age** of Data Visualization

1786 1900

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



The Commercial and Political Atlas, William Playfair 1786
Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



Statistical Breviary, William Playfair 1801

 $\mathbf{O}$ 



1786 1826(?) Illiteracy in France, Pierre Charles Dupin



1786

1856 "Coxcomb" of Crimean War Deaths, Florence Nightingale



1864 British Coal Exports, Charles Minard

1786

### Consommations approximatives de la Houille dans la Grande Bretagne de 1850 à 1864.

Les abscisses représentent les années et les ordonnées les quantités annuelles de houille consommée. Les couleurs indiquent les espèces de consommations. Les longueurs d'ordonnées comprises dans une couleur sont les quantités de houille consommées à raison de deux millimètres pour un million de tonnes.



ourg

Données admisés pour former le Tableau ci-contre. Consommations. \_\_\_\_ Sources des Renseignements. Exportations .\_ Mineral statistics 1865 page 214 et Renseignements Parlementaires. District de Londres. \_\_\_\_\_ id. \_\_\_\_\_ \_ page 213 Produits de la Fonte. \_\_\_\_\_ id \_\_\_\_\_ page 215 et pour les années avant 1855 calculée à raison de 3.º de houille pour 1.º de fonte, en admettant les quantilés annuelles de fonte du Coal question page 192. Production du fer \_ Mineral statistics \_ page 215 et pour les années asont 1855\_ calculée à raison de 31.35 de houille pour 1 tonne de fonte convertie en fer, et admettant 20 es de la fonte produite convertis en fer Foyers domestiques : \_\_\_ En y comprenant les petites manufactures. On l'estimait en 1848 à 19 millions de tonnes, (A) qu'on peut réduire à 18 millions to. pour les foyers seuls, mais qu'on peut porter à 20 millions pour la population de 1864. Eclairage au Gaz. \_\_ Consommation estimée généralement du 3º au 8º de la production totale.

Exploitation des Chemins de Fer. \_ En supposant pour consommation totale 10 <sup>e</sup> par Kilomètre parcouru par les trains d'après les renseignements parlementaires.

Navigntion à vapeur. \_ Calculée à raison de 5<sup>\*</sup> houille par cheval vapeur et par heure, le nombre de chevaux étant celui du Steam Vessels pour 1864, et les steamens étant supposés marcher la moitié de l'aunée;

Avant 1864 j'ai supposé les consommations proportionnelles aux tonnages annuels des steamers du statistical abstract et du Board of trade.

(A) Voir l'excellent article houille de M." Lamé Fleury, Dictionnaire du Commerce Page III.



1884 Rail Passengers and Freight from Paris

1786

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1890 Statistical Atlas of the Eleventh U.S. Census

1786

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 $\mathbf{O}$ 

# The Rise of Statistics

Rise of **formal methods** in statistics and social science – Fisher, Pearson, ...

Little innovation in graphical methods

A period of **application and popularization** Graphical methods enter textbooks, curricula, and **mainstream use** 



Data Analysis & Statistics, Tukey 1962

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Four major influences act on data analysis today: 1. The formal theories of statistics. 2. Accelerating developments in computers and display devices. 3. The challenge, in many fields, of more and larger bodies of data. 4. The emphasis on quantification in a wider variety of disciplines.



The last few decades have seen the rise of formal theories of statistics, "legitimizing" variation by confining it by assumption to random sampling, often assumed to involve tightly specified distributions, and restoring the appearance of security by emphasizing narrowly optimized techniques and claiming to make statements with "known" probabilities of error.

While some of the influences of statistical theory on data analysis have been helpful, others have not.

**Exposure**, the effective laying open of the data to display the unanticipated, is to us a major portion of data analysis. Formal statistics has given almost no guidance to exposure; indeed, it is not clear how the informality and flexibility appropriate to the exploratory character of exposure can be fitted into any of the structures of formal statistics so far proposed.

Nothing - not the careful logic of mathematics, not statistical models and theories, not the awesome arithmetic power of modern computers - nothing can substitute here for the **flexibility of the informed human mind**.

Accordingly, both approaches and techniques need to be structured so as to facilitate human involvement and intervention.

