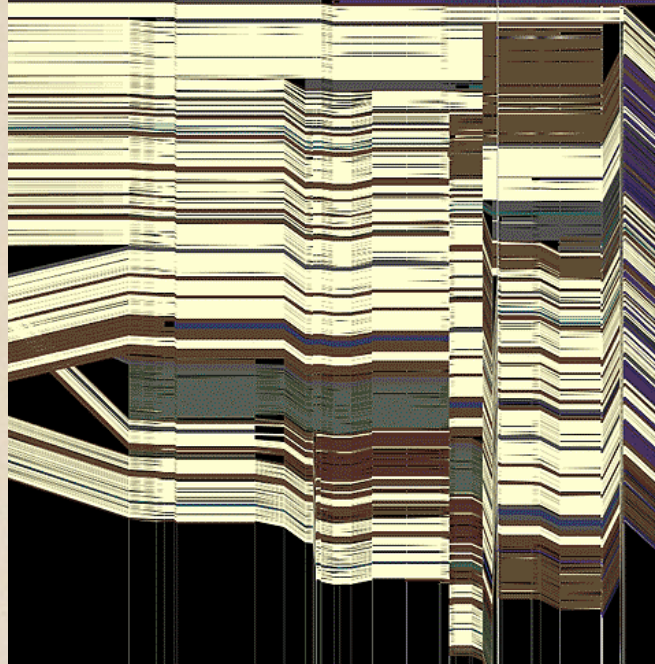
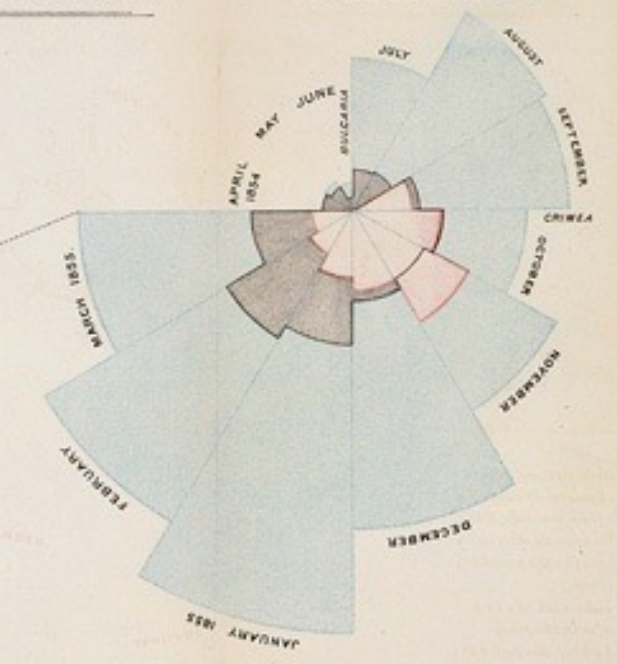


CSE 442 - Data Visualization

# Using Space Effectively



Jeffrey Heer University of Washington

# Effectiveness Rankings [Mackinlay 86]

## QUANTITATIVE

Position  
Length  
Angle  
Slope  
Area (Size)  
Volume  
Density (Value)  
Color Sat  
Color Hue  
Texture  
Connection  
Containment  
Shape

## ORDINAL

Position  
Density (Value)  
Color Sat  
Color Hue  
Texture  
Connection  
Containment  
Length  
Angle  
Slope  
Area (Size)  
Volume  
Shape

## NOMINAL

Position  
Color Hue  
Texture  
Connection  
Containment  
Density (Value)  
Color Sat  
Shape  
Length  
Angle  
Slope  
Area  
Volume

# Effectiveness Rankings [Mackinlay 86]

## QUANTITATIVE

### **Position** .....

Length  
Angle  
Slope  
Area (Size)  
Volume  
Density (Value)  
Color Sat  
Color Hue  
Texture  
Connection  
Containment  
Shape

## ORDINAL

### **Position** .....

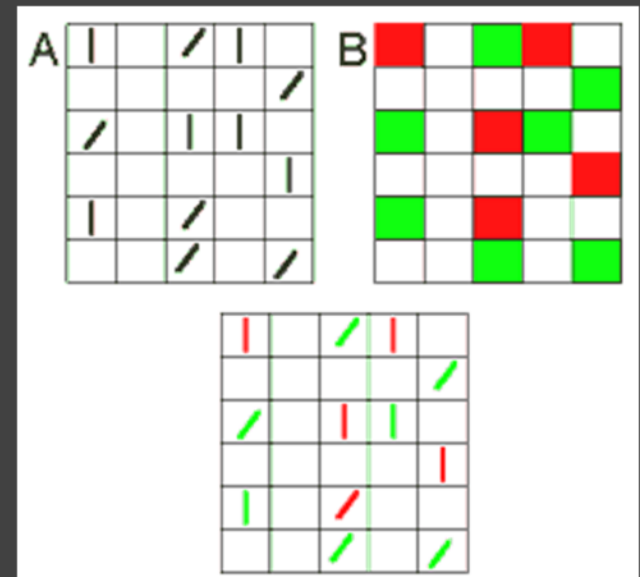
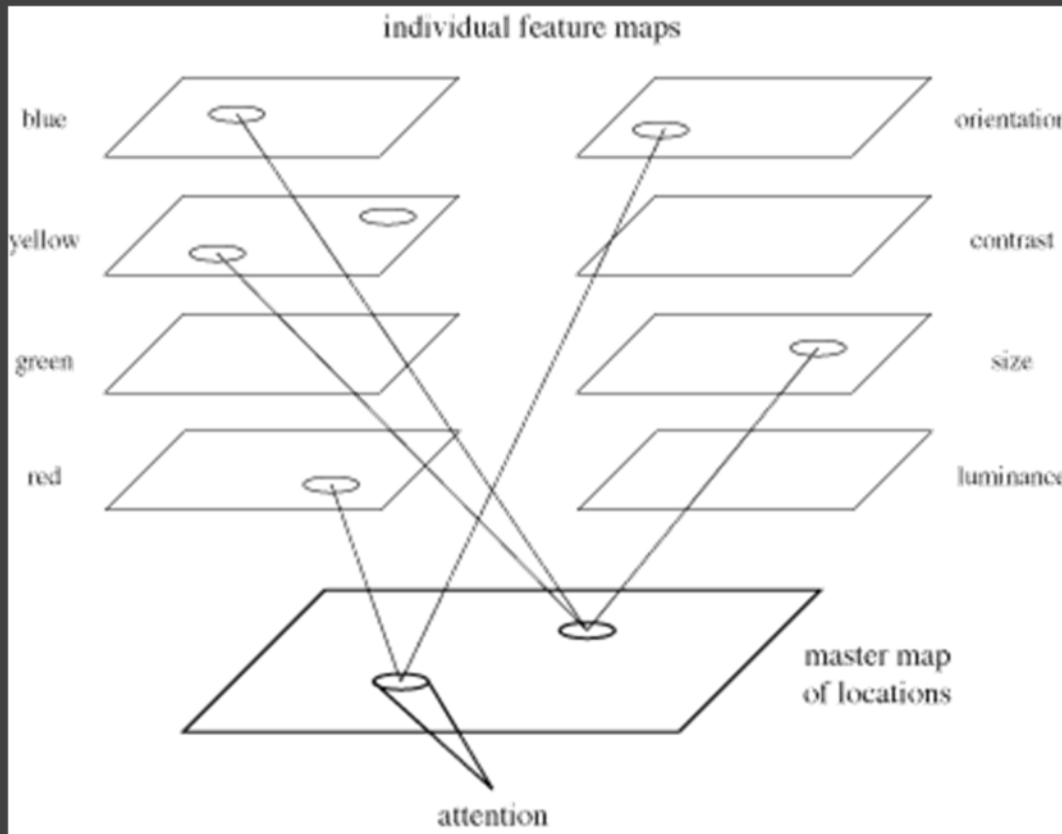
Density (Value)  
Color Sat  
Color Hue  
Texture  
Connection  
Containment  
Length  
Angle  
Slope  
Area (Size)  
Volume  
Shape

## NOMINAL

### **Position**

Color Hue  
Texture  
Connection  
Containment  
Density (Value)  
Color Sat  
Shape  
Length  
Angle  
Slope  
Area  
Volume

# Feature Integration Theory



Feature maps for orientation & color [Green]

Treisman's feature integration model [Healey 04]

**Indexed by *Position!***



Space  $(x, y)$  is the most important encoding channel.

**But are you in the *right* space?**

# Topics

Scales & Axes

Data Space, Model Space

Optimizing Chart Design

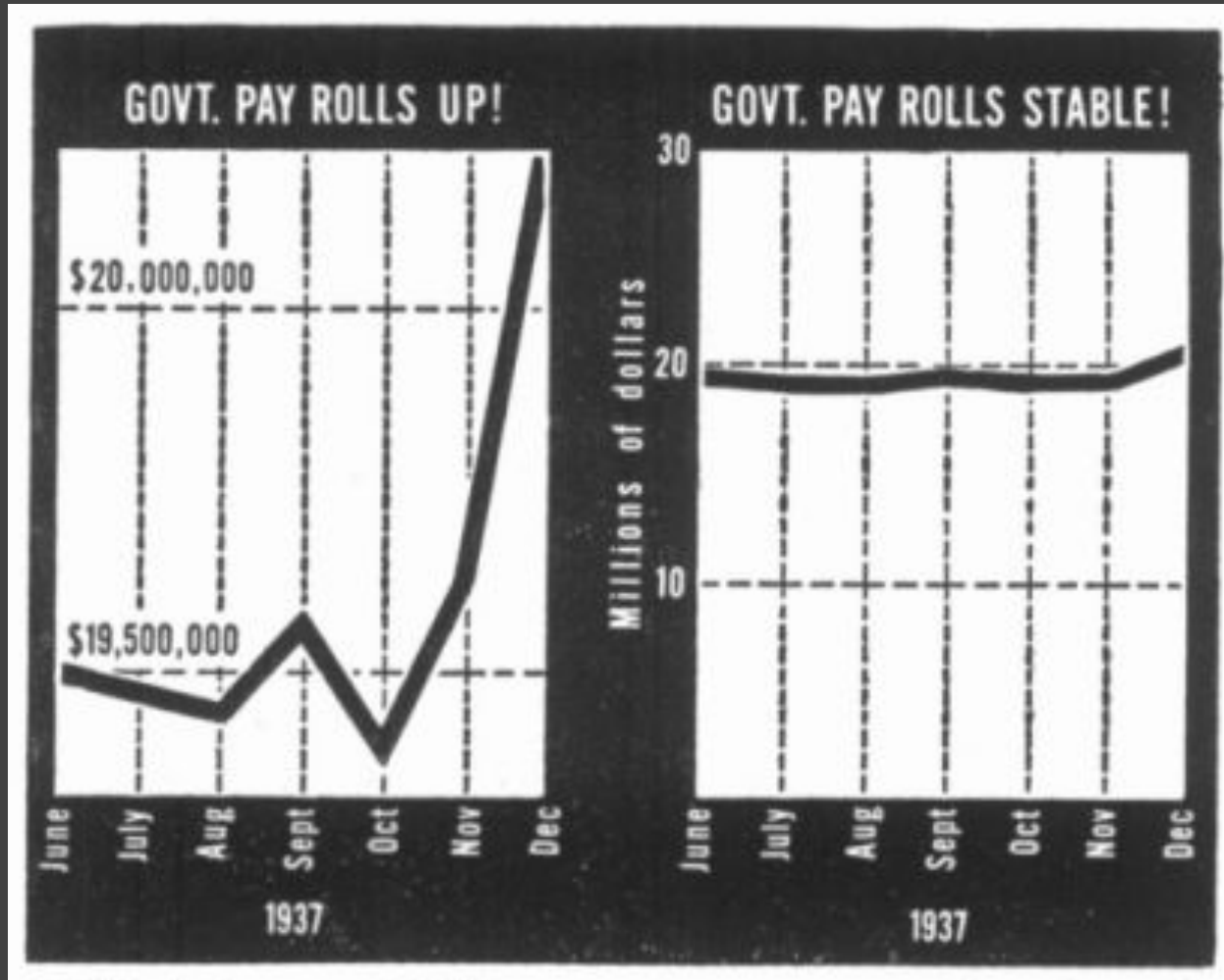
Zooming

Focus + Context

Dimensionality Reduction

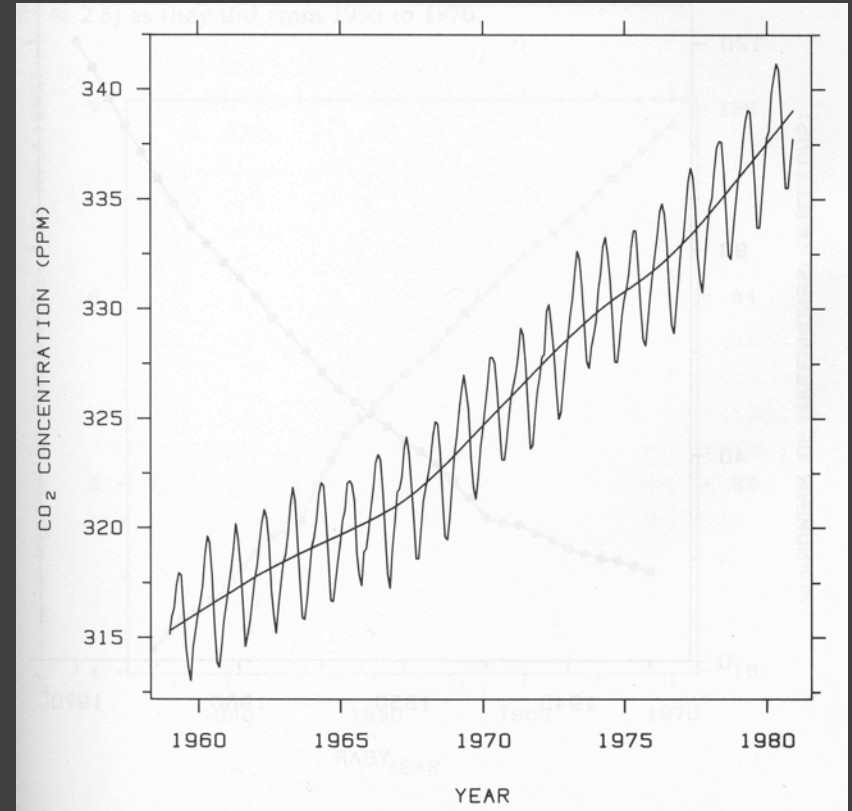
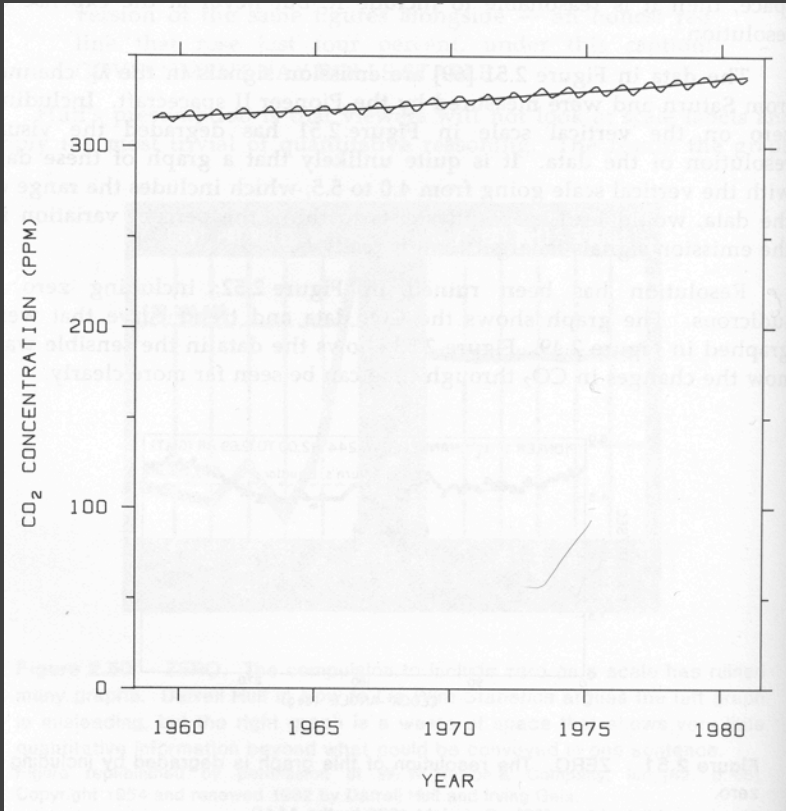
# Scales & Axes

# Include Zero in Axis Scale?



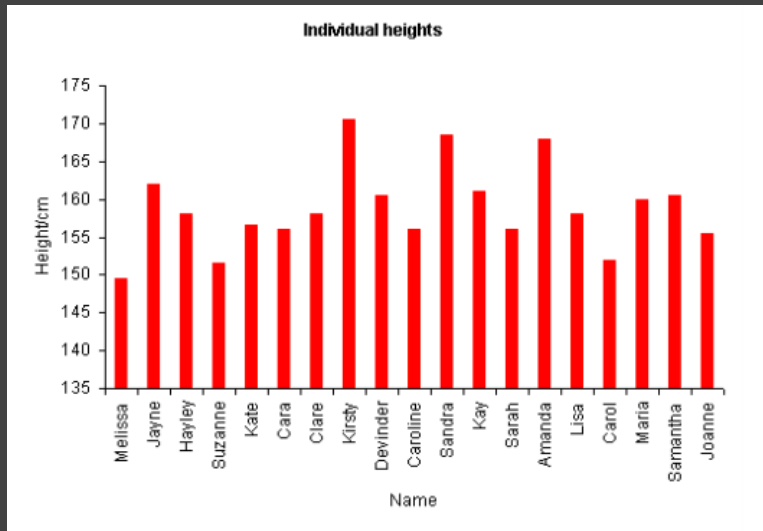
Government payrolls in 1937 [How To Lie With Statistics. Huff]

# Include Zero in Axis Scale?



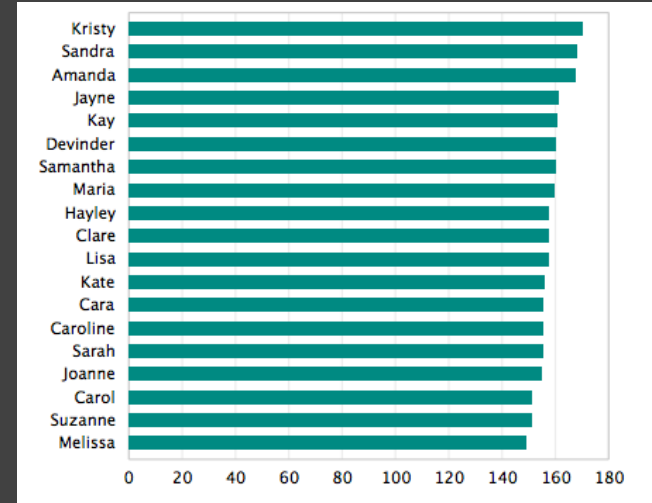
Yearly CO<sub>2</sub> concentrations [Cleveland 85]

# Include Zero in Axis Scale?

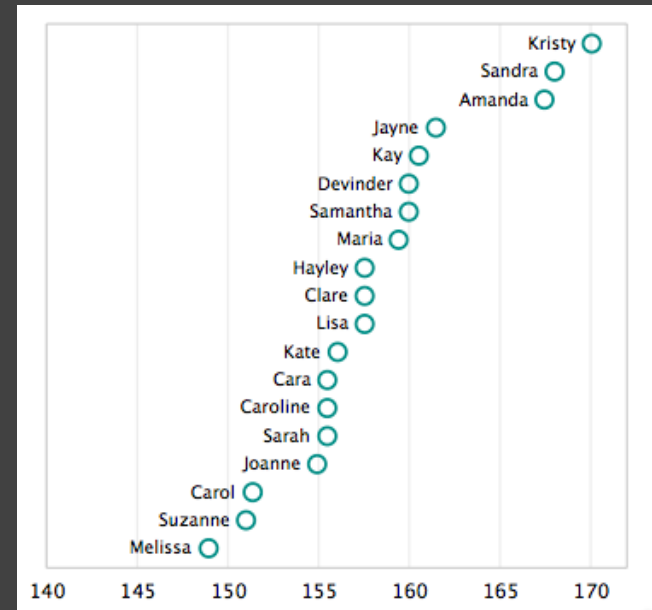


Violates Expressiveness Principle!

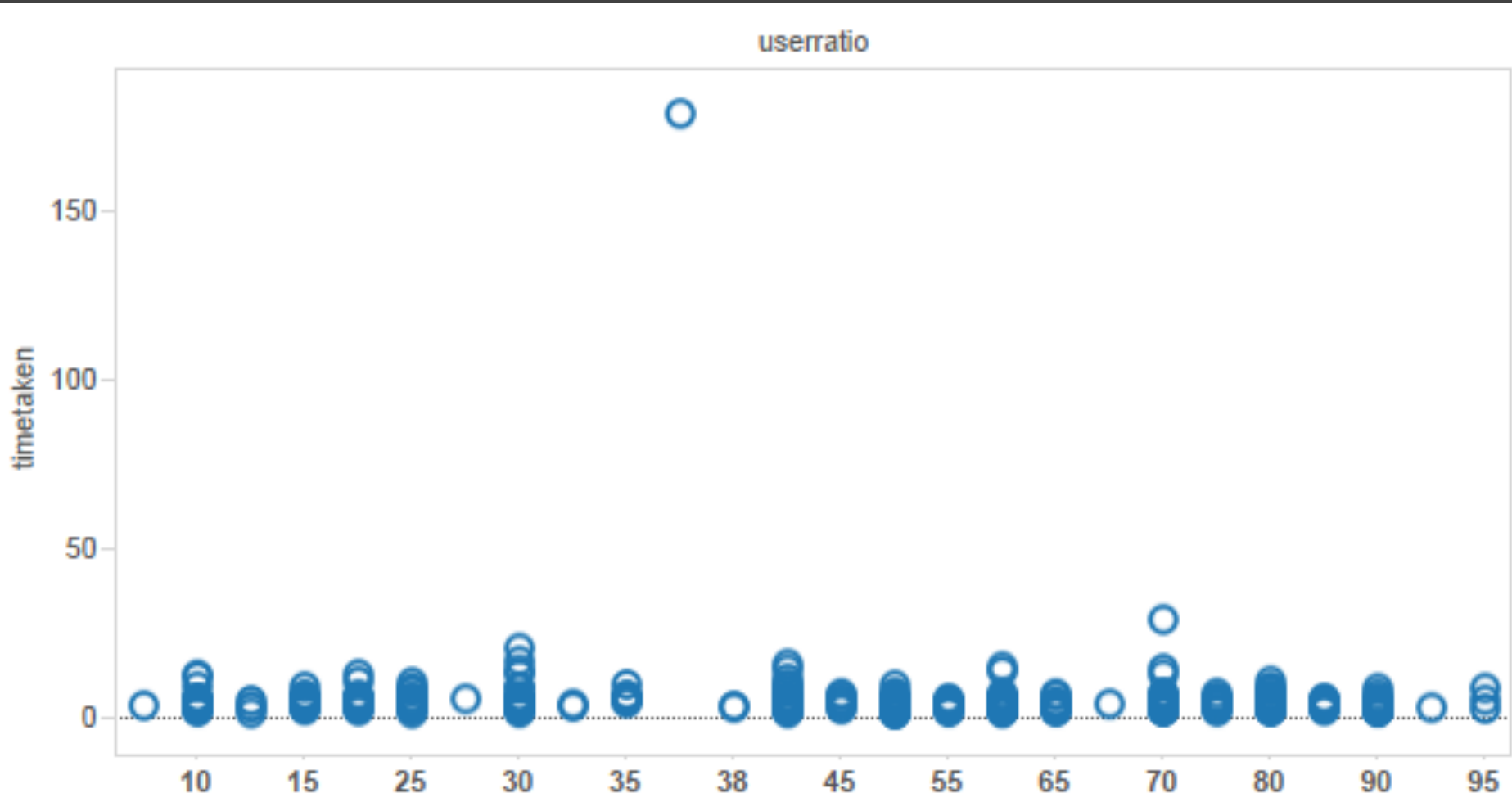
Compare Proportions (Q-Ratio)