Y
-
6.58
5.76
7.71
8.84
8.47
7.04
5.25
12.5
5.56
7.91
6.89

Summai	y Statistics
$u_{X} = 9.0$	$\sigma_{\chi} = 3.317$
$u_{y} = 7.5$	$\sigma_{\rm Y} = 2.03$

**Linear Regression** Y = 3 + 0.5 X  $R^2 = 0.67$ 

[Anscombe 1973]

Set A

Set B



Set C







## **Exploratory Data Analysis**

Data Wrangling Exploratory Analysis Examples Polaris / Tableau

# Data Wrangling

I spend more than half of my time integrating, cleansing and transforming data without doing any actual analysis. Most of the time I'm lucky if I get to do any "analysis" at all.

> Anonymous Data Scientist [Kandel et al. '12]







# In Data Science, 80% of time spent prepare data, 20% of time spent complain about need for prepare data.

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Bureau of Justice Statistics – Data Online http://bjs.ojp.usdoj.gov/							
Reported crime in Alabama							
Year 2004 2005 2006 2007 2008	Population 4525375 4029.3 4548327 3900 4599030 3937 4627851 3974.9 4661900 4081.9	Property crime rate 987 2732.4 309.9 955.8 2656 289 968.9 2645.1 322.9 980.2 2687 307.7 1080.7 2712.6 288.6	Burglary rate	Larceny-theft rate	Motor vehicle theft rate		
Reported crime in Alaska							
Year 2004 2005 2006 2007 2008	Population 657755 3370.9 663253 3615 670053 3582 683478 3373.9 686293 2928.3	Property crime rate 573.6 2456.7 340.6 622.8 2601 391 615.2 2588.5 378.3 538.9 2480 355.1 470.9 2219.9 237.5	Burglary rate	Larceny-theft rate	Motor vehicle theft rate		
Reported crime in Arizona							
Year 2004 2005 2006 2007 2008	Population 5739879 5073.3 5953007 4827 6166318 4741.6 6338755 4502.6 6500180 4087.3	Property crime rate 991 3118.7 963.5 946.2 2958 922 953 2874.1 914.4 935.4 2780.5 786.7 894.2 2605.3 587.8	Burglary rate	Larceny-theft rate	Motor vehicle theft rate		
Reported crime in Arkansas							
Year 2004 2005 2006 2007 2008	Population 2750000 4033.1 2775708 4068 2810872 4021.6 2834797 3945.5 2855390 3843.7	Property crime rate 1096.4 2699.7 237 1085.1 2720 262 1154.4 2596.7 270.4 1124.4 2574.6 246.5 1182.7 2433.4 227.6	Burglary rate	Larceny-theft rate	Motor vehicle theft rate		
Reported crime in California							
Year 2004 2005 2006 2007 2008	Population 35842038 36154147 36457549 36553215 36756666	Property crime rate 3423.9 686.1 2033.1 3321 692.9 1915 3175.2 676.9 1831.5 3032.6 648.4 1784.1 2940.3 646.8 1769.8	Burglary rate 704.8 712 666.8 600.2 523.8	Larceny-theft rate	Motor vehicle theft rate		
Reported crime in Colorado							
Year 2004	Population 4601821 3918.5	Property crime rate 717.3 2679.5 521.6	Burglary rate	Larceny-theft rate	Motor vehicle theft rate		

## **DataWrangler**

Suggestions	rows: 408 prev next	
	# Year	🛊 🎁 Property_crime_rate 🔶
Delete array 0.10	1 Reported crime in Alabama	
Delete rows 8,10	2	
Delete empty rows	3 2004	4029.3
	4 2005	3900
Delete rows where Property_crime_rate	5 2006	3937
is null	6 2007	3974.9
	7 2008	4081.9
Delete rows where Year is null	8	
Context Events	9 Reported crime in Alaska	
Script Export	10	
Split data repeatedly on newline into	11 2004	3370.9
rows	12 2005	3615
Split data repeatedly on '.'	13 2006	3582
	14 2007	3373.9

### Wrangler: Interactive Visual Specification of Data Transformation Scripts

Sean Kandel et al. CHI'11

## Demo: https://vimeo.com/19185801

## Data Wrangling

One often needs to manipulate data prior to analysis. Tasks include reformatting, cleaning, quality assessment, and integration.

Approaches include: Manual manipulation in spreadsheets Custom code (e.g., dplyr in R, Pandas in Python) Trifacta Wrangler <u>http://www.trifacta.com/products/wrangler/</u> Open Refine <u>http://openrefine.org/</u>

## Data Quality

"The first sign that a visualization is good is that it shows you a problem in your data...

...every successful visualization that I've been involved with has had this stage where you realize, "Oh my God, this data is not what I thought it would be!" So already, you've discovered something."