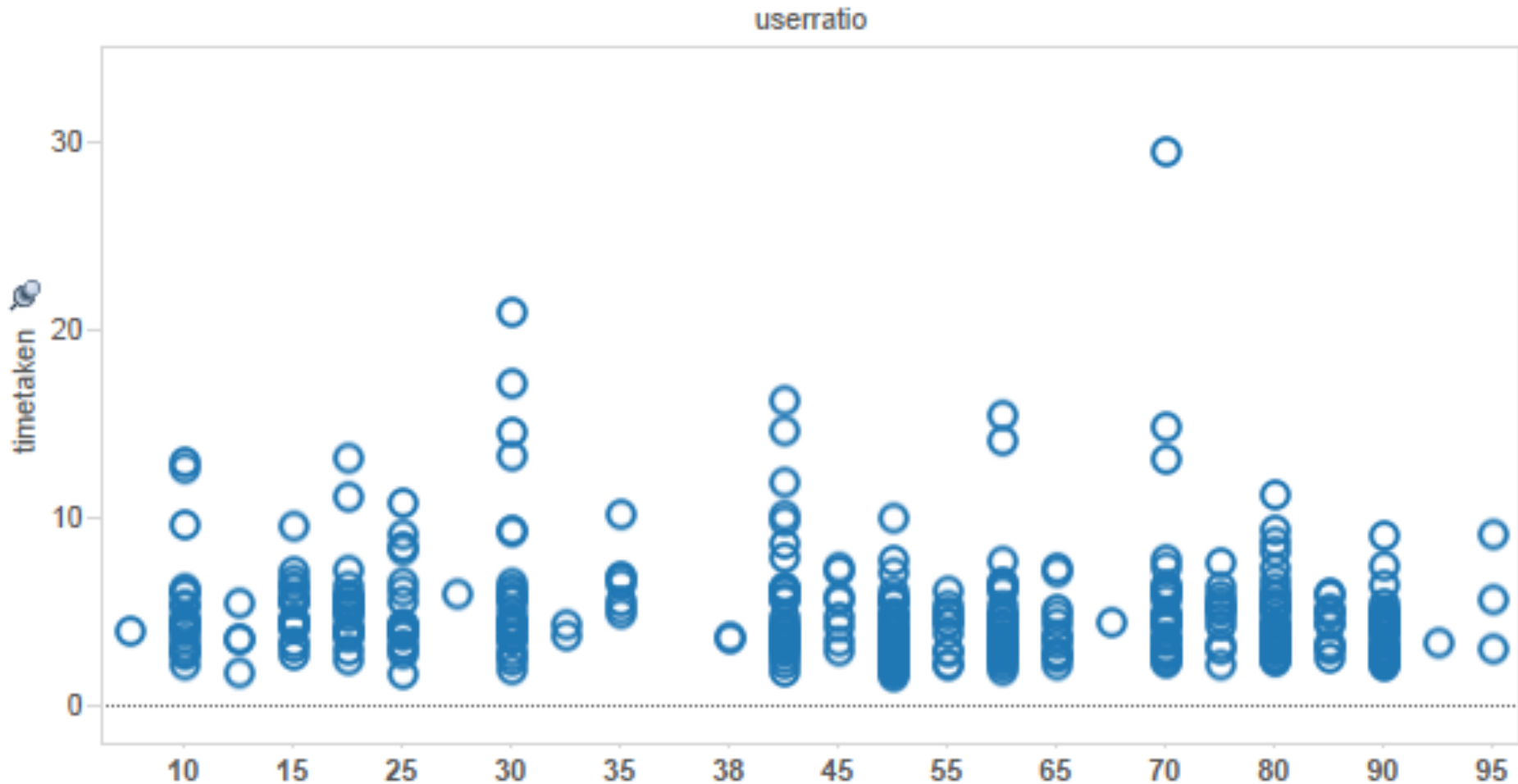
Compare Relative Position (Q-Interval)



# How to Scale the Axis?



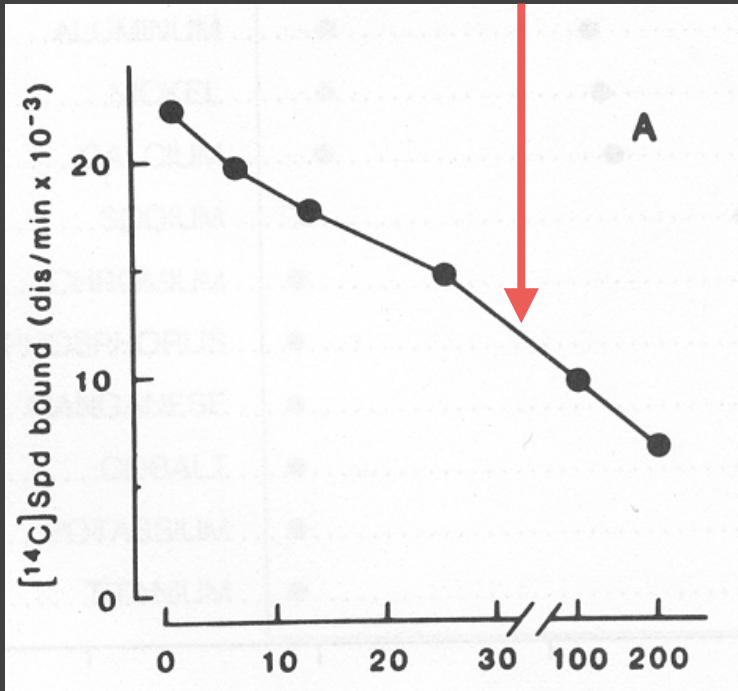
# One Option: Clip Outliers



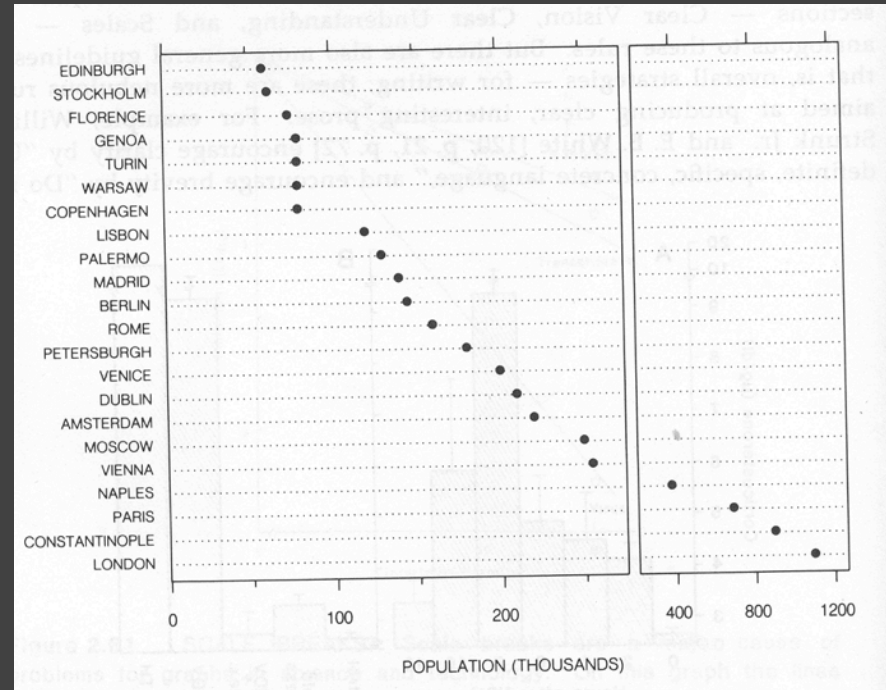


# Clearly Mark Scale Breaks

Violates Expressiveness Principle!

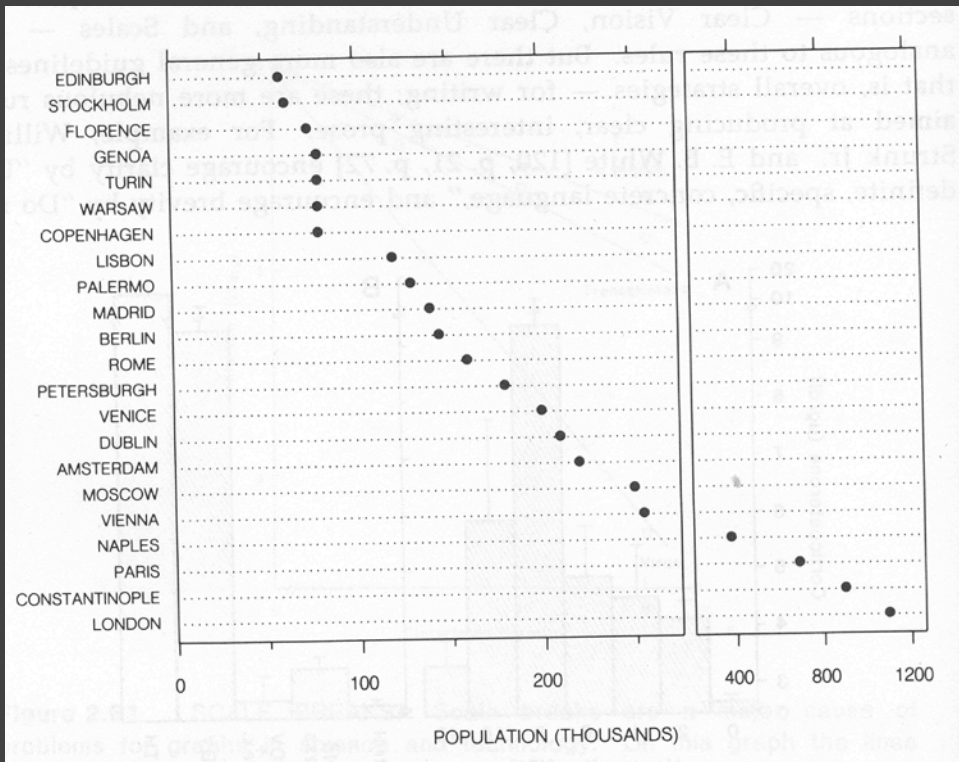


Poor scale break [Cleveland 85]

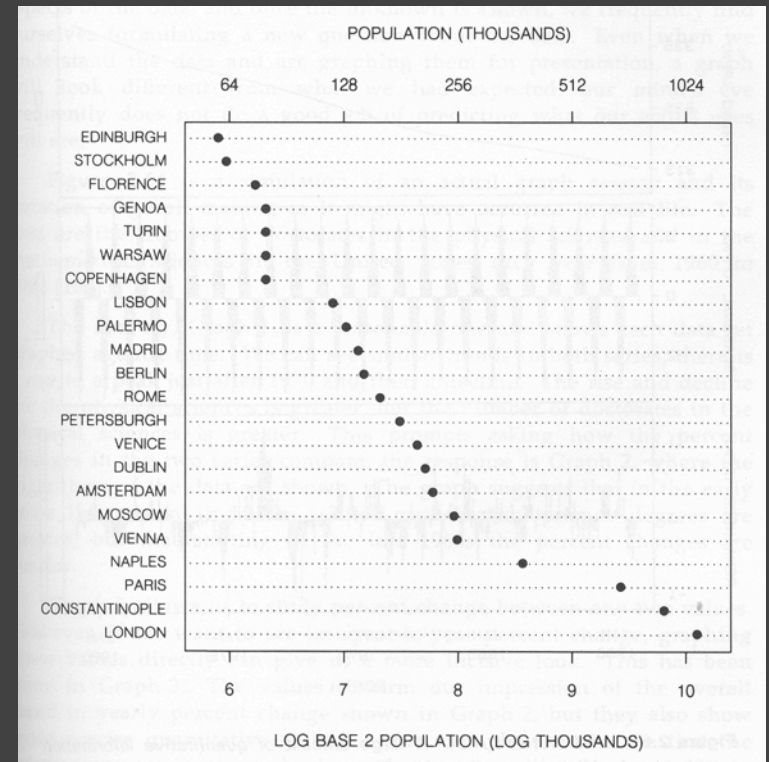


Well-marked scale break [Cleveland 85]

# Scale Break vs. Log Scale

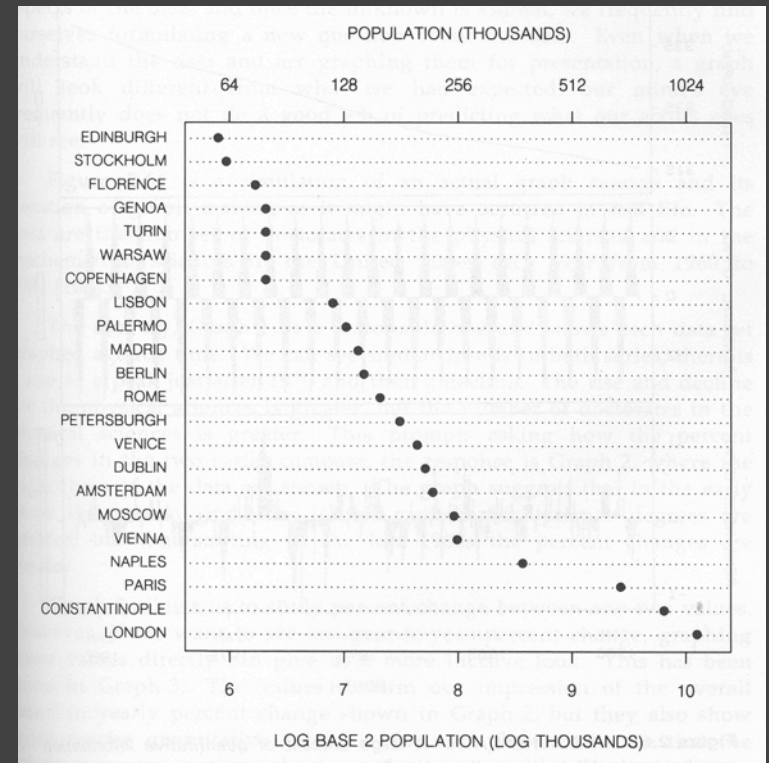
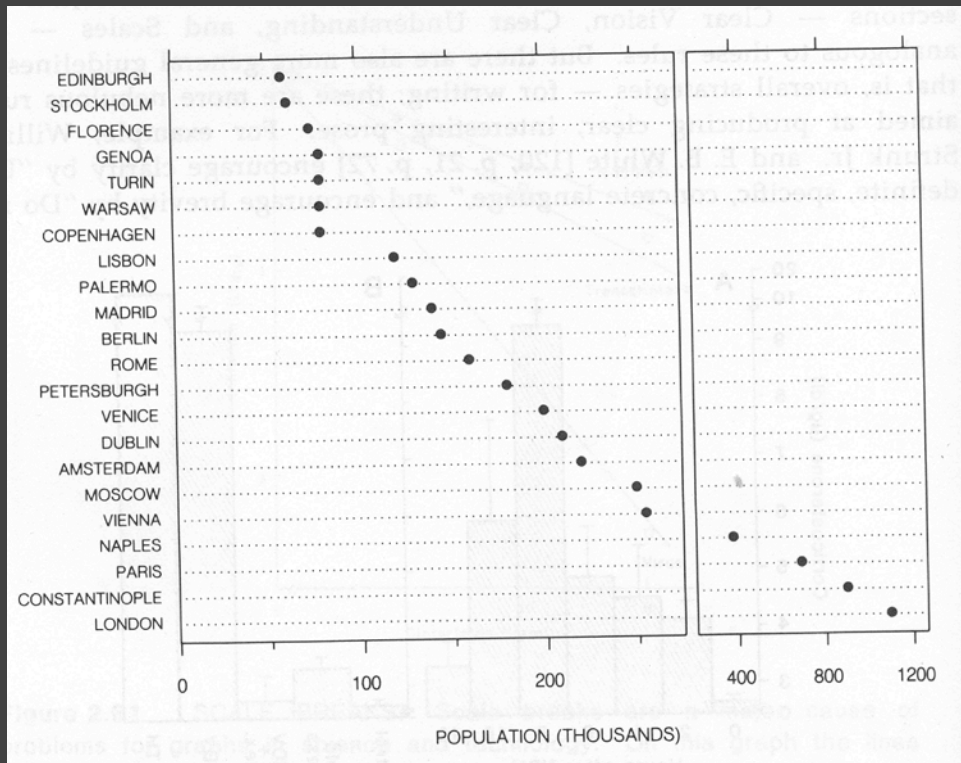


Scale Break



Log Scale

# Scale Break vs. Log Scale



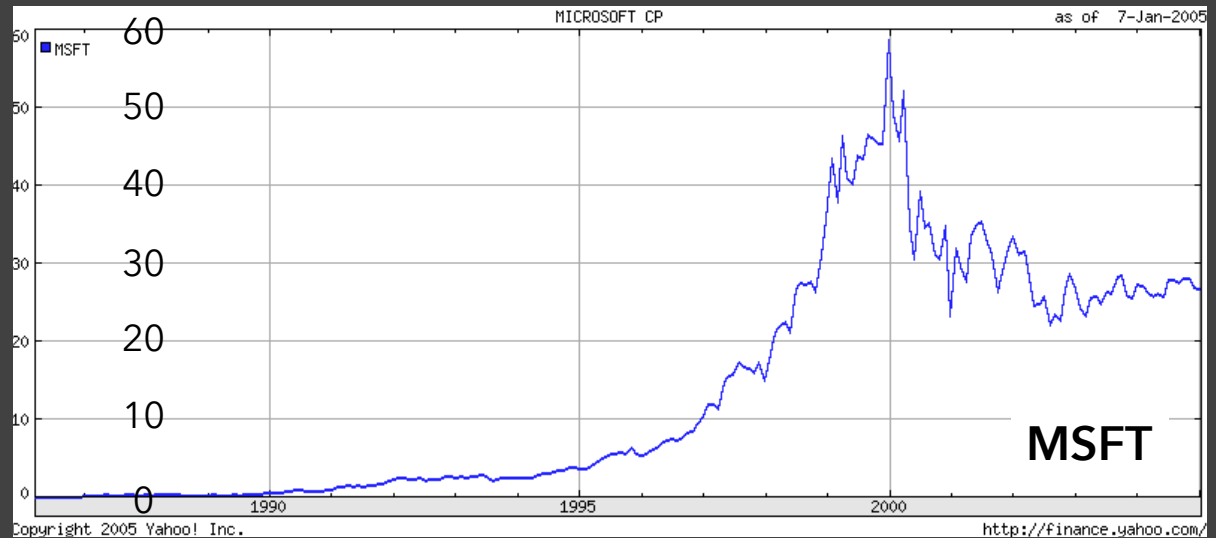
**Both increase visual resolution**

Scale break: difficult to compare (*cognitive* – not *perceptual* – work)

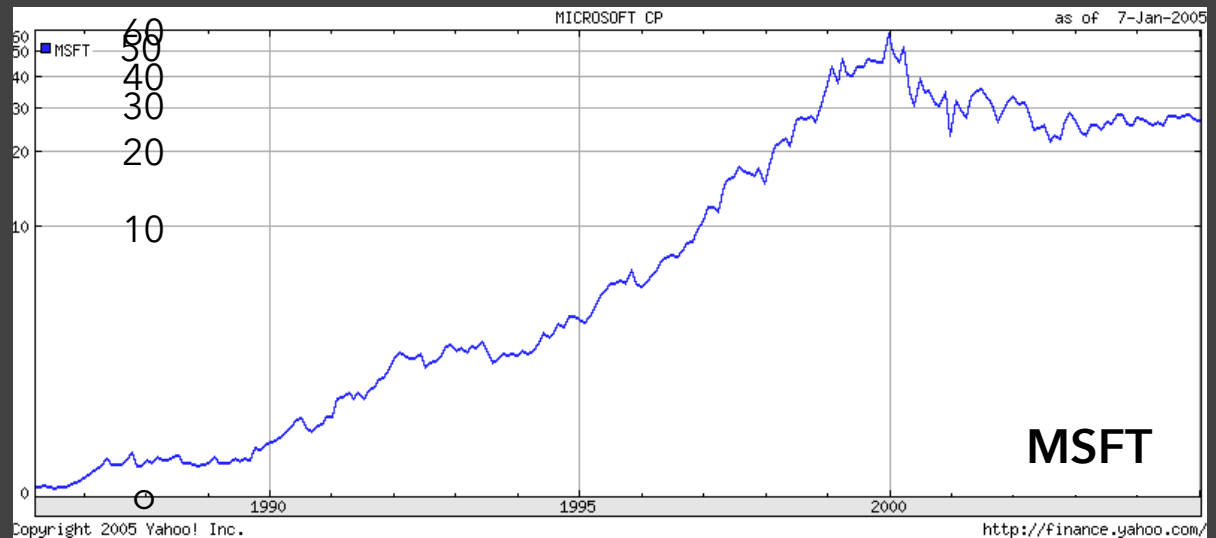
Log scale: direct comparison of all data

# Linear Scale vs. Log Scale

Linear Scale



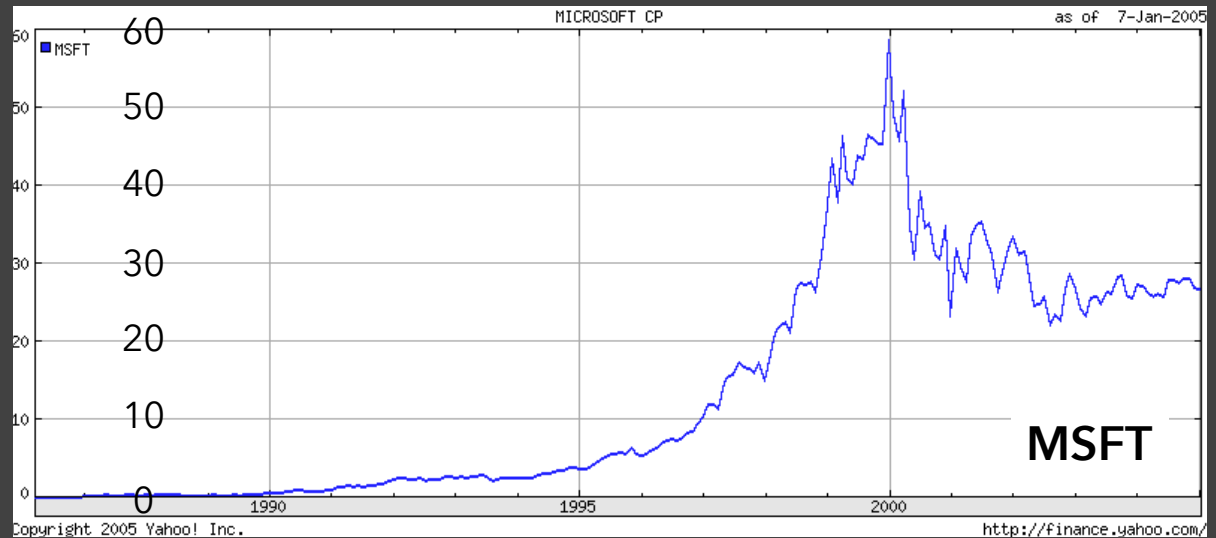
Log Scale



# Linear Scale vs. Log Scale

## Linear Scale

Absolute change

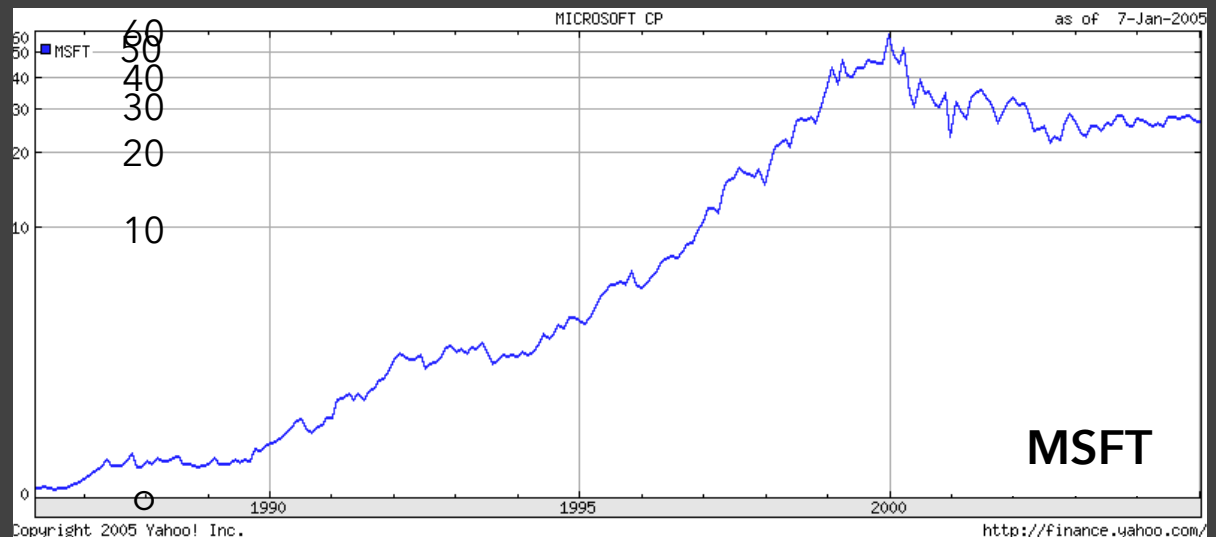


## Log Scale

Small fluctuations

Percent change

$$d(10,20) = d(30,60)$$



# When To Apply a Log Scale?

**Address data skew** (e.g., dominant outliers)

Enables comparison within and across multiple orders of magnitude.

**Focus on multiplicative factors** (not additive)

Recall that the logarithm transforms  $\times$  to  $+$ !

Percentage change, not absolute value.

Constraint: **positive, non-zero values**

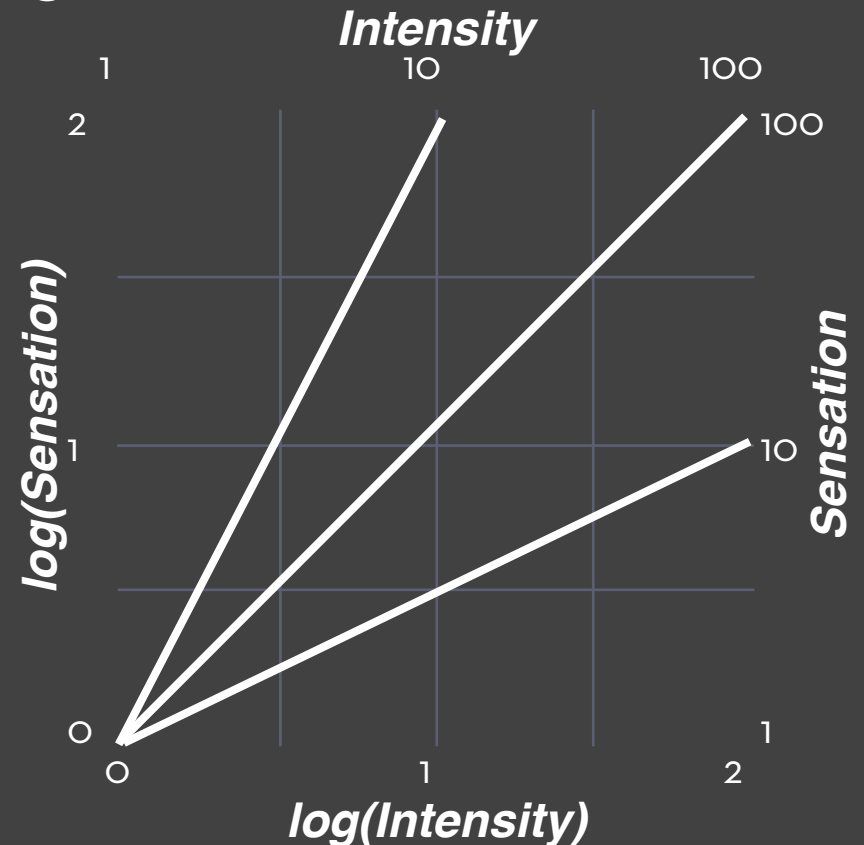
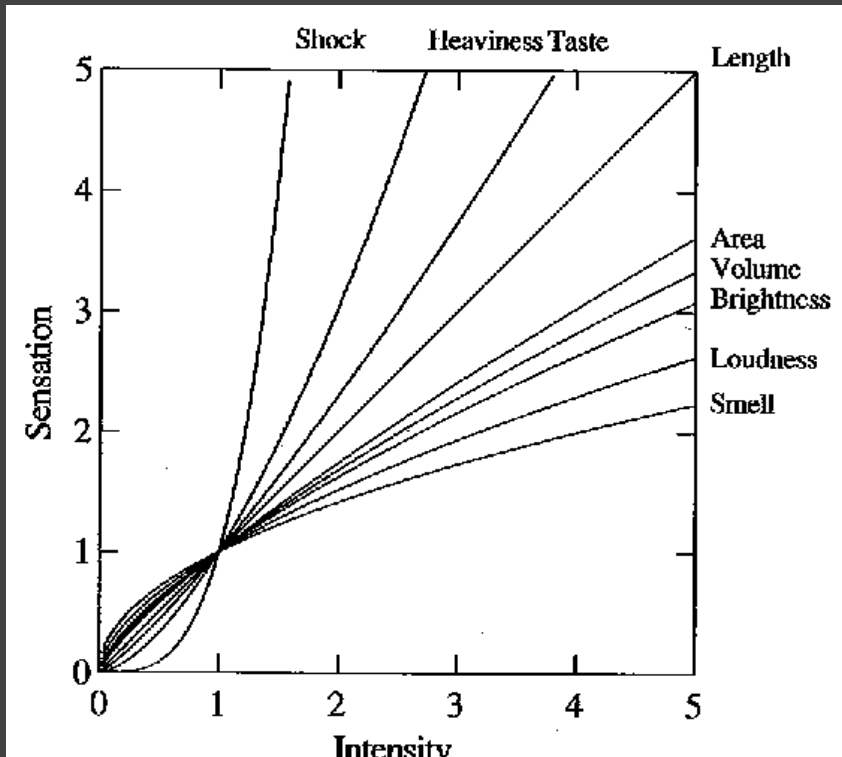
Constraint: **audience familiarity?**

# Log-Log Plot

Power functions ( $y = kx^a$ ) transform into lines

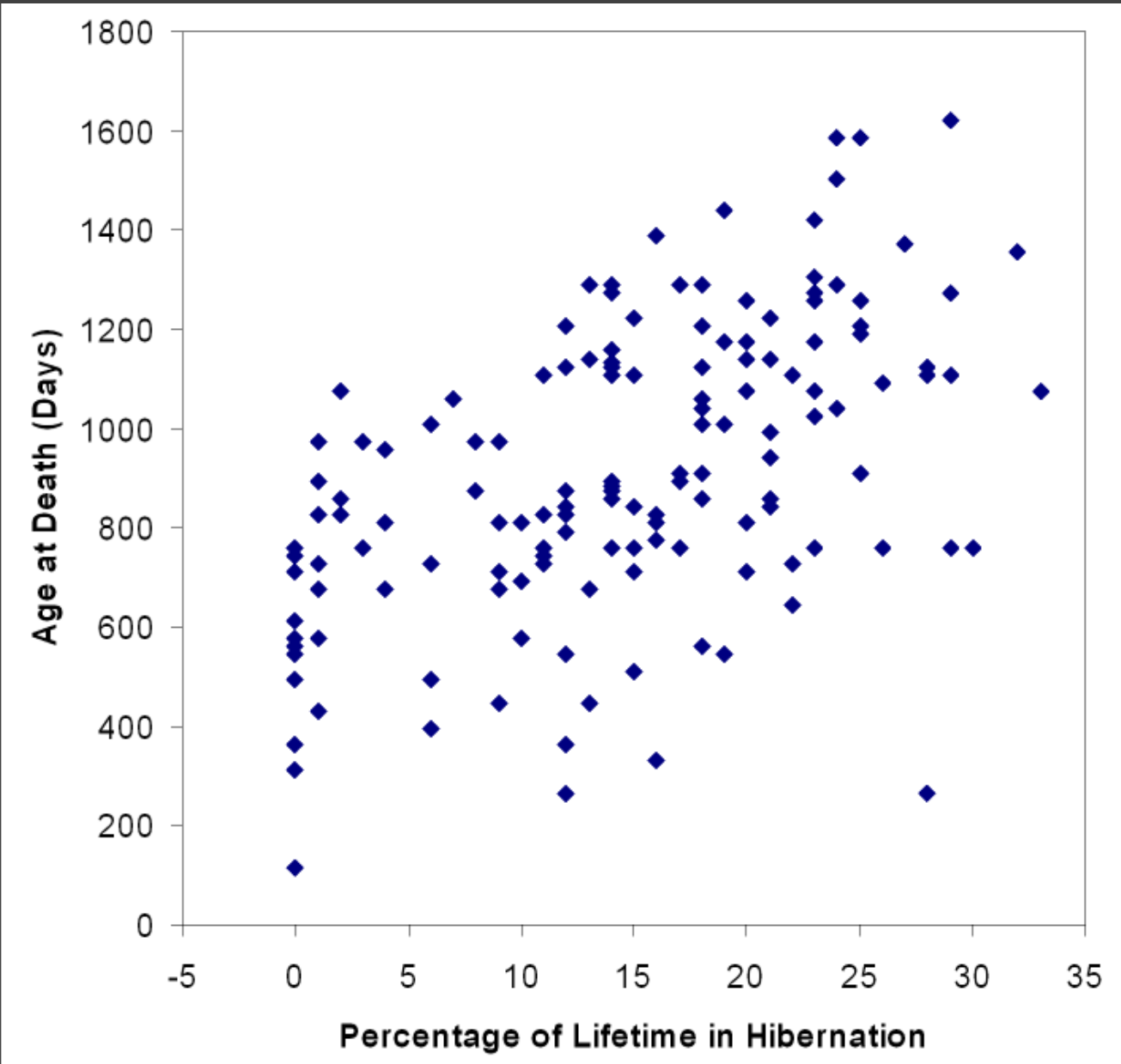
Example - Steven's Power Law:

$$S = kI^p \rightarrow \log S = \log k + p \log I$$

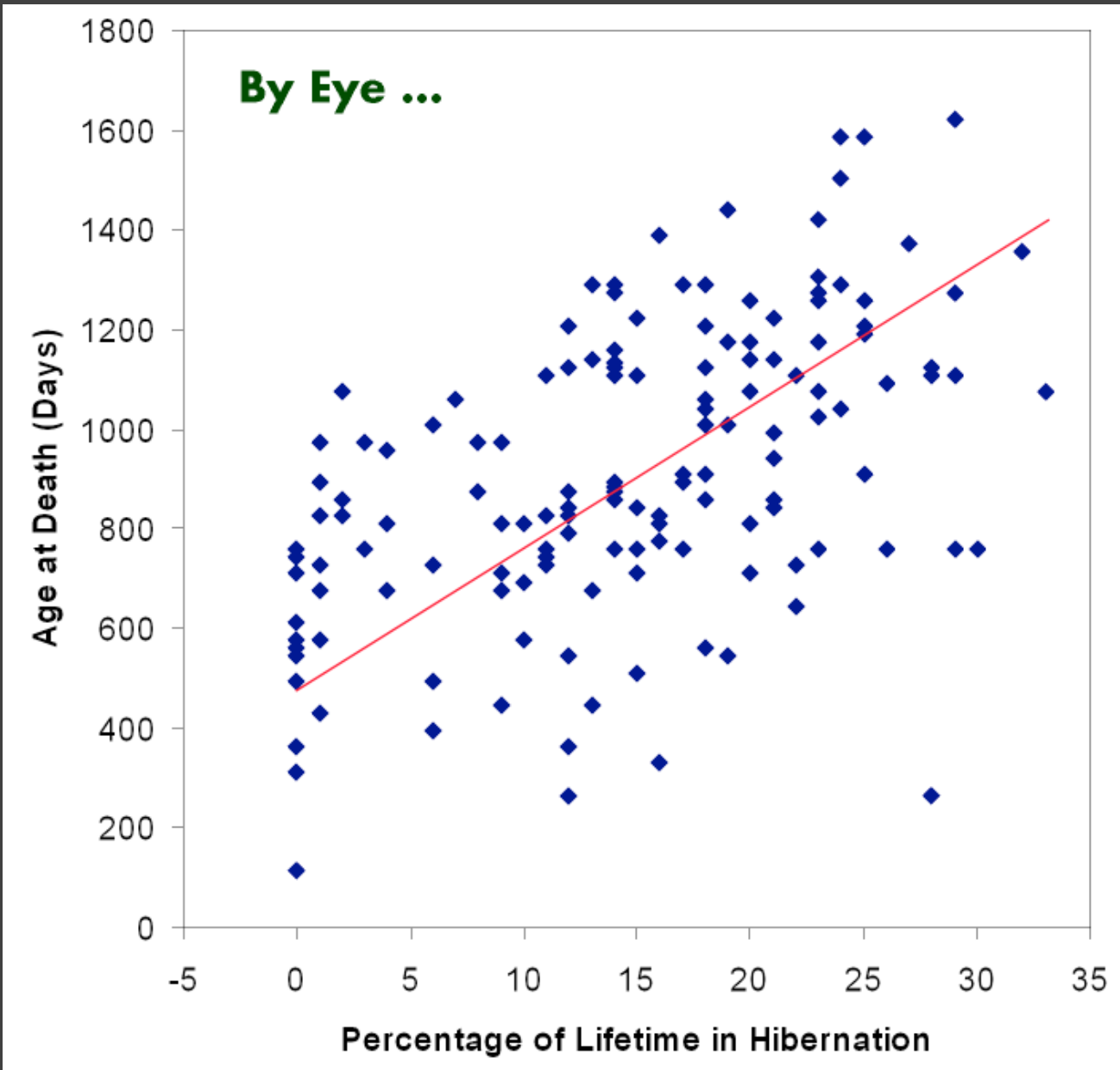


# Data Space, Model Space

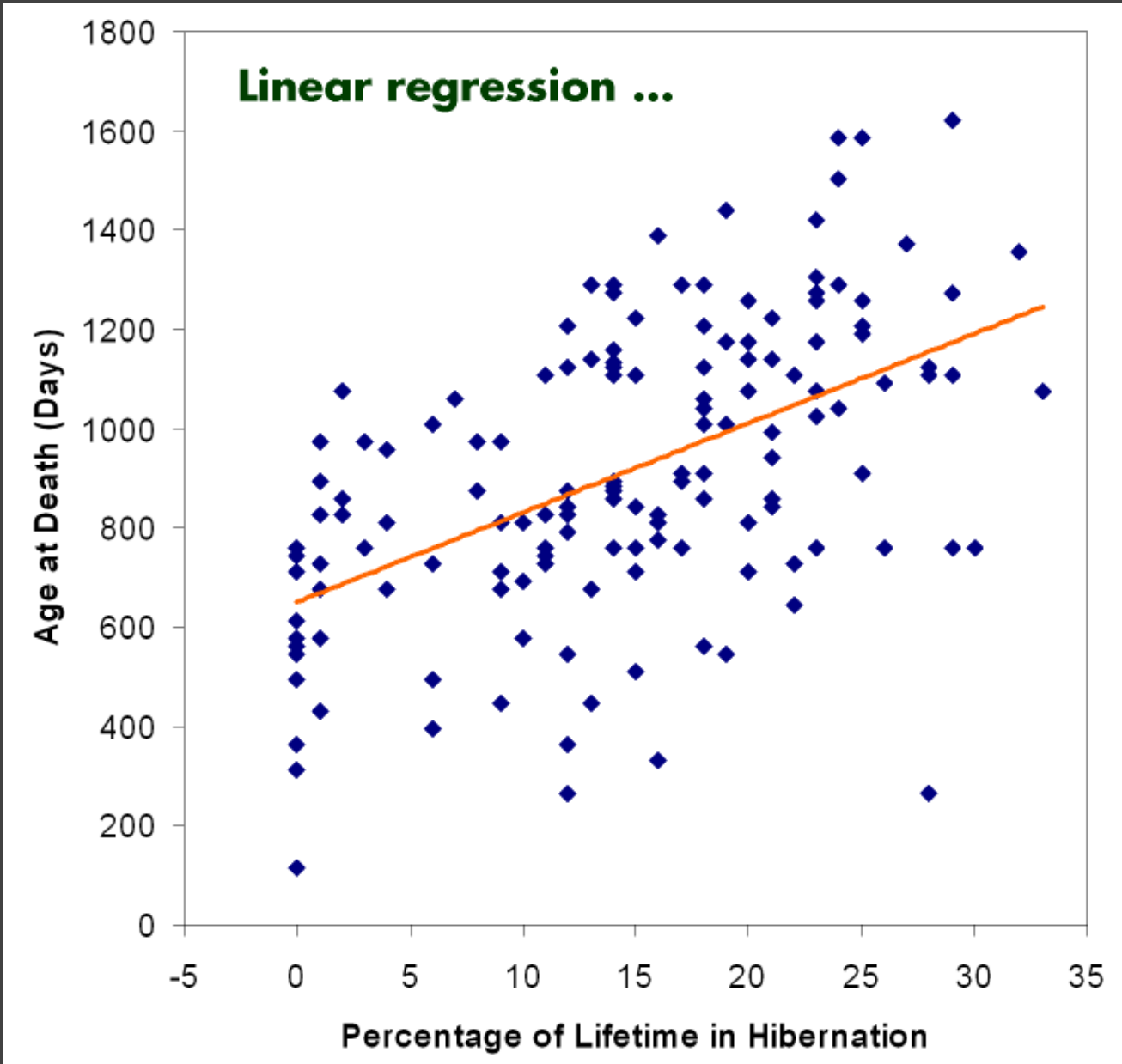




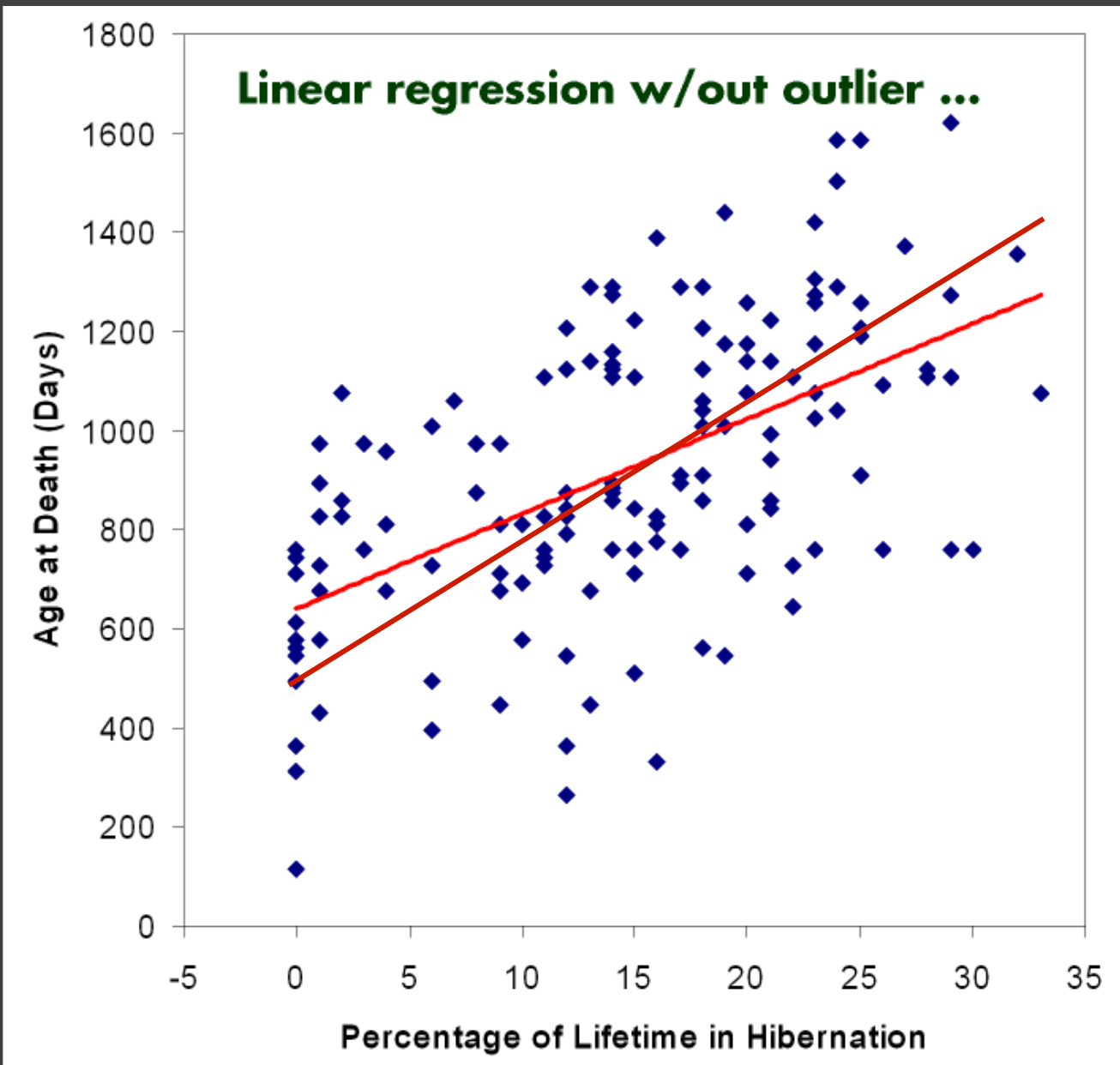
[The Elements of Graphing Data. Cleveland 94]



[The Elements of Graphing Data. Cleveland 94]



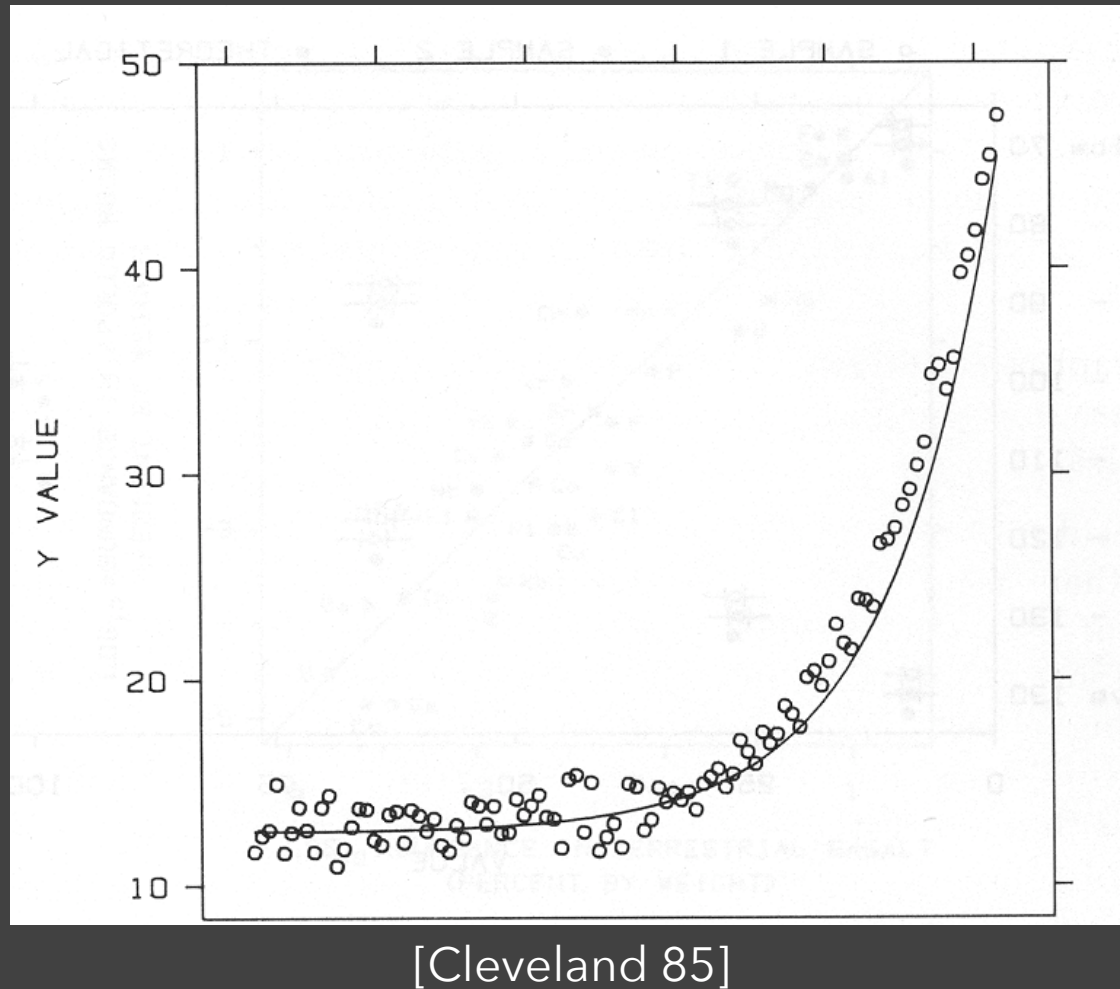
[The Elements of Graphing Data. Cleveland 94]