## Martin Wattenberg





☐ Images
✓ Animate

#### 000

#### Graph Viewer

#### Roll-up by:

All

Visualization:

Matrix

Sort by:

Linkage

Edge centrality filters:





Graph Viewer

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## Θ C **Graph Viewer** Graph Viewer х. Roll-up by: + All Visualization: ÷ Matrix Sort by: + None Edge centrality filters:

# Visualize Friends by School?

Berkeley Cornell Harvard Harvard University Stanford Stanford University UC Berkeley UC Davis University of California at Berkeley University of California, Berkeley University of California, Davis

# Data Quality Hurdles

Missing Data Erroneous Values Type Conversion Entity Resolution Data Integration

no measurements, redacted, ...? misspelling, outliers, ...? e.g., zip code to lat-lon diff. values for the same thing? effort/errors when combining data

*LESSON*: Anticipate problems with your data. Many research problems around these issues!

# Analysis Example: Motion Pictures Data

## **Motion Pictures Data**

Title IMDB Rating Rotten Tomatoes Rating MPAA Rating Release Date String (N) Number (Q) Number (Q) String (O) Date (T) IMDB Rating (bin)




Rotten Tomatoes Rating (bin)









### Lesson: Exercise Skepticism

Check data quality and your assumptions.

Start with **univariate summaries**, then start to consider **relationships among variables**.

Avoid premature fixation!

# Analysis Example: Antibiotic Effectiveness

### Data Set: Antibiotic Effectiveness

Genus of BacteriaString (N)Species of BacteriaString (N)Antibiotic AppliedString (N)Gram-Staining?Pos / Neg (N)Min. Inhibitory Concent. (g)Number (Q)

Collected prior to 1951.

## What questions might we ask?

Table 1: Burtin's data.				
Bacteria	Penicillin	Streptomycin	Neomycin	Gram Staining
Aerobacter aerogenes	870	1	1.6	negative
Brucella abortus	1	2	0.02	negative
Brucella anthracis	0.001	0.01	0.007	positive
Diplococcus pneumoniae	0.005	11	10	positive
Escherichia <i>coli</i>	100	0.4	0.1	negative
Klebsiella pneumoniae	850	1.2	1	negative
Mycobacterium tuberculosis	800	5	2	negative
Proteus vulgaris	3	0.1	0.1	negative
Pseudomonas aeruginosa	850	2	0.4	negative
Salmonella (Eberthella) typhosa	1	0.4	0.008	negative
Salmonella schottmuelleri	10	0.8	0.09	negative
Staphylococcus albus	0.007	0.1	0.001	positive
Staphylococcus aureus	0.03	0.03	0.001	positive
Streptococcus <i>fecalis</i>	1	1	0.1	positive
Streptococcus hemolyticus	0.001	14	10	positive
Streptococcus viridans	0.005	10	40	positive



Bacteria	Penicillin	Antibiotic Streptomycin	Neomycin	Gram stain
Aerobacter aerogenes	870	1	1.6	_
Brucella abortus	1	2	0.02	-
Bacillus anthracis	0.001	0.01	0.007	+
Diplococcus pneumoniae	0.005	11	10	+
Escherichia coli	100	0.4	0.1	-
Klebsiella pneumoniae	850	1.2	1	-
Mycobacterium tuberculosis	800	5	2	-
Proteus vulgaris	3	0.1	0.1	-
Pseudomonas aeruginosa	850	2	0.4	-
Salmonella (Eberthella) typhosa	1	0.4	0.008	-
Salmonella schottmuelleri	10	0.8	0.09	-
Staphylococcus albus	0.007	0.1	0.001	+
Staphylococcus aureus	0.03	0.03	0.001	+
Streptococcus fecalis	1	1	0.1	+
Streptococcus hemolyticus	0.001	14	10	+
Streptococcus viridans	0.005	10	40	+

Original graphic by Will Burtin, 1951



Bacteria	Penicillin	Antibiotic Streptomycin	Neomycin	Gram stain
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Brucella abortus	1	2	0.02	-
Bacillus anthracis	0.001	0.01	0.007	+
Diplococcus pneumoniae	0.005	11	10	+
Escherichia coli	100	0.4	0.1	-
Klebsiella pneumoniae	850	1.2	1	-
Mycobacterium tuberculosis	800	5	2	-
Proteus vulgaris	3	0.1	0.1	-
Pseudomonas aeruginosa	850	2	0.4	-
Salmonella (Eberthella) typhosa	1	0.4	0.008	-
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Staphylococcus aureus	0.03	0.03	0.001	+
Streptococcus fecalis	1	1	0.1	+
Streptococcus hemolyticus	0.001	14	10	+
Streptococcus viridans	0.005	10	40	+

Radius: 1 / log(MIC) Bar Color: Antibiotic Background Color: Gram Staining



Mike Bostock Stanford CS448B, Winter 2009





X-axis: Antibiotic | log(MIC) Y-axis: Gram-Staining | Species Color: Most-Effective?