[The Elements of Graphing Data. Cleveland 94]

# Transforming Data

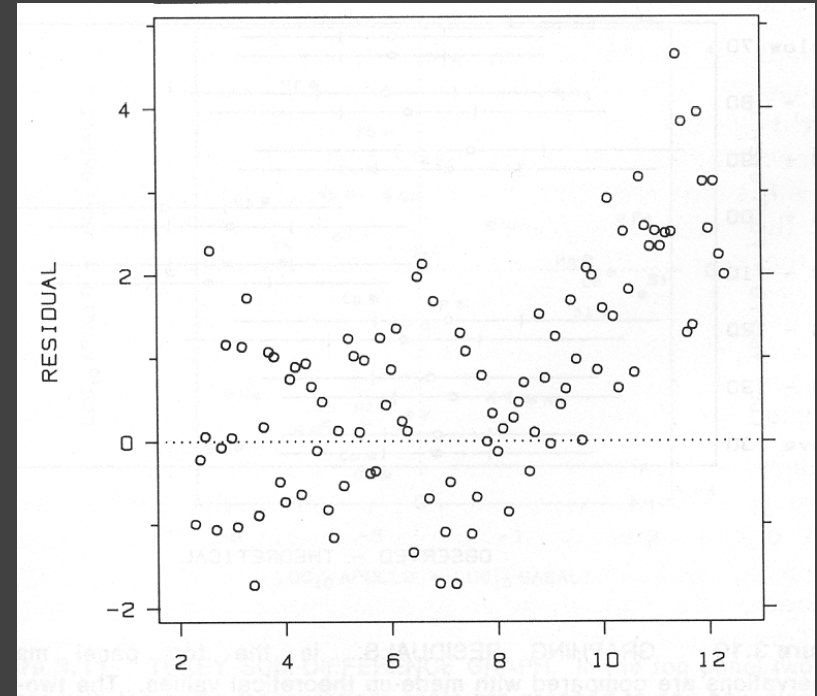
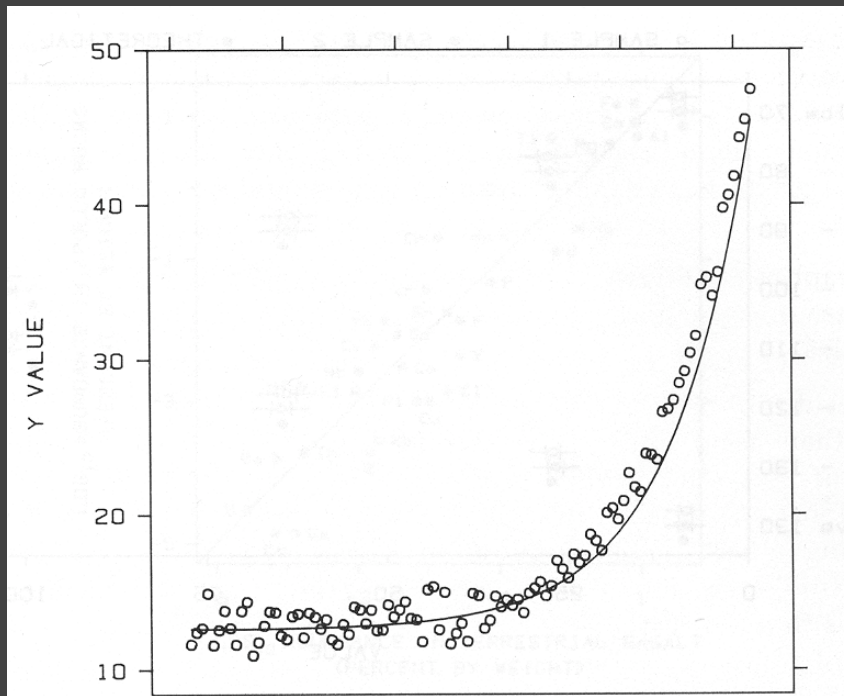
How well does the curve fit the data?



# Plot the Residuals

Plot vertical distance from best fit curve

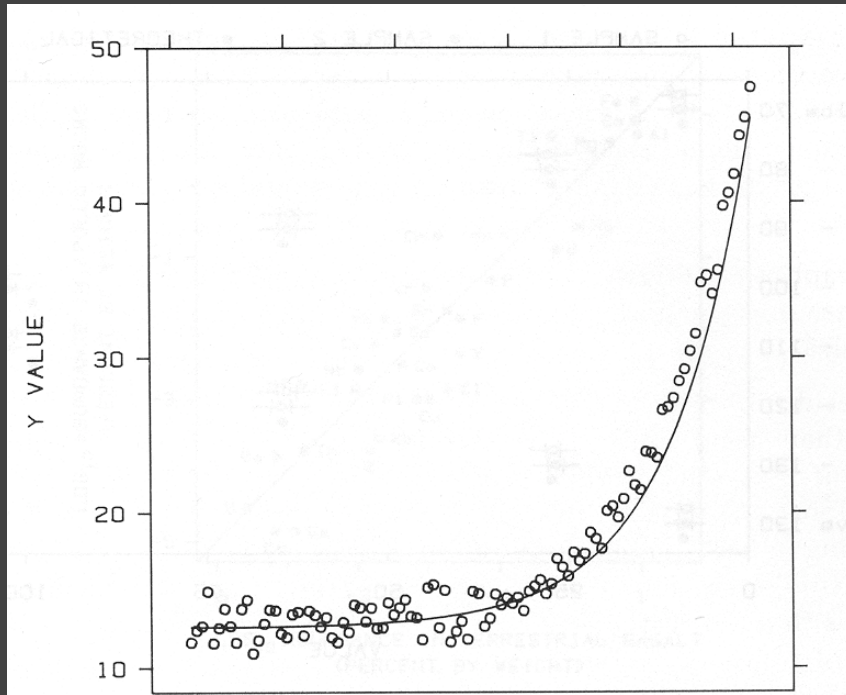
Residual graph shows accuracy of fit



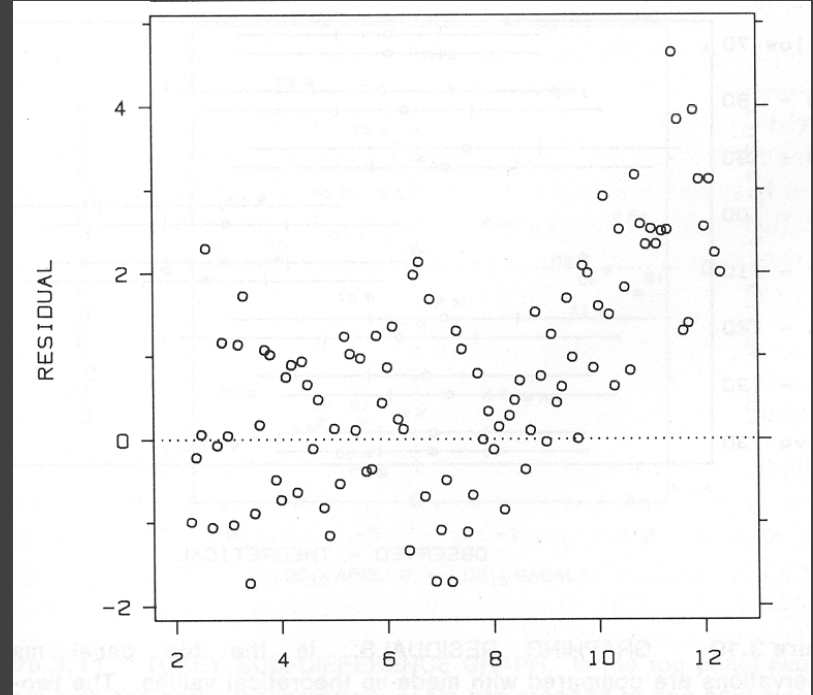
[Cleveland 85]

# Multiple Plotting Options

Plot model in data space



Plot data in model space



[Cleveland 85]

# Optimizing Chart Design



# Chart Design Parameters

Given a visual encoding (e.g., line chart), what aspects might affect graphical perception?

Physical Size

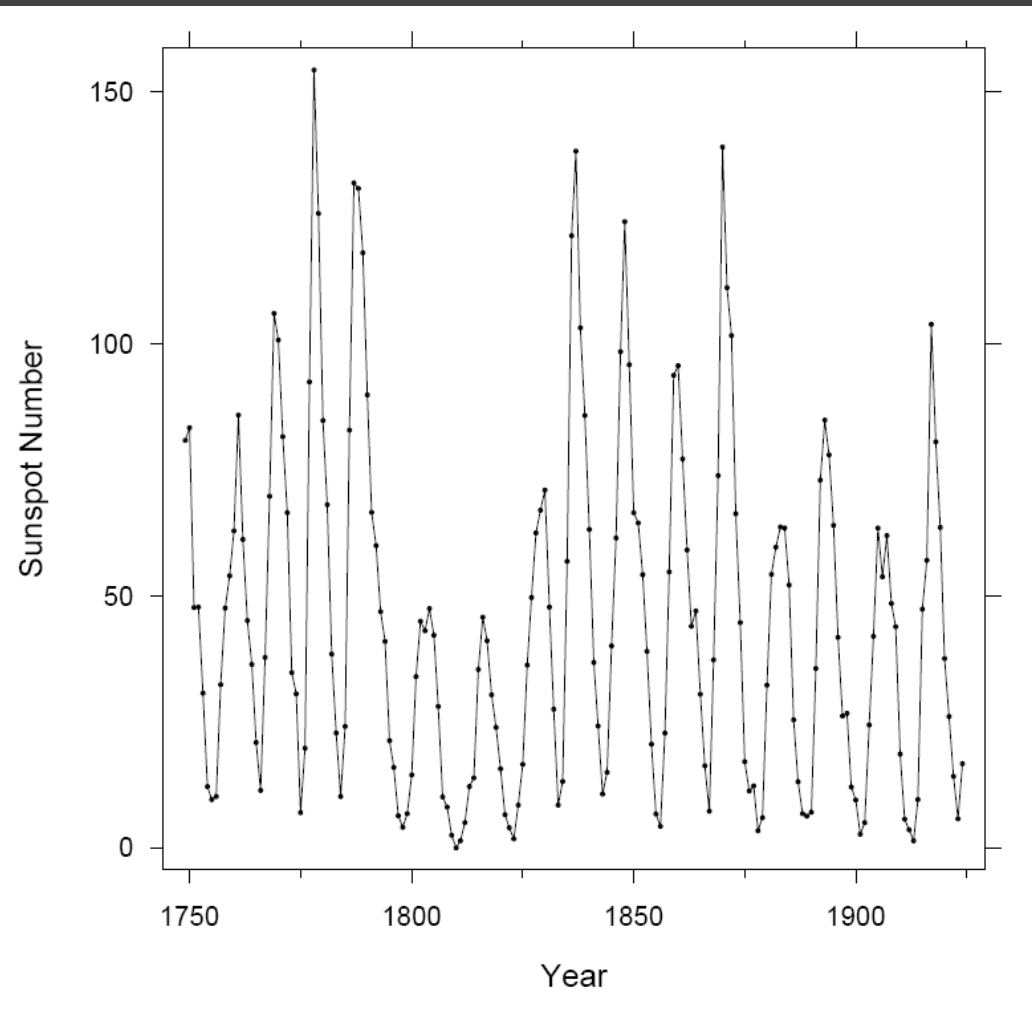
Aspect Ratio

Ticks, Labels, Gridlines

Line Width

Data Points (e.g., dots)

How might we determine optimized choices?



# Optimization-Based Design

Determine *error* or *energy* functions for measuring the “quality” of a visualization.

Treat as an optimization *objective* and then *solve* (or *search*) for better chart parameters.

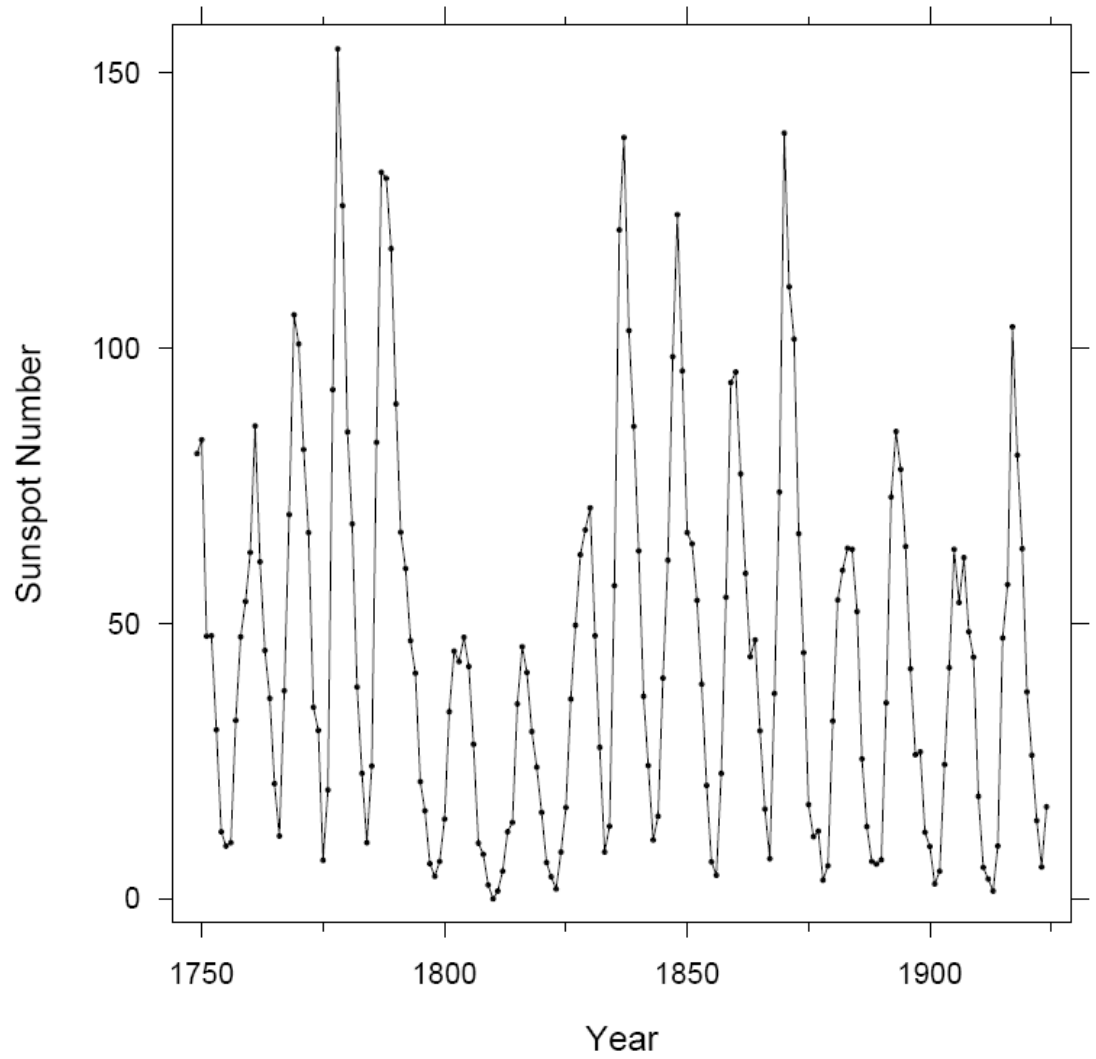
## **Examples:**

Determining chart aspect ratio

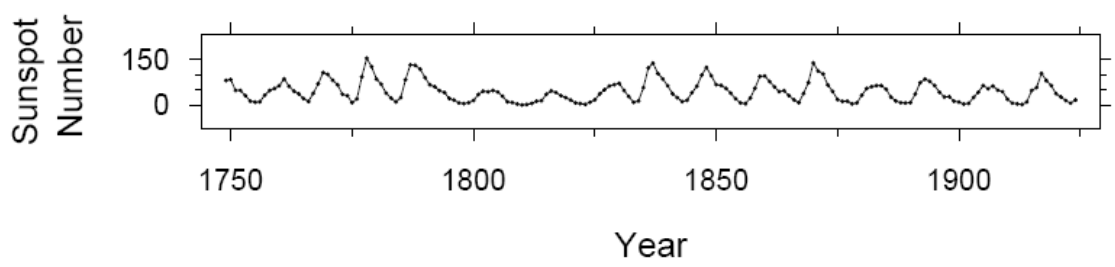
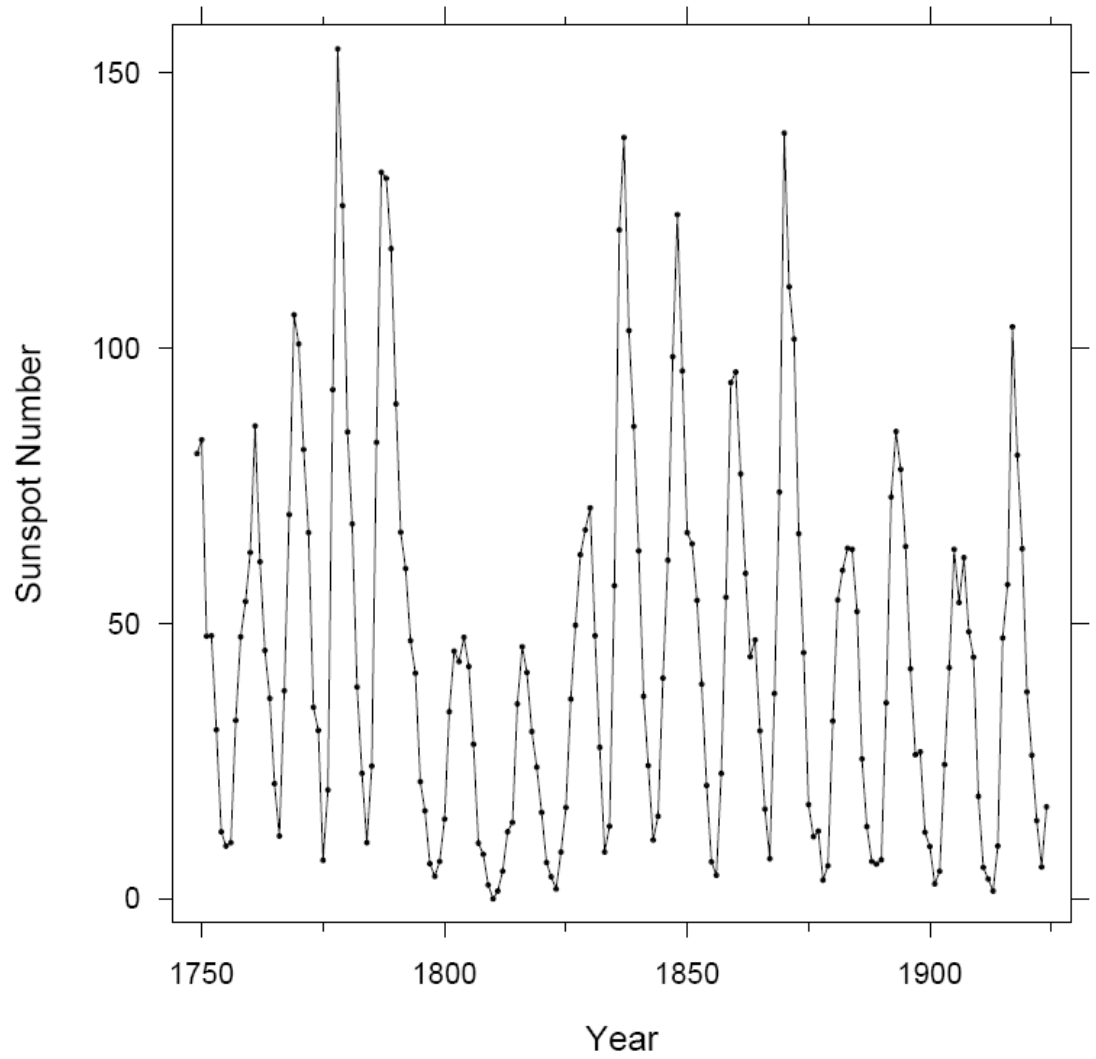
Selecting axis ticks

Streamgraph layout

# Aspect Ratio



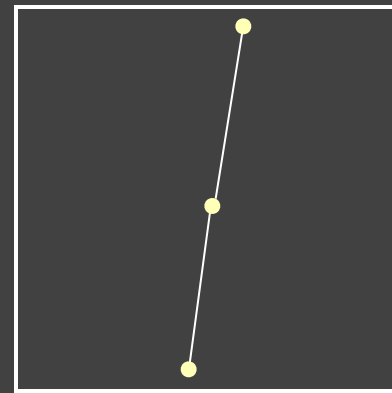
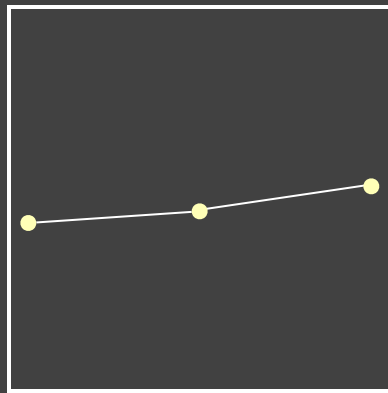
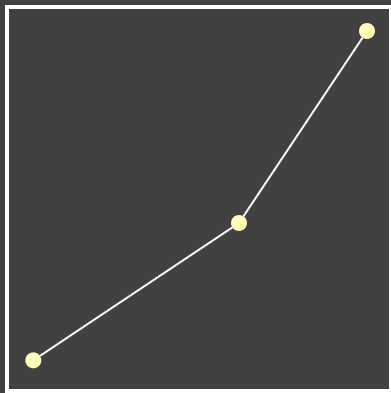
William S. Cleveland  
*The Elements of  
Graphing Data*



William S. Cleveland  
*The Elements of  
Graphing Data*

# Banking to 45° [Cleveland]

To facilitate perception of trends, maximize the discriminability of line segment orientations



Two line segments are maximally discriminable when their average absolute angle is 45°

Insight: to optimize the aspect ratio, bank to 45°

# Aspect Ratio Banking Methods

## Median-Absolute-Slope

$$\alpha = \text{median} |s_i| R_x / R_y$$

## Average-Absolute-Slope

$$\alpha = \text{mean} |s_i| R_x / R_y$$

## Avg-Absolute-Orientation

Unweighted

$$\sum_i \frac{|\theta_i(\alpha)|}{n} = 45^\circ$$

## Max-Orientation-Resolution

Global (over all  $i, j$  s.t.  $i \neq j$ )

$$\sum_i \sum_j |\theta_i(\alpha) - \theta_j(\alpha)|^2$$

Weighted

$$\frac{\sum_i |\theta_i(\alpha)| l_i(\alpha)}{\sum_i l_i(\alpha)} = 45^\circ$$

Local (over adjacent segments)

$$\sum_i |\theta_i(\alpha) - \theta_{i+1}(\alpha)|^2$$

## Closed Form Solutions

### Median-Absolute-Slope

$$\alpha = \text{median } |s_i| R_x / R_y$$

### Average-Absolute-Slope

$$\alpha = \text{mean } |s_i| R_x / R_y$$

---

### Avg-Absolute-Orientation

Unweighted

$$\sum_i \frac{|\theta_i(\alpha)|}{n} = 45^\circ$$

### Max-Orientation-Resolution

Global (over all  $i, j$  s.t.  $i \neq j$ )

$$\sum_i \sum_j |\theta_i(\alpha) - \theta_j(\alpha)|^2$$

Weighted

$$\frac{\sum_i |\theta_i(\alpha)| l_i(\alpha)}{\sum_i l_i(\alpha)} = 45^\circ$$

Local (over adjacent segments)

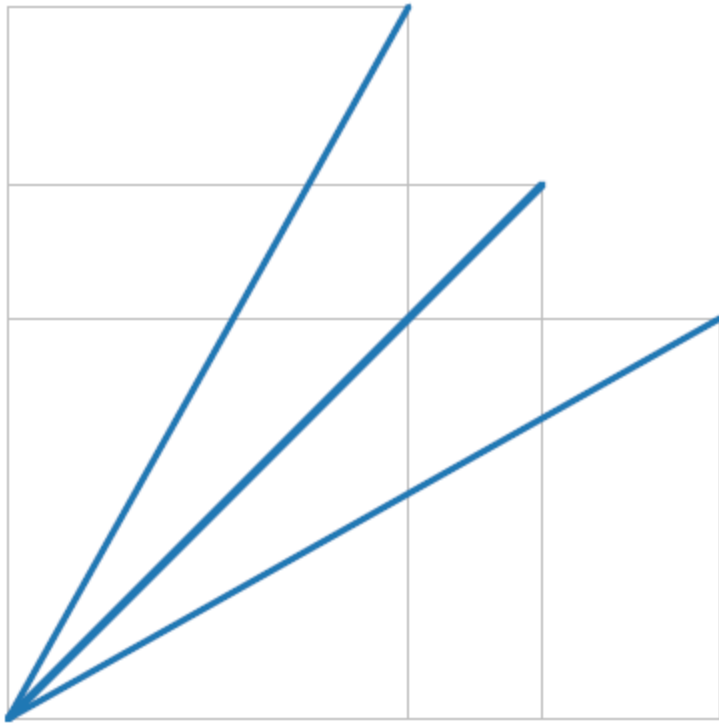
$$\sum_i |\theta_i(\alpha) - \theta_{i+1}(\alpha)|^2$$

Requires Iterative Optimization

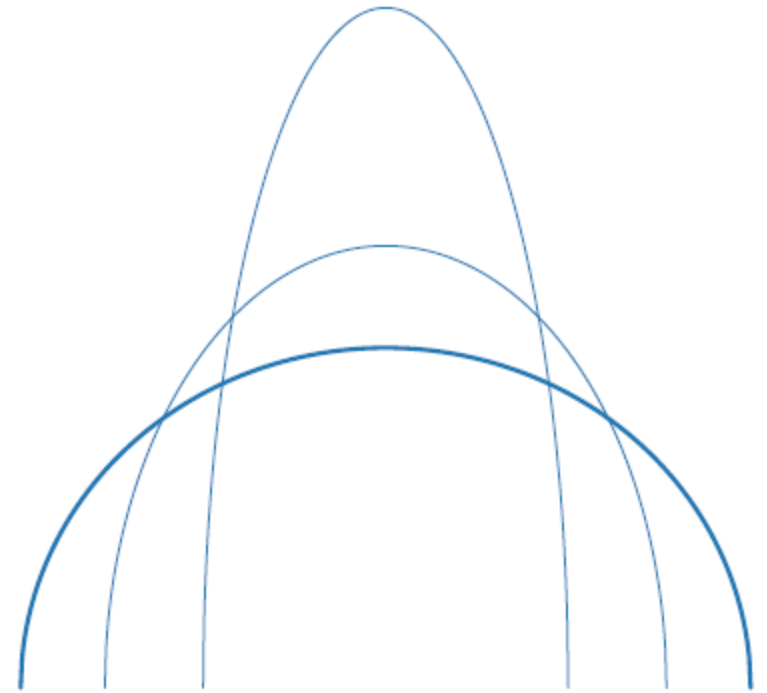


# Alternative: Minimize Arc Length

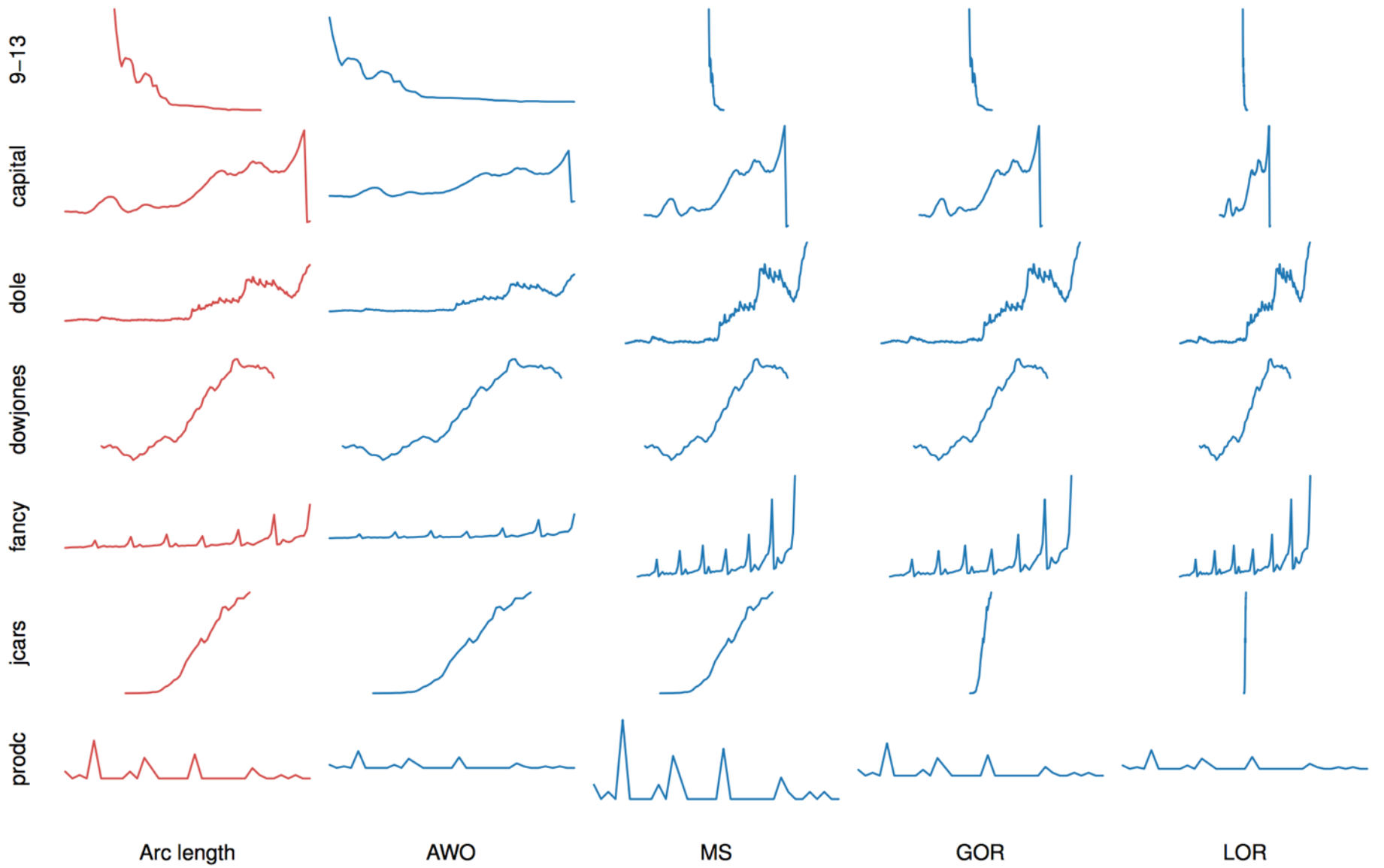
while holding area constant [Talbot et al. 2011]



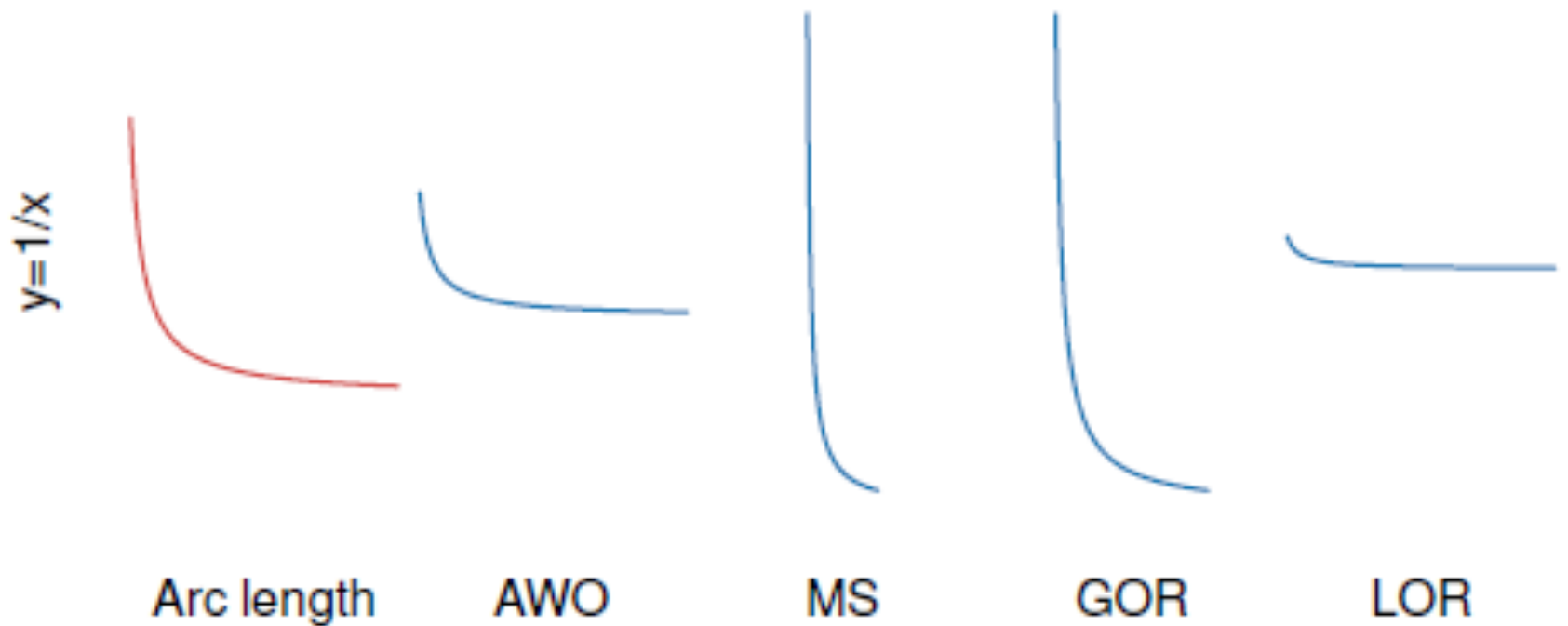
Straight line  $\rightarrow 45^\circ$

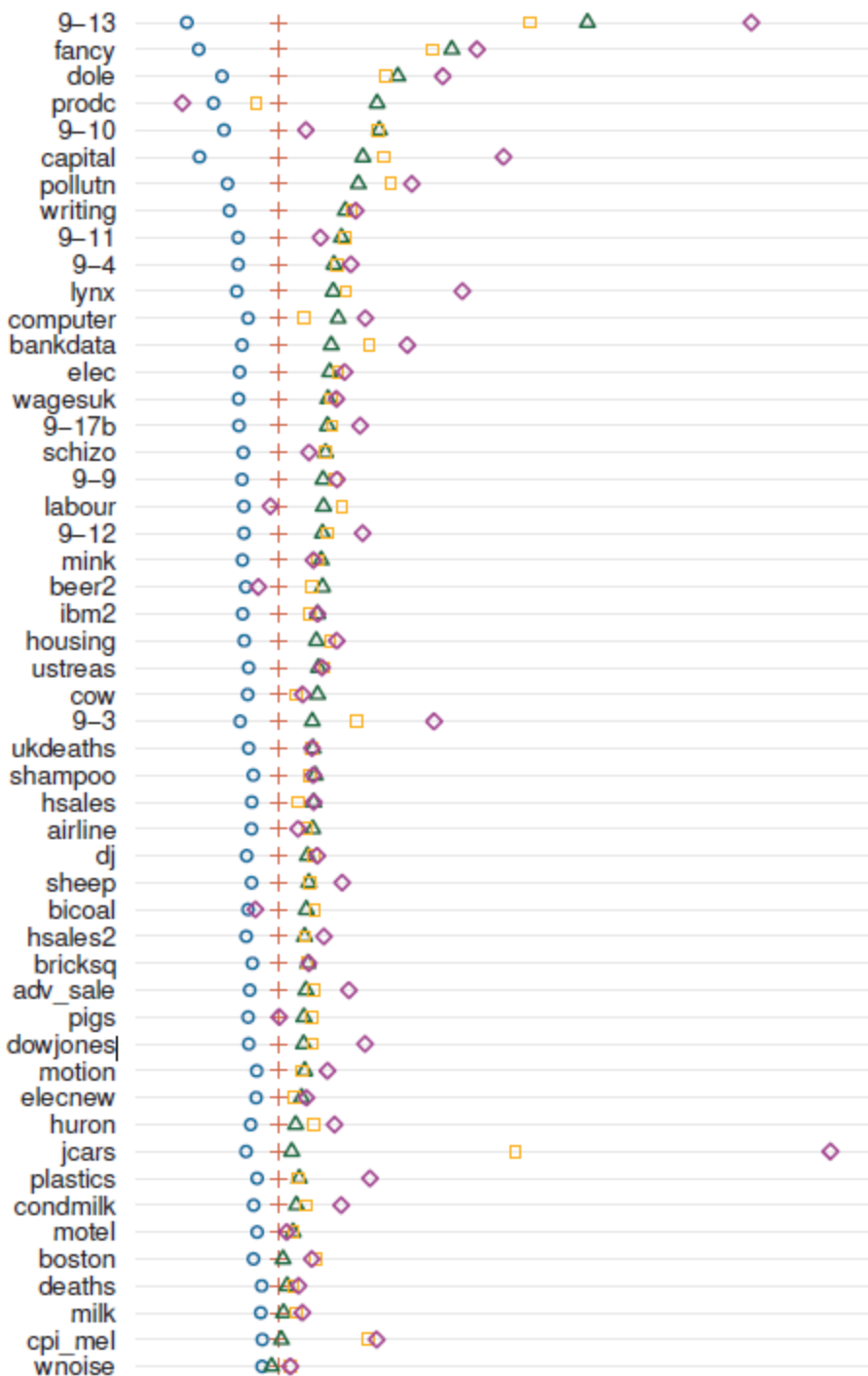


Ellipse  $\rightarrow$  Circle



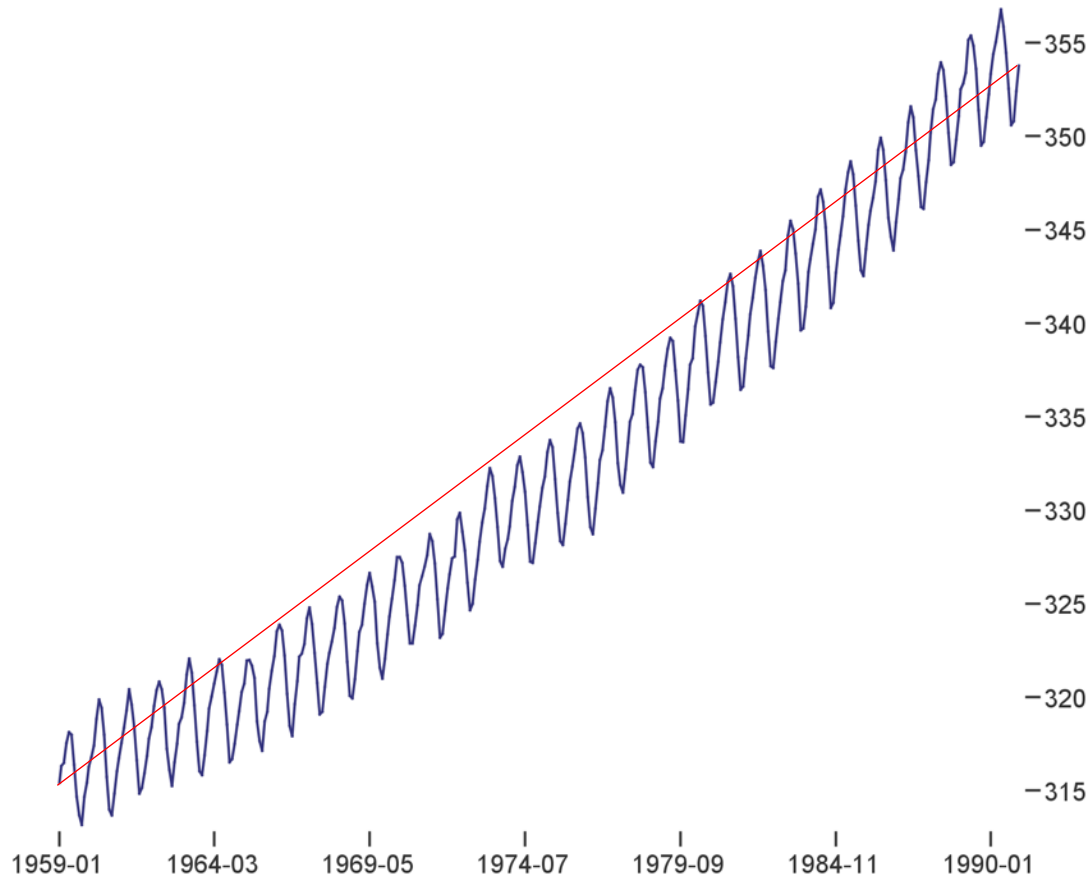
# Robustness: Banking $y = 1/x$



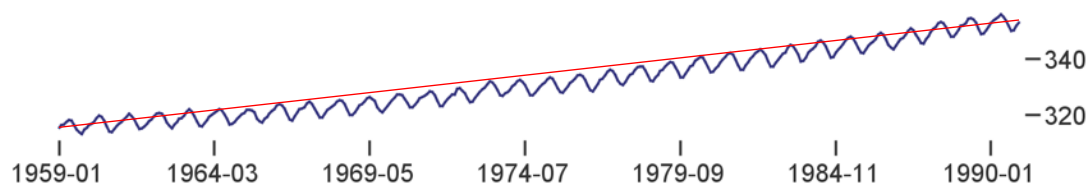


## A Good Compromise

Arc-length banking  
produces aspect  
ratios in-between  
those produced by  
other methods.



Aspect Ratio = 1.17



Aspect Ratio = 7.87

**Trends may occur  
at different scales!**

Apply banking to  
the original data *or*  
to fitted trend lines.

[Heer & Agrawala '06]

**CO<sub>2</sub> Measurements**

William S. Cleveland

*Visualizing Data*

# Discussion

**Arc-length banking preferable to prior methods**

Parameterization invariant

Robust (corner cases, compromise results)

Applicable to plotted curves and contour lines

Fast-converging iterative optimization

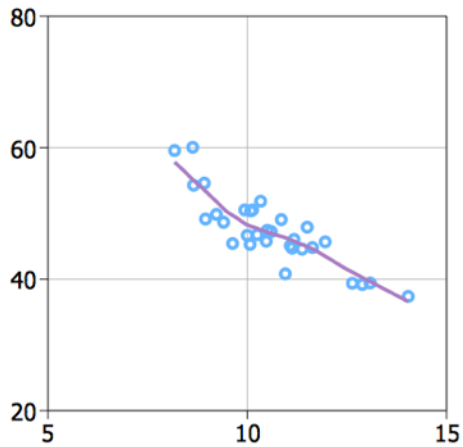
**But what about perceptual effectiveness?**

We lack theory to motivate aspect ratio selection.

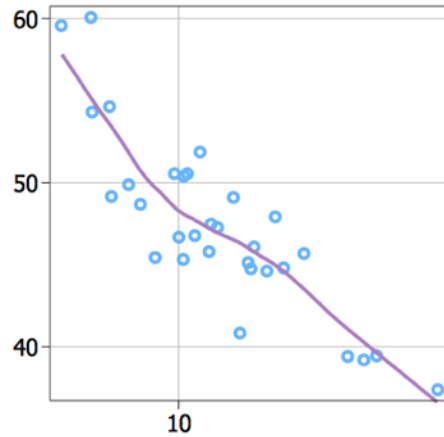
Perceptual experiments needed to assess?

# Axis Ticks

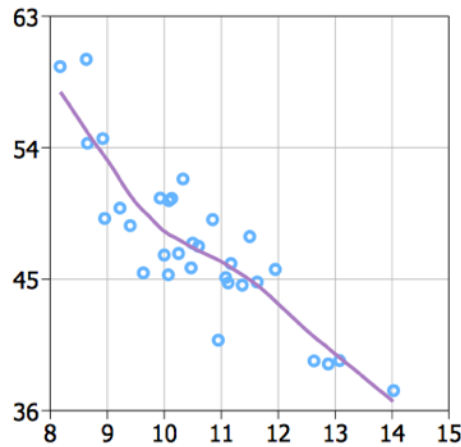
# Tick Mark Selection



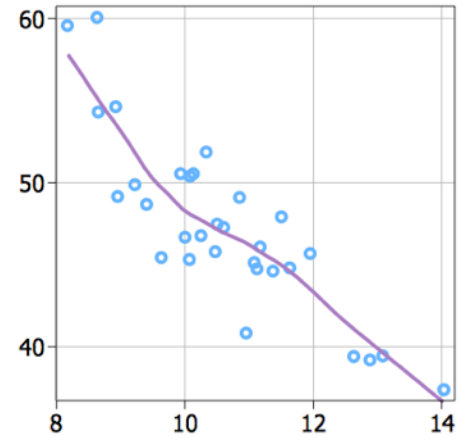
(a) Heckbert



(b) R's pretty



(c) Wilkinson



(d) Extended

What are some properties of "good" tick marks?



# Tick Mark Criteria

**Simplicity** - numbers are multiples of 10, 5, 2

**Coverage** - ticks near the ends of the data

**Density** - not too many, nor too few

**Legibility** - whitespace, horizontal text, size

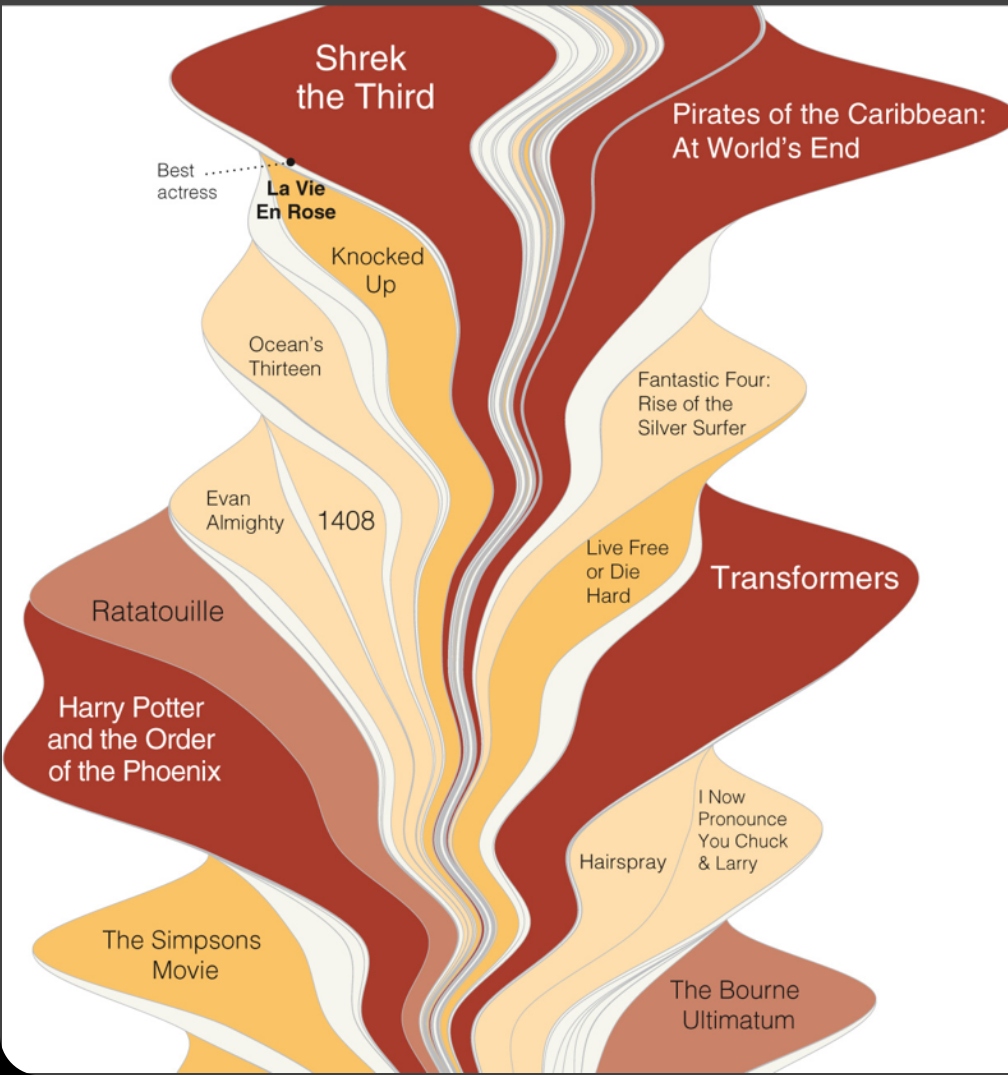
## Optimization

Talbot et al '10 use a search procedure that optimizes criteria in turn (e.g., find simple numbers first, then adjust coverage, etc.).

$$S = 0.2 \text{ simplicity} + 0.25 \text{ coverage} + 0.5 \text{ density} + 0.05 \text{ legibility}$$

# Streamgraph Layout

# Streamgraph Layout [Byron & Wattenberg '08]



Thickness encodes values, stack extent encodes sum of values.