Stanford CS448B, Fall 2009



0.0001

MIC

(ug/uL)

Neomycin

Streptomycin

darker colors: more effective

S. viridans

Penicillin









#### Not a streptococcus! (realized ~30 yrs later)



Not a streptococcus! (realized ~30 yrs later) Really a streptococcus!

(realized ~20 yrs later)

Do the bacteria group by resistance? Do different drugs correlate?



Do the bacteria group by resistance? Do different drugs correlate?

### Lesson: Iterative Exploration

#### **Exploratory Process**

Construct graphics to address questions
 Inspect "answer" and assess new questions
 Repeat...

Transform data appropriately (e.g., invert, log)

Show data variation, not design variation [Tufte]

# Administrivia

## Updates

A1 due tonight. We'll be discussing the submission in class on Thursday

A2 will be released on Thursday. This will require you to conduct an exploratory analysis and present your findings visually.

Lectures recorded starting with this one \o/

# Tableau / Polaris

### **Dataset Schema**

Year (Qi) Candidate Code (N) Candidate Name (N) Incumbent / Challenger / Open-Seat (N) Party Code (N) [1=Dem,2=Rep,3=Other] Party Name (N) Total Receipts (Qr) State (N) District (N)

This is a subset of the larger data set available from the FEC.

# Hypotheses?

#### What might we learn from this data?

# Hypotheses?

What might we learn from this data? Correlation between receipts and winners? Do receipts increase over time? Which states spend the most? Which party spends the most? Margin of victory vs. amount spent? Amount spent between competitors?

# Tableau Demo





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36 marks 12 rows by 3 columns SUM(Sales): \$2,297,201

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# Quantity	Furniture					
# Sales Description						
Longitude (generated)						
# Number of Records						
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Abc Customer Name		Region	Segment	Techno	ology	Office Su	pplies	Furni	iture
Order		Central	Consumer						
► A Location	Marks		Corporate						
▼ 品 Product	All II		Home Office						
Abc Category	Automatic A	East	Consumer						
Abc Sub-Category			Corporate						
Manufacturer	😓 🕐 Abs	0 11	Home Office						
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Init Profit (bin)		Corporate							
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incusure manes		West	Consumer						
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# Discount			Home Onice						
# Profit	SUM(Sales)		:	\$0 \$100,000 Sales	\$0 \$20,000 \$0 Profit	0 \$100,000 Sales	\$0 \$20,000 Profit	\$0 \$100,000 Sales	\$0 \$20,000 Profit
# Quantity	SUM(Profit)			Galos	1 Ion	Galos	Tion	Galos	1 Ion
# Sales	Category								
Latitude (generated)	Technology								
Longitude (generated)	Office Supplies								
# Number of Records	Furniture								
# Measure Values									
Data Source Sheet 1	to 🖽 to								

72 marks 12 rows by 6 columns SUM(Profit): \$286,397



### A Detective Story

You have accounting records for two firms that are in dispute. One is lying. *How to tell?* 

Firm A		Firm B	LIARS!
283.08	25.23	283.08	75.23
153.86	385.62	353.86	185.25
1448.97	12371.32	5322.79	9971.42
18595.91	1280.76	8795.64	4802.43
21.33	257.64	61.33	57.64
Amt. Paid:	\$34823.72	Amt. Rec'd:	\$29908.67
## Benford's Law (Benford 1938, Newcomb 1881)

The *logarithms* of the values (not the values themselves) are uniformly randomly distributed.



Hence the leading digit **1** has a ~30% likelihood. Larger digits are increasingly less likely.

## Benford's Law (Benford 1938, Newcomb 1881)

The *logarithms* of the values (not the values themselves) are uniformly randomly distributed. Holds for many (but certainly not all) real-life data sets: Addresses, Bank accounts, Building heights, ... Data must span multiple orders of magnitude. Evidence that records do not follow Benford's Law is admissible in a court of law!