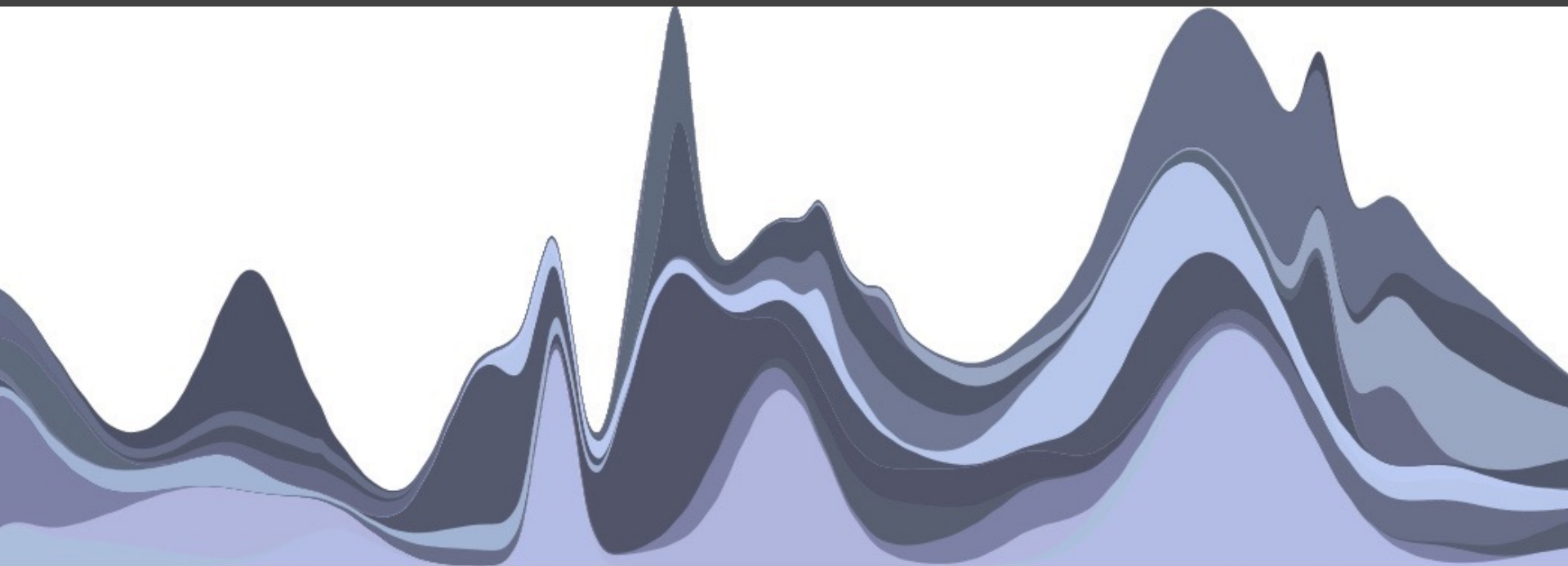
## Issues

Layers bias each other  
Perceived thickness  
Slope (bank to 45?)

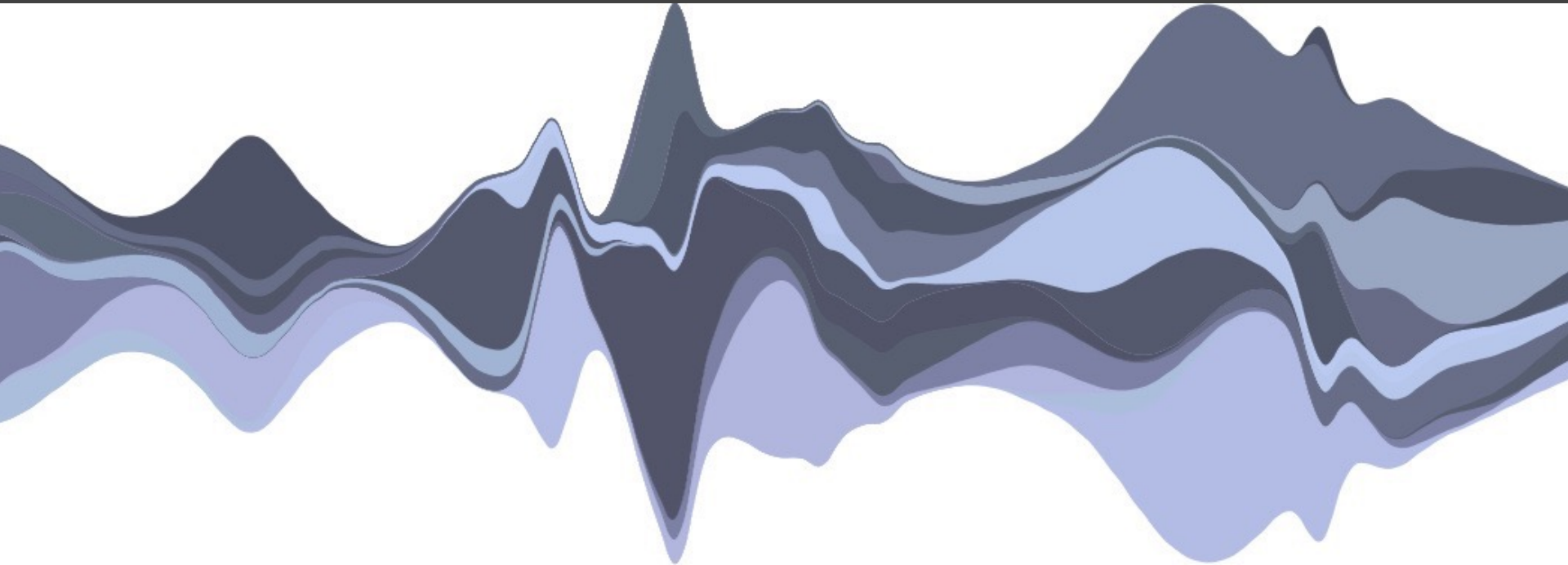
## Design Parameters

Layer baseline  
Layer order

# Zero Baseline



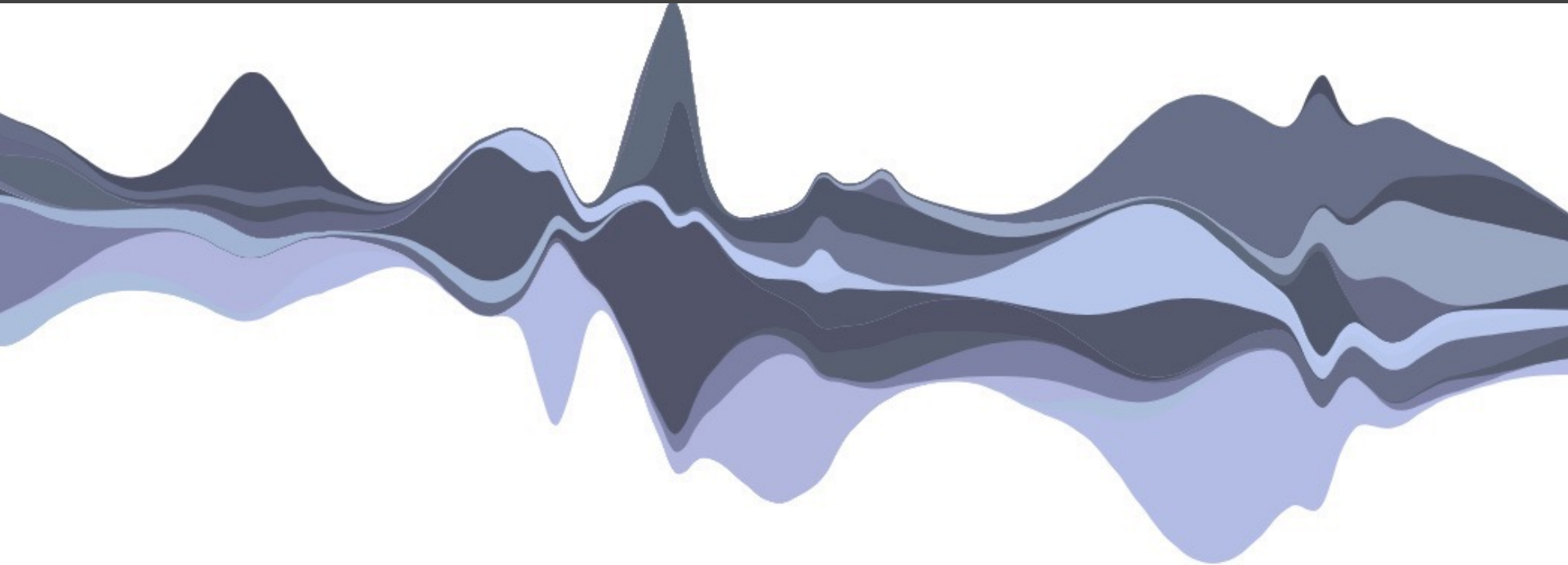
# Centered Baseline



**Pros:** reduces "spikiness", minimizes slope (in the least squares sense) for bottom and top layers.

**Cons:** removes fixed baseline for sum comparison.

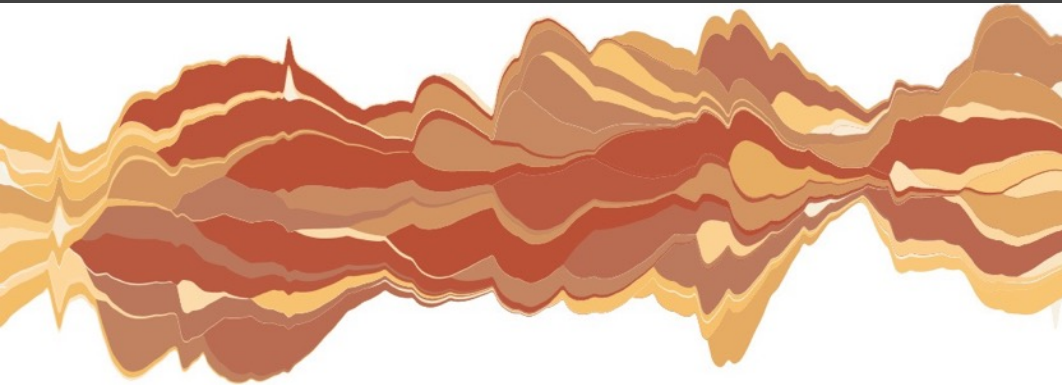
# “Wiggle” Baseline [Byron & Wattenberg '08]



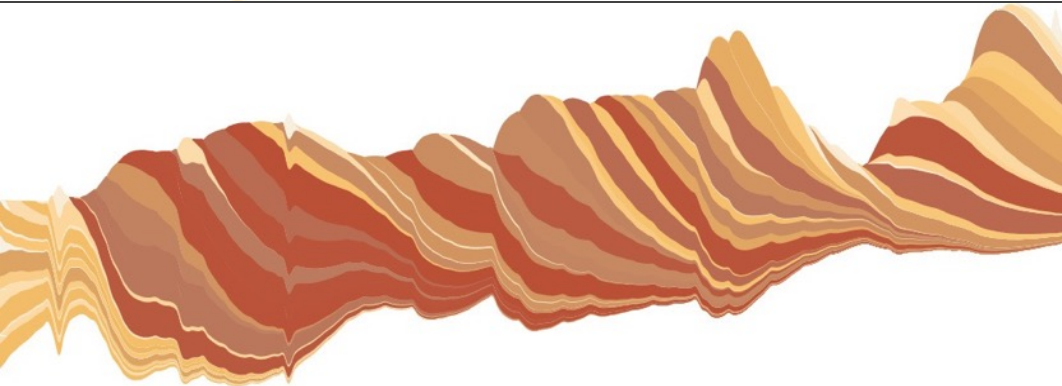
**Pros:** minimizes *overall* slope (in the least squares sense), weighted by layer thickness.

**Cons:** chart may require more total area.

# Layer Sorting



Unsorted

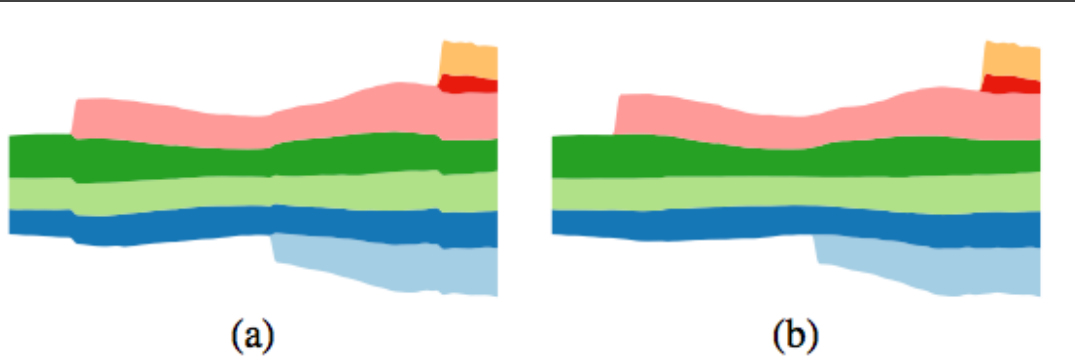


Sort by First Non-Zero Value



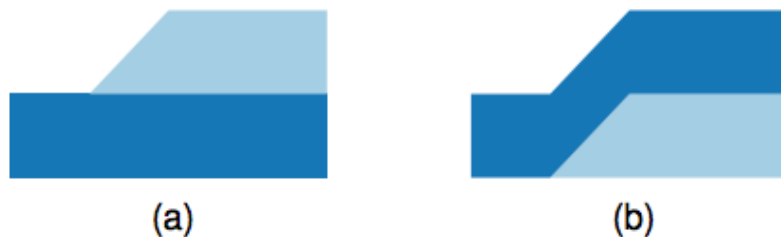
**Sort "Inside-Out"**: Place layers alternately at top or bottom, in a thickness balancing manner.

# Streamgraphs Redux [Di Bartolomeo & Hu '16]



**Figure 2:** *Different baselines: (a) with 2-norm, distortions are present; (b) with 1-norm, all layers are smooth.*

Minimize slope using **L1-norm**, not (least-squares) **L2-norm**.



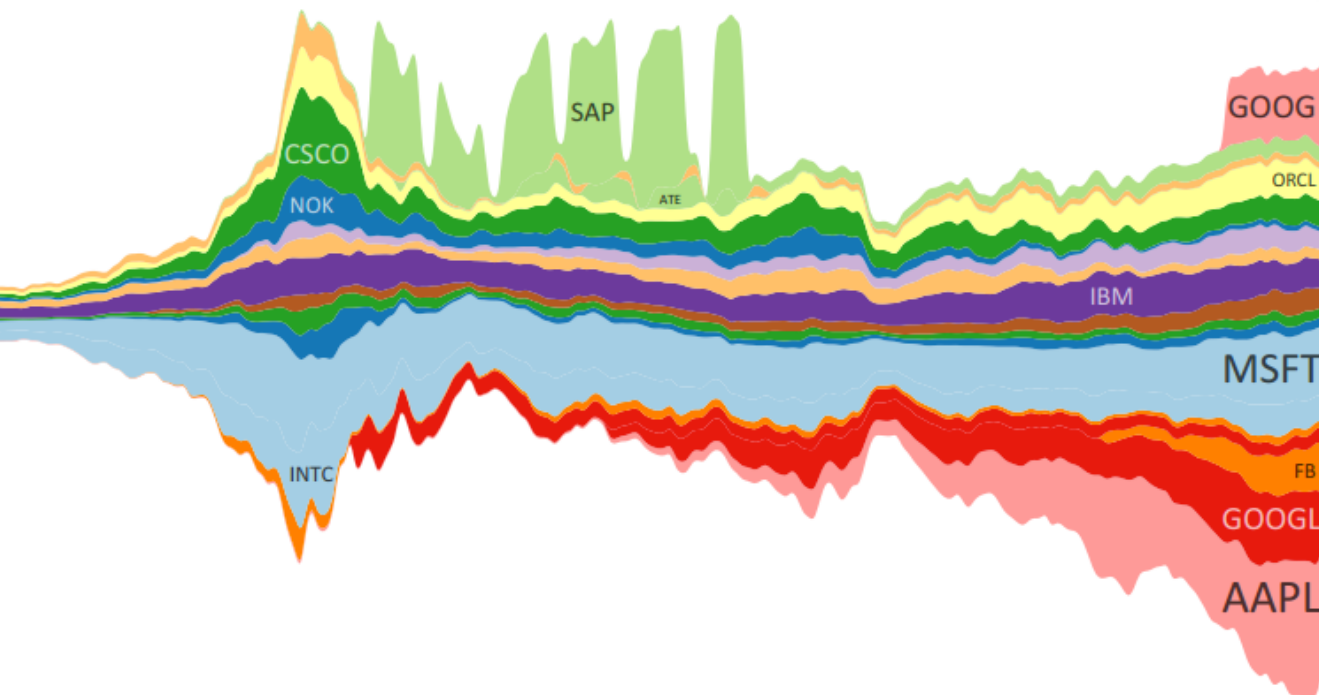
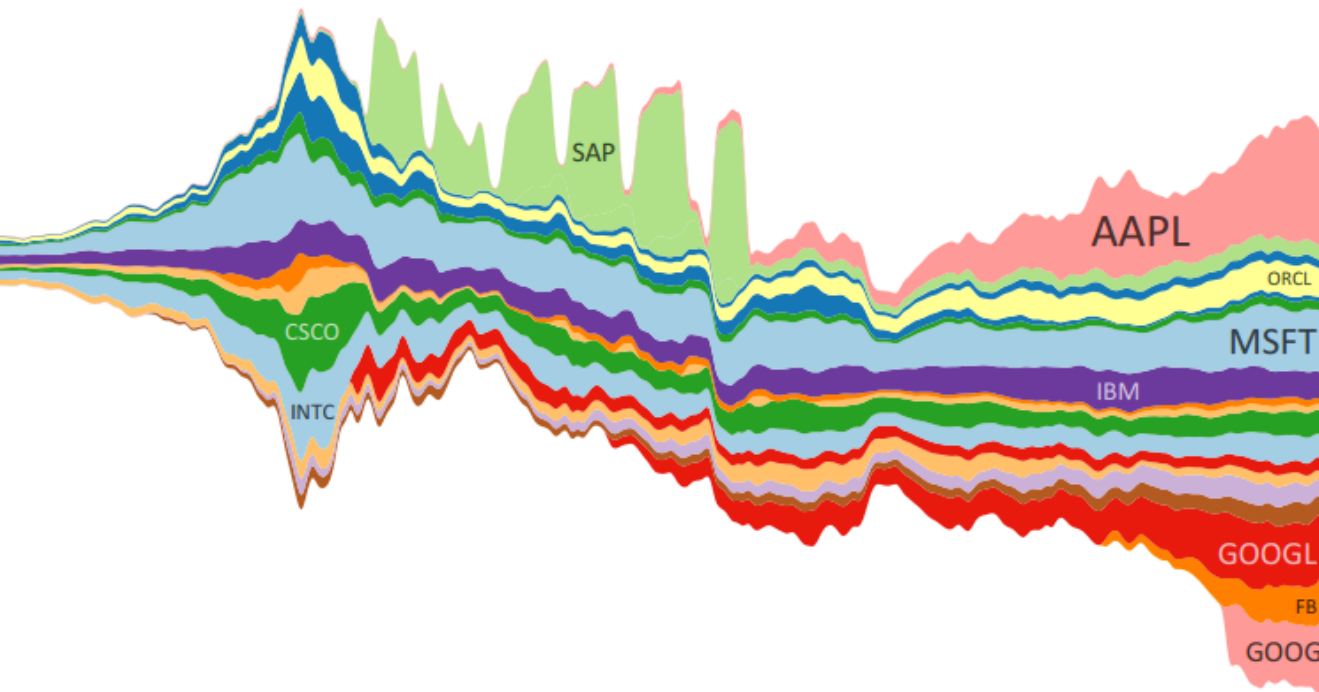
**Figure 4:** *Two orderings of two layers on a flat baseline. Case (b) produces a distortion, case (a) is preferred.*

Perform **iterative optimization**, swapping adjacent layers if it improves the overall layout.



# Results

Byron & Wattenberg '08  
(via D3's stacked layout)



Di Bartolomeo & Hu '16

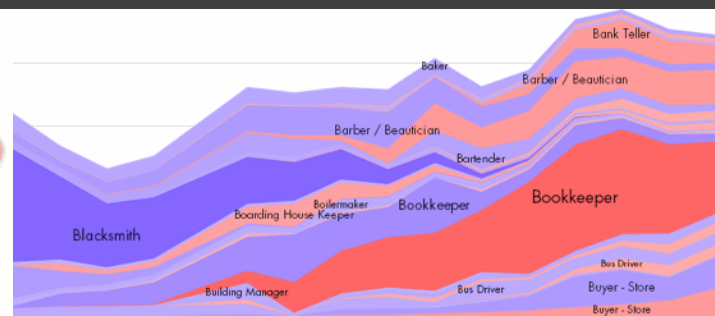
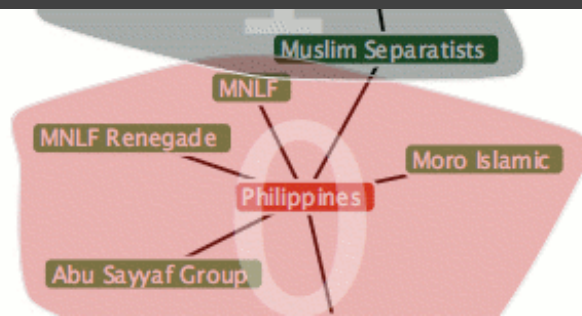
# Administrivia

# FP: Interactive Prototype

Create an interactive visualization. Choose a driving question in your topic area and develop an appropriate visualization + interaction techniques, then deploy your visualization on the web.

Due by 5pm on **Wednesday, May 3.**

We will assign your GitHub repos shortly!

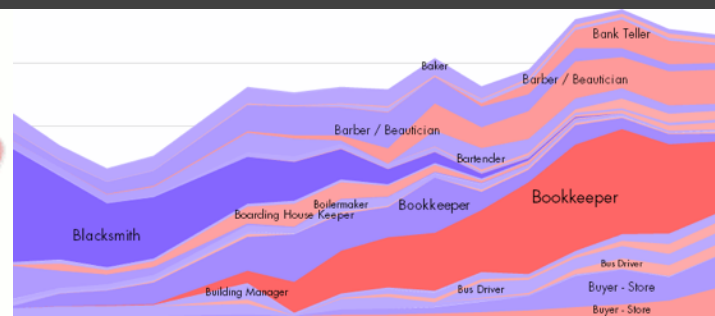
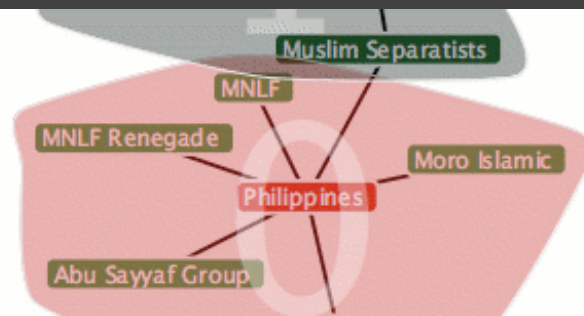


# Requirements

**Interactive.** You must implement interaction methods! However, this is not only selection / filtering / tooltips. Also consider annotations or other narrative features to draw attention and provide additional context

**Web-based.** D3 is encouraged, but not required. Deploy your visualization using GitHub pages.

**Write-up.** Provide design rationale on your web page.

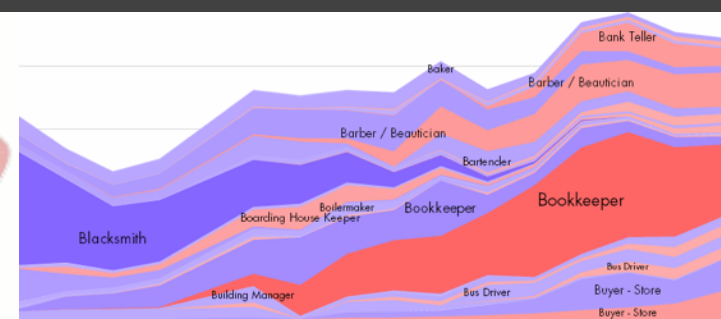
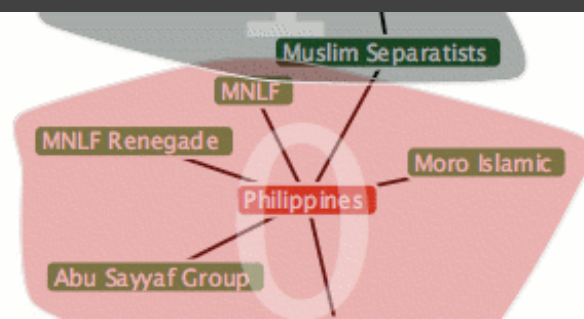


# Interactive Prototype Tips

**Start now.** It will take longer than you think.

**Keep it simple.** Choose a minimal set of interactions that enables users to explore and generate interesting insights. Do not feel obligated to convey *everything* about the data: focus on a compelling subset.

**Promote engagement.** How do your chosen interactions reveal interesting observations?



# Team Member Roles

We encourage you to structure team responsibilities!

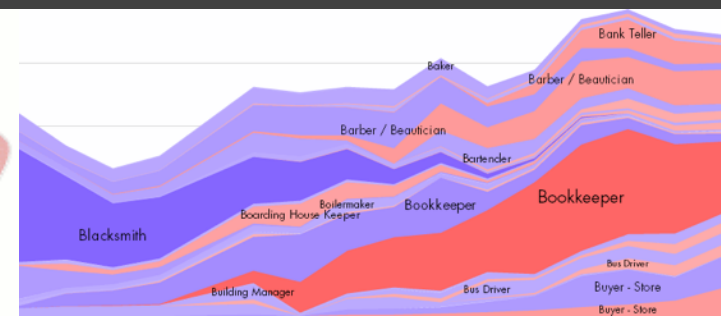
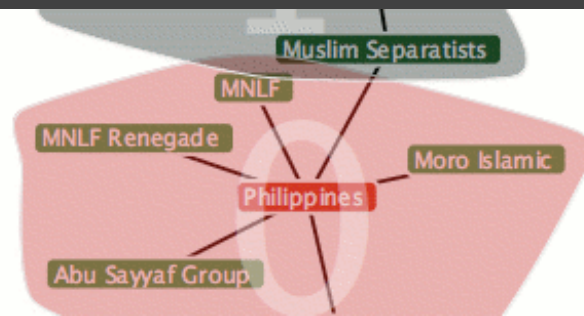
**Coordinator:** Organize meetings, track deadlines, etc.

**Data Lead:** Data wrangling, management, distillation

**Tech Lead:** Manage code integration, GitHub repo

**UX Lead:** Visualization/interaction design & evaluation

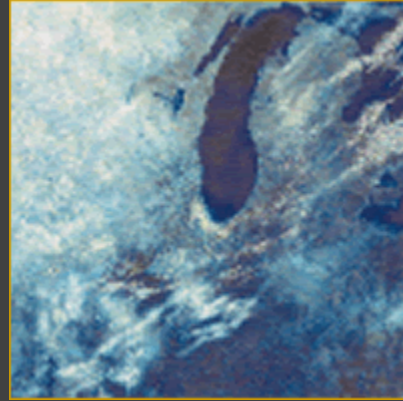
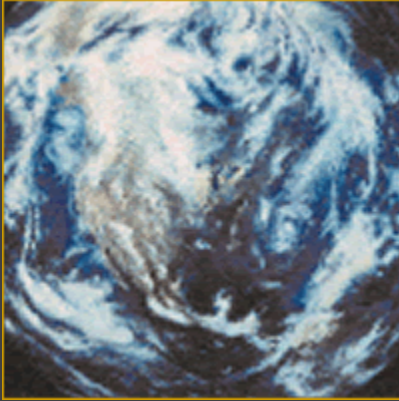
*One may have multiple roles, share work across roles...*



**Zooming**



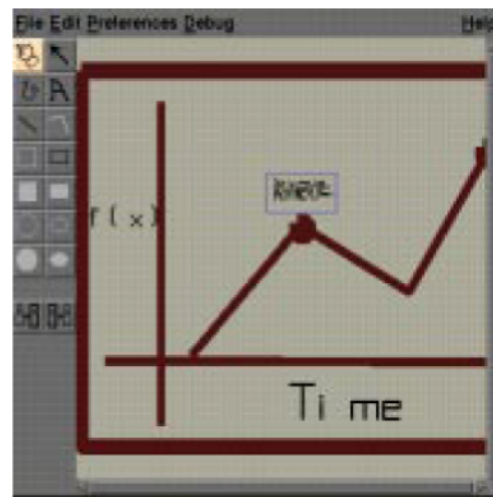
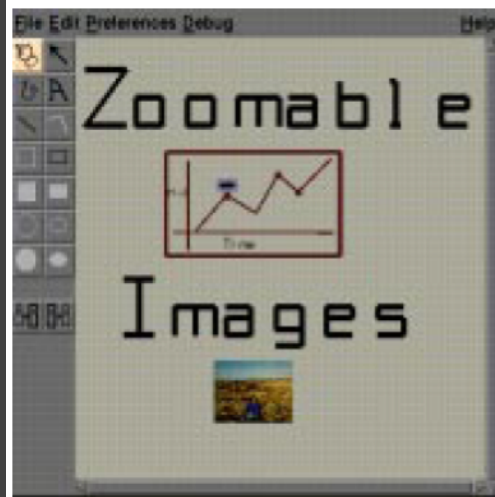
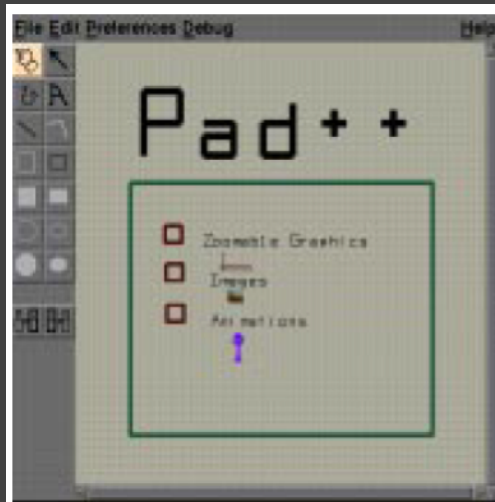
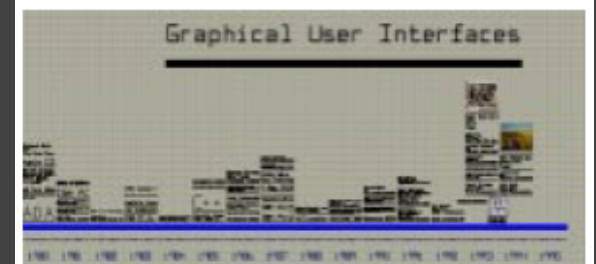
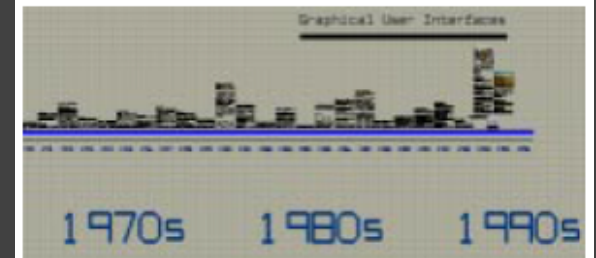
# Zooming



Eames' Powers of Ten



# Interactive Zooming



Pad++ [Bederson and Hollan 94]

# Semantic Zooming

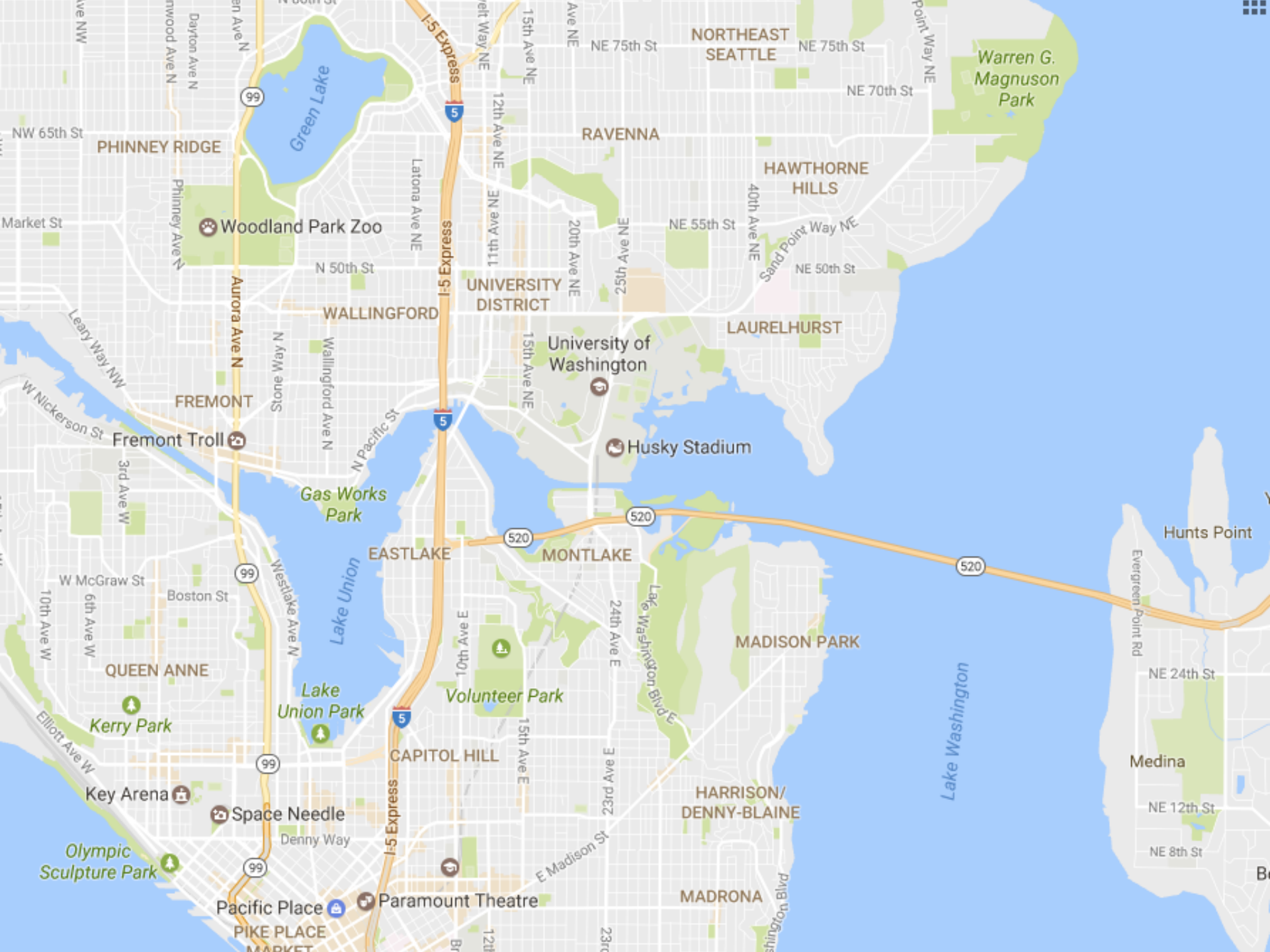
Change representations as zoom level changes

1990	1991	1992	1993	1994
1995	1996	1997	1998	1999

Figure 2: As you approach the calendar object the large scale display items fade out and disappear.

*1992*			
Jan	Feb	Mar	Apr
May	Jun	Jul	Aug
Sep	Oct	Nov	Dec
*1997*			
Jan	Feb	Mar	Apr
May	Jun	Jul	Aug
Sep	Oct	Nov	Dec

		Tuesday
6	7	Tuesday
3	14	Tuesday



Warren G. Magnuson Park

Woodland Park Zoo

University of Washington

Husky Stadium

Gas Works Park

Lake Union Park

Volunteer Park

Kerry Park

Space Needle

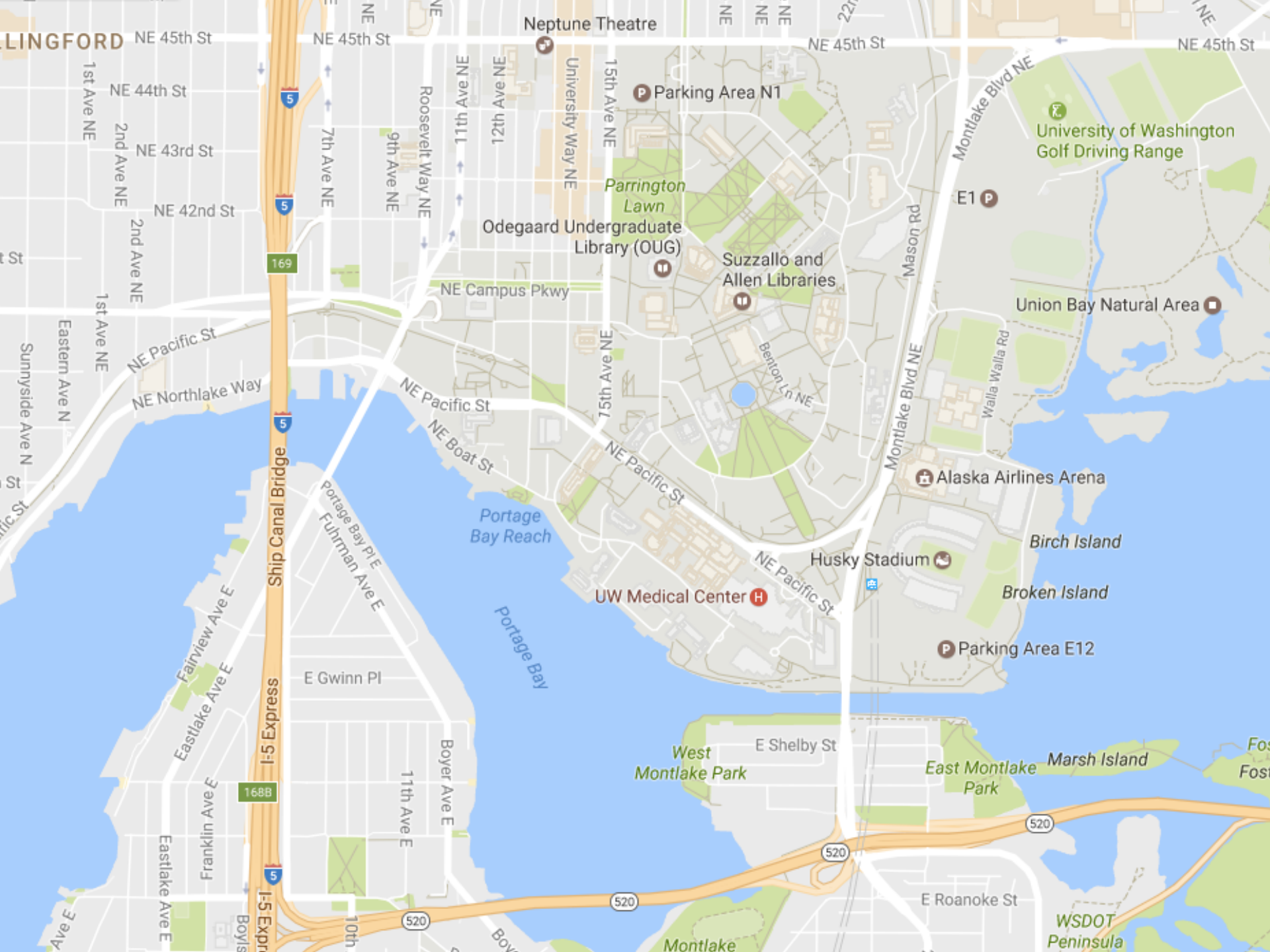
Olympic Sculpture Park

Pacific Place

Paramount Theatre

HARRISON/DENNY-BLAINE

MADRONA



LINGFORD

NE 45th St

NE 45th St

Neptune Theatre

NE 45th St

NE 45th St

1st Ave NE

NE 44th St

NE 45th St

Roosevelt Way NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 43rd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

Montlake Blvd NE

University of Washington  
Golf Driving Range

2nd Ave NE

NE 42nd St

7th Ave NE

9th Ave NE

11th Ave NE

12th Ave NE

University Way NE

15th Ave NE

15th Ave NE

Parking Area N1

NE 45th St

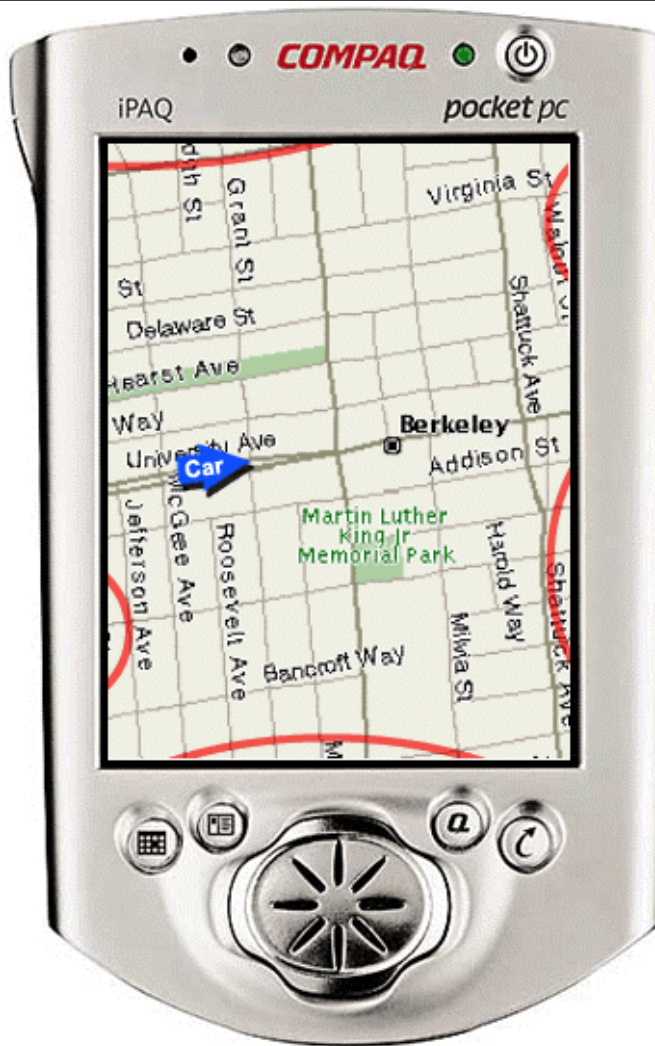
Montlake Blvd NE

University of Washington  
Golf Driving Range



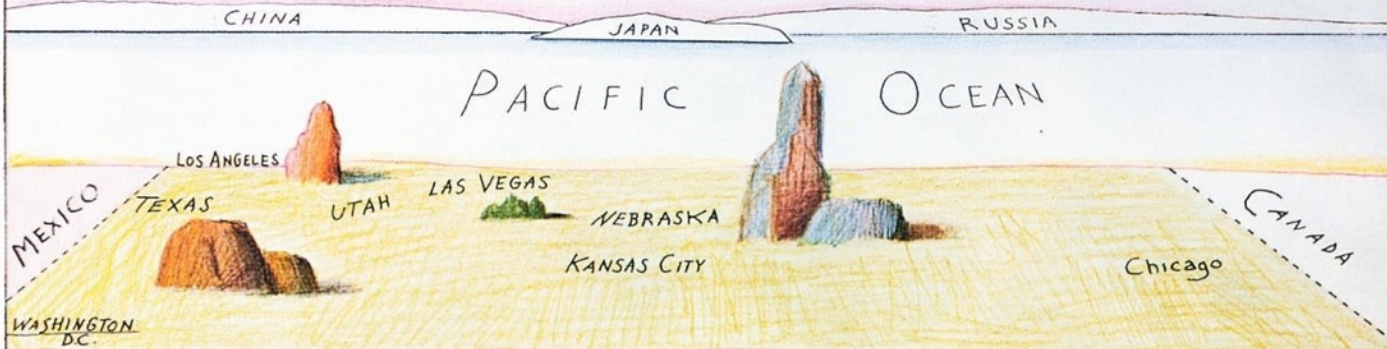


# Halo [Baudisch et al '03]



**Focus + Context**

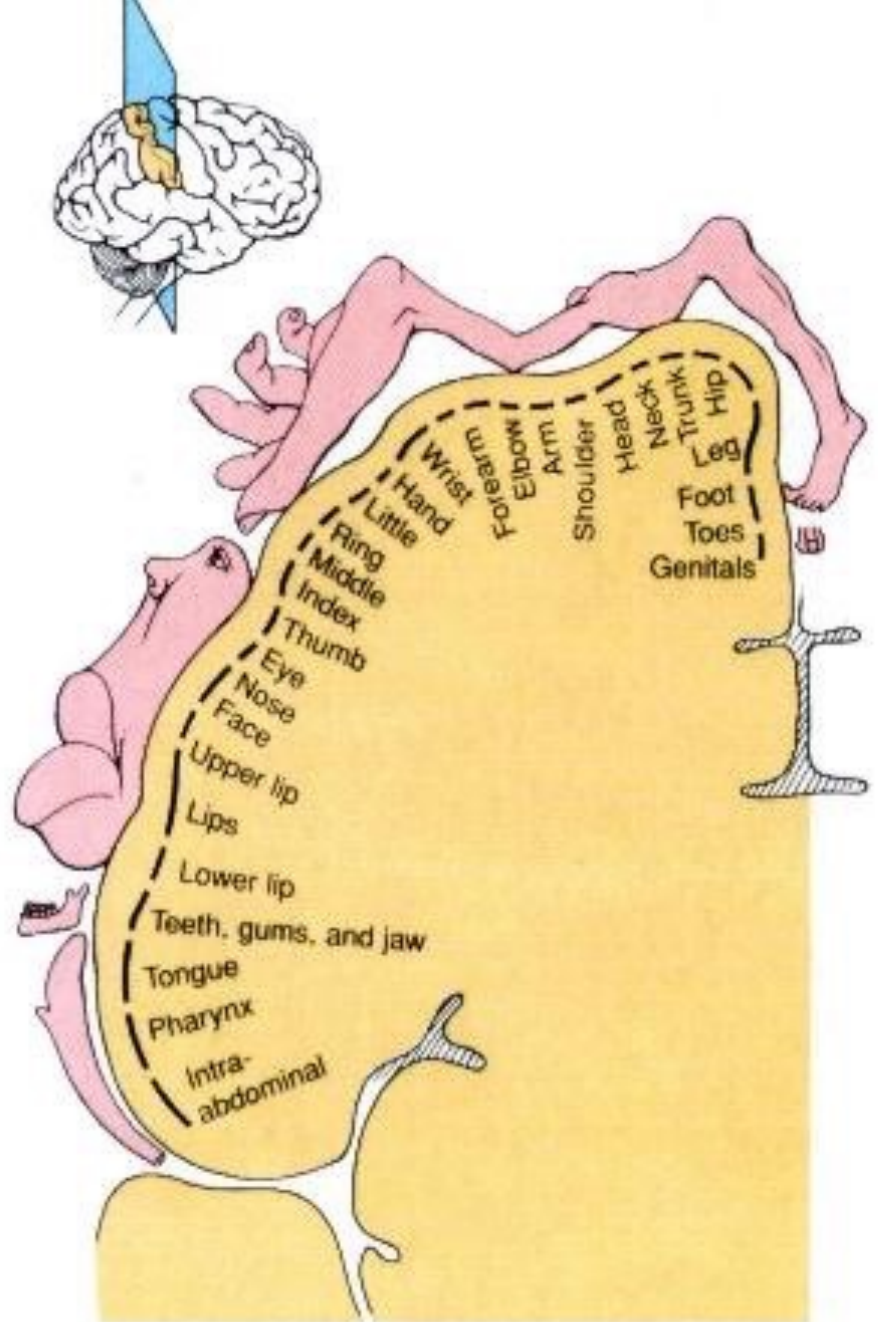








This model shows what a man's body would look like if each part grew in proportion to the area of the cortex of the brain concerned with its sensory perception. The hands and lips dominate – but the feet are also disproportionately large, indicating their sensory importance.



(a) Somatosensory cortex in right cerebral hemisphere

# Degree-of-Interest [Furnas '81, '06]

Estimate the saliency of information to display.  
Can affect **what** to show and/or **how** to show it!

*DOI ~ f (Current Focus, A Priori Importance)*

*Example: Google Search*

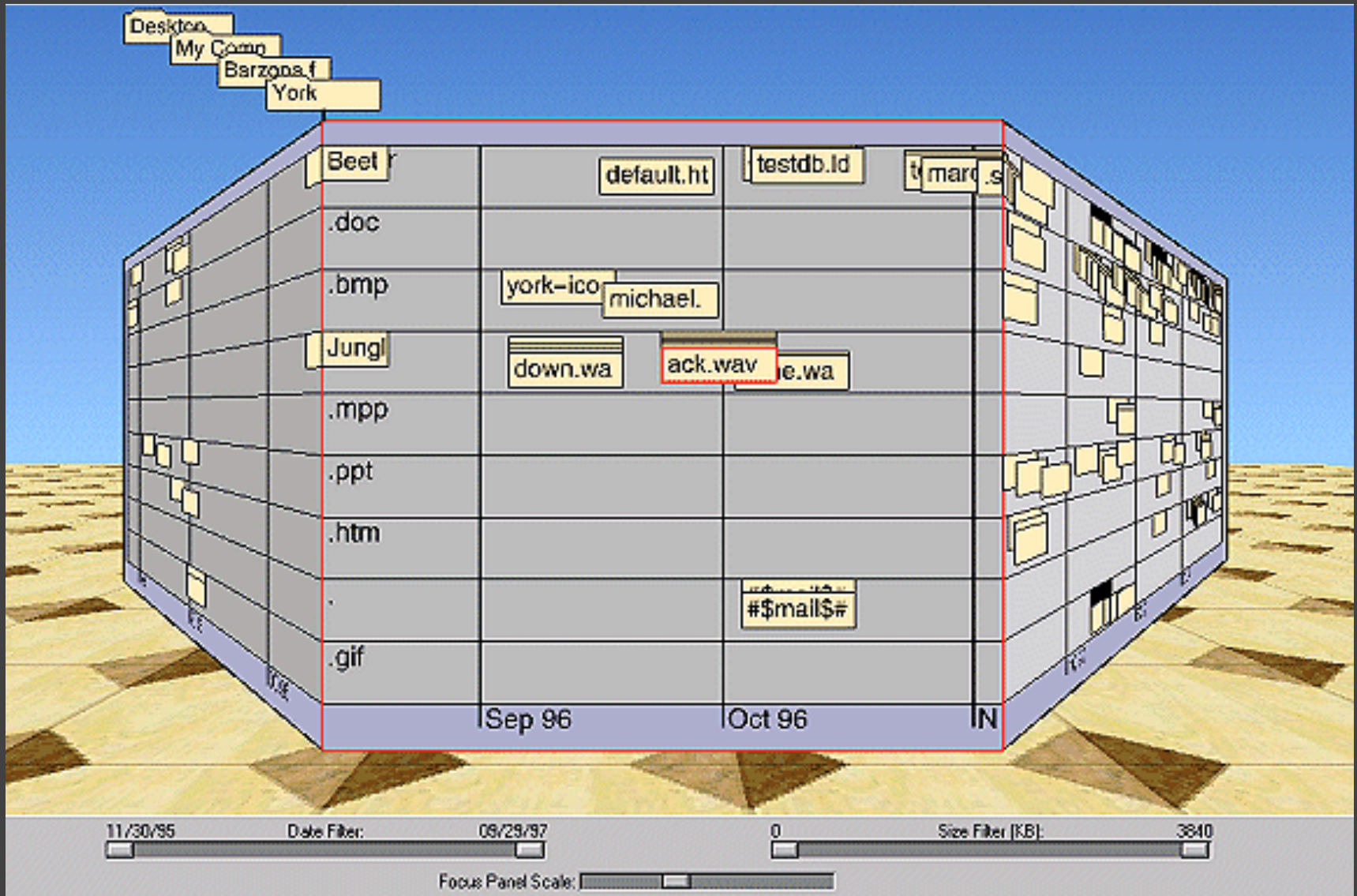
Current Focus = Query Hits (e.g., text matching)

A Priori Importance = PageRank

What to show: Top N results

How to show it: List

# Perspective Wall [Mackinlay et al. '91]





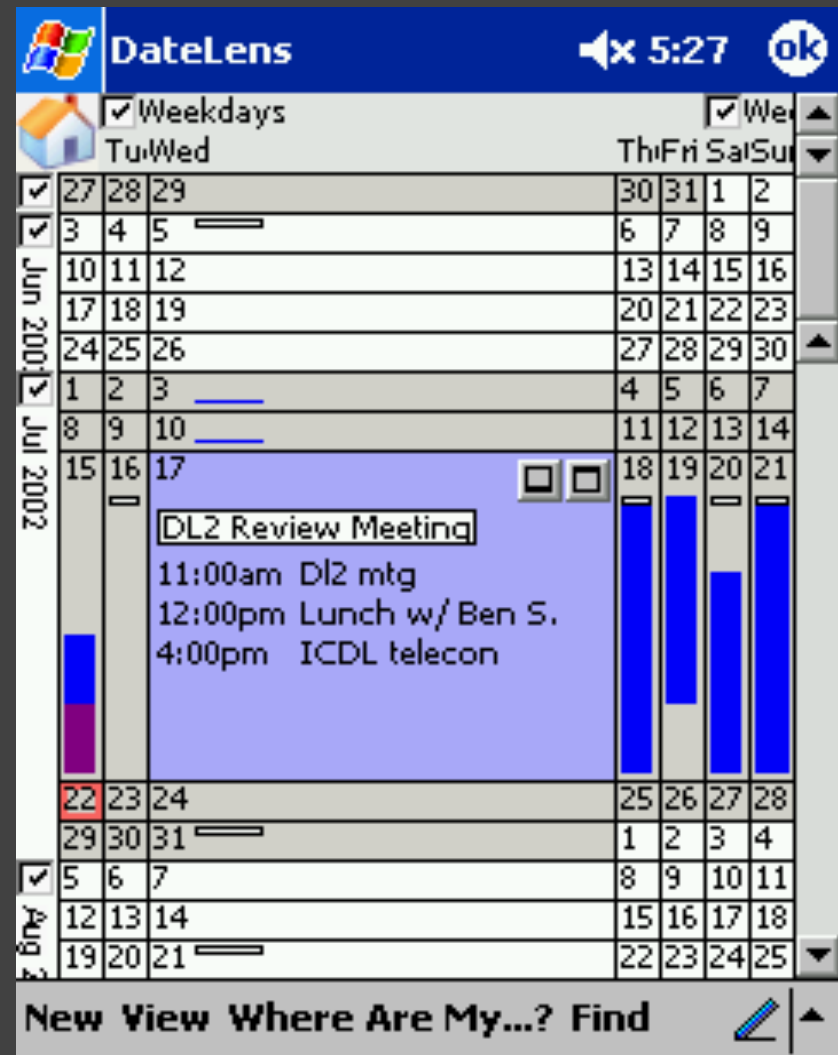
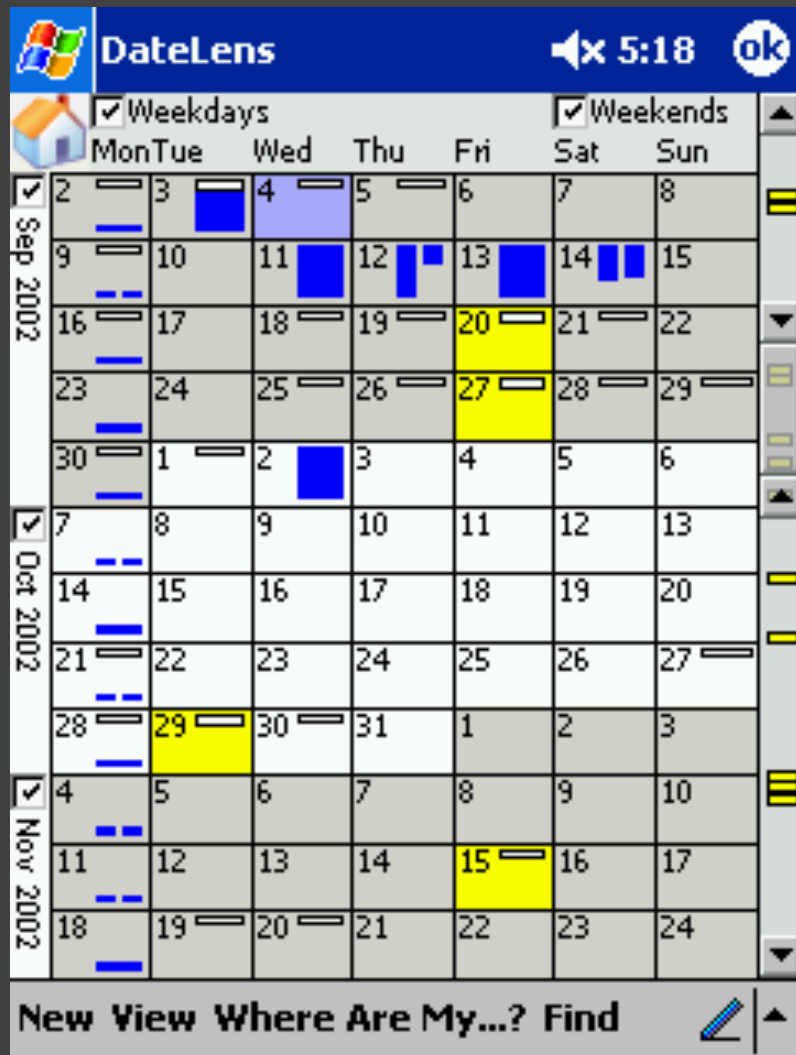
# TableLens [Rao & Card '94]

The screenshot shows the TableLens application window titled "Baseball.txt - TLDemo". The interface includes a menu bar (File, Edit, View, Options, Help), a toolbar with various icons, and the "inxight" logo. The main data area displays a table with columns: League, Players, At Bats, Hits, Home Runs, Runs, and Rbi. The table is divided into two sections, "N" and "A". The "N" section contains rows 52-55, and the "A" section contains row 191. A red border highlights the row for Reggie J... (row 191). The background of the table is a heatmap where darker blue indicates higher values. The status bar at the bottom shows "Row 79: 35", "Col: Assists", and "Entry: 35".

League ...	Players	At Bats	Hits	Home Runs	Runs	Rbi	
N	52	Andres ...	321	87	10	39	42
	53	Jose Cruz	479	133	10	48	72
	54	Bo Diaz	474	129	10	50	56
	55	Tony Pena	510	147	10	56	52
A	191	Reggie J...	419	101	18	65	58

Row 79: 35      Col: Assists      Entry: 35

# DateLens [Bederson et al. '04]

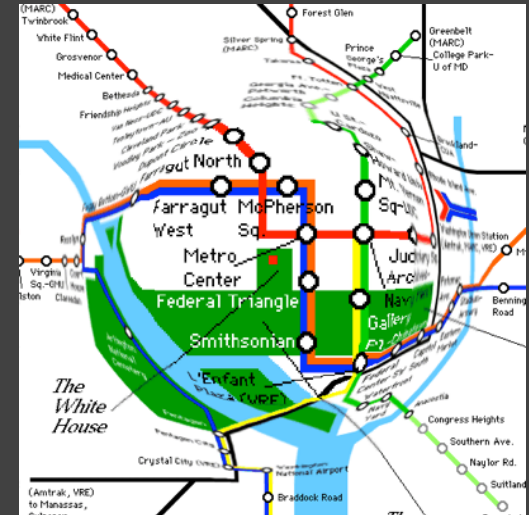
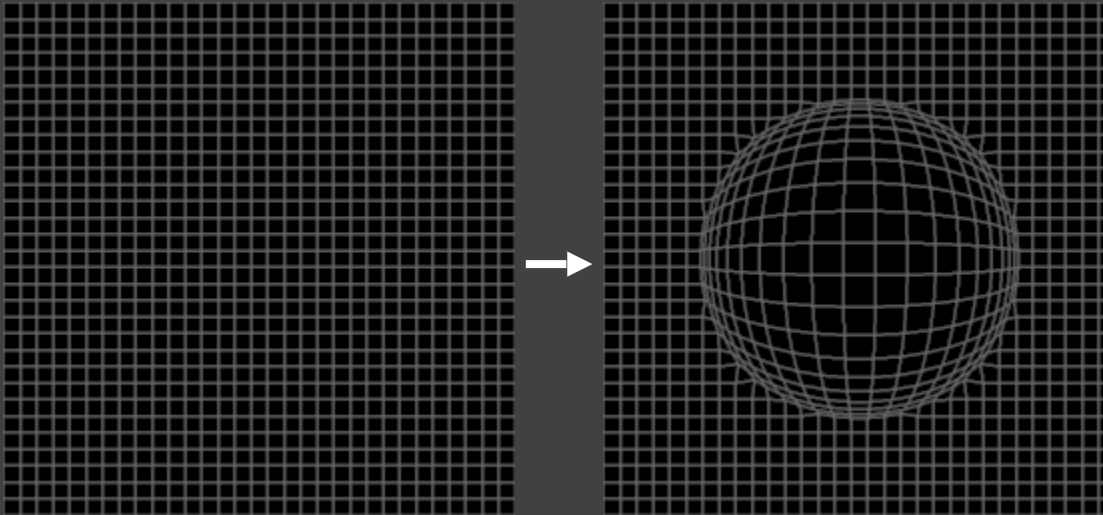


# Focus + Context Distortion

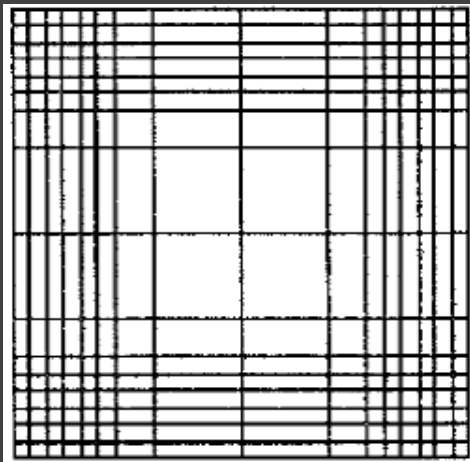
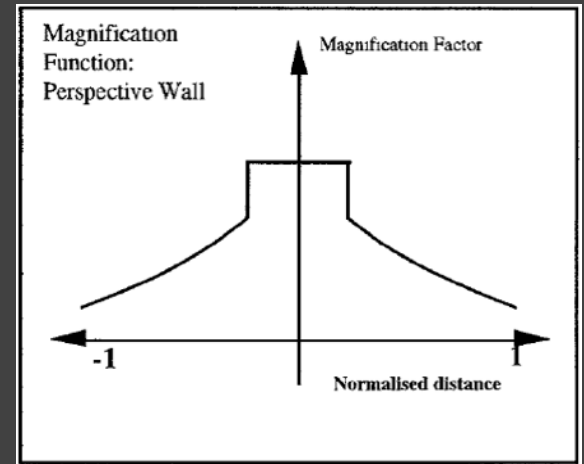
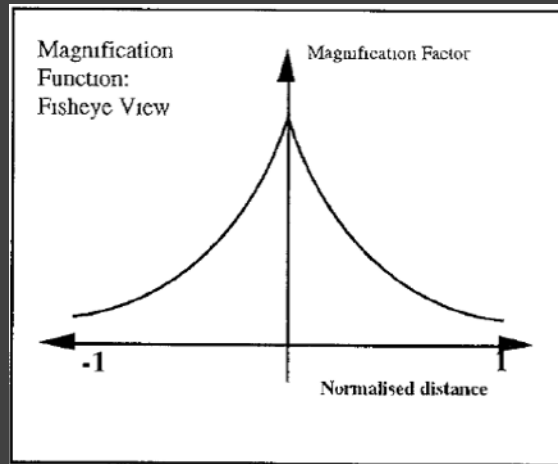
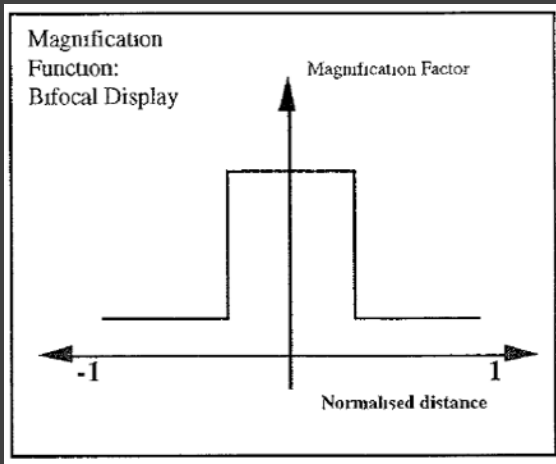
Focus area - local details

De-magnified area - surrounding context

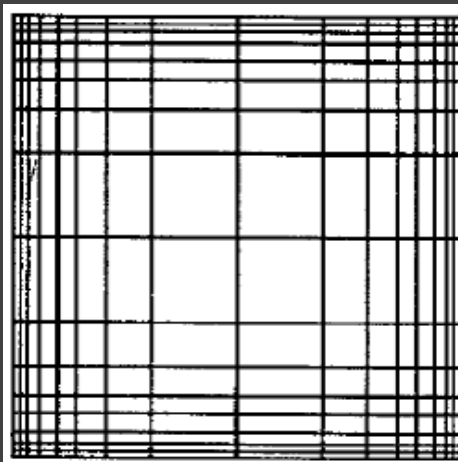
Like a rubber sheet with borders tacked down



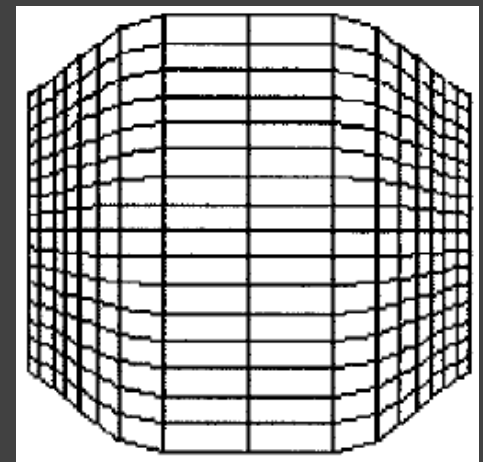
# Distortion Functions



Bifocal



Fisheye



Perspective Wall

# Uses (and Abuses) of Distortion

Often more harm than help...

Pan & zoom more familiar – and visually stable – than “rubber sheet” navigation.

Consider F+C over **data** rather than just **view**

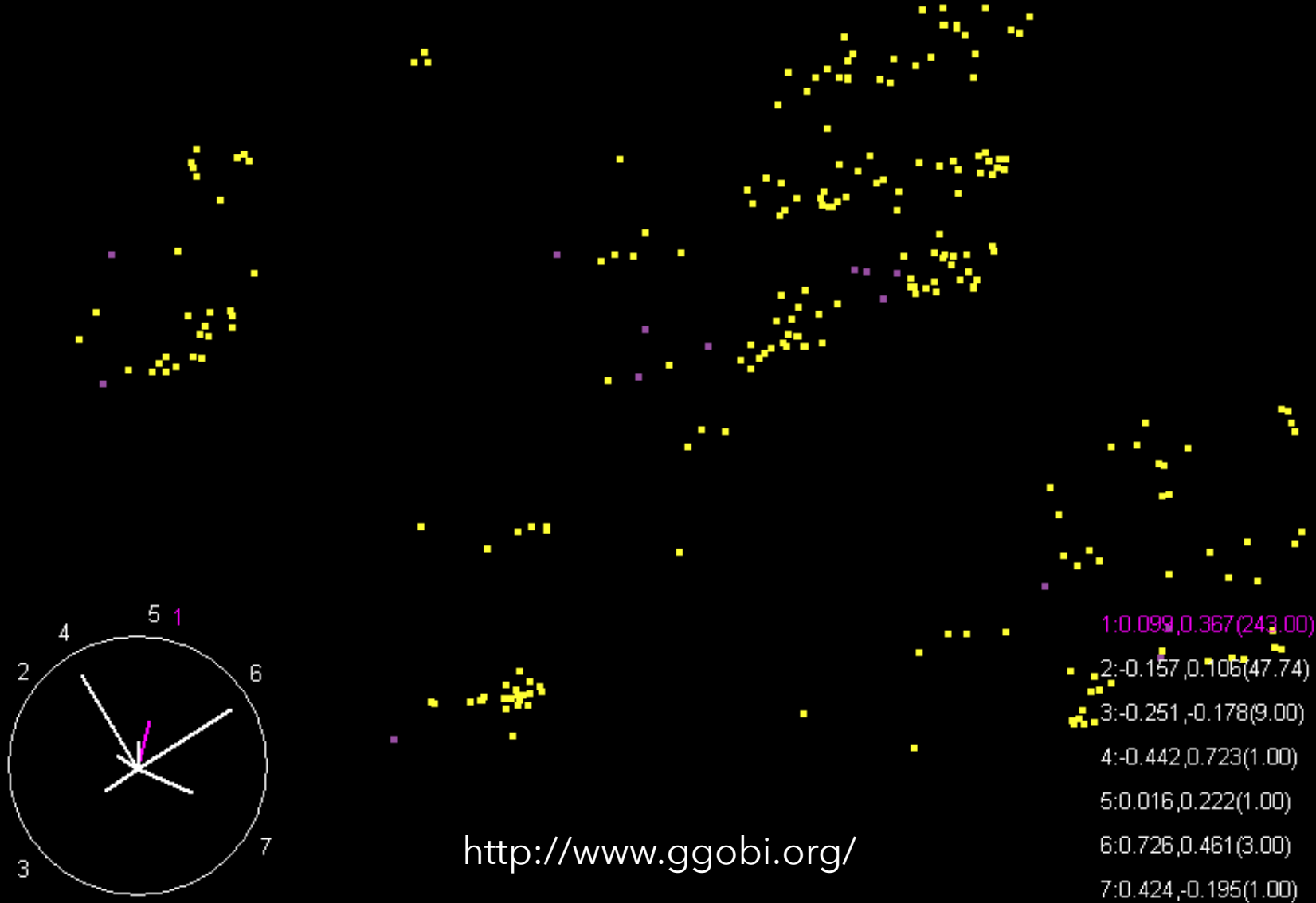


# Dimensionality Reduction

**Project from nD down to 2D.**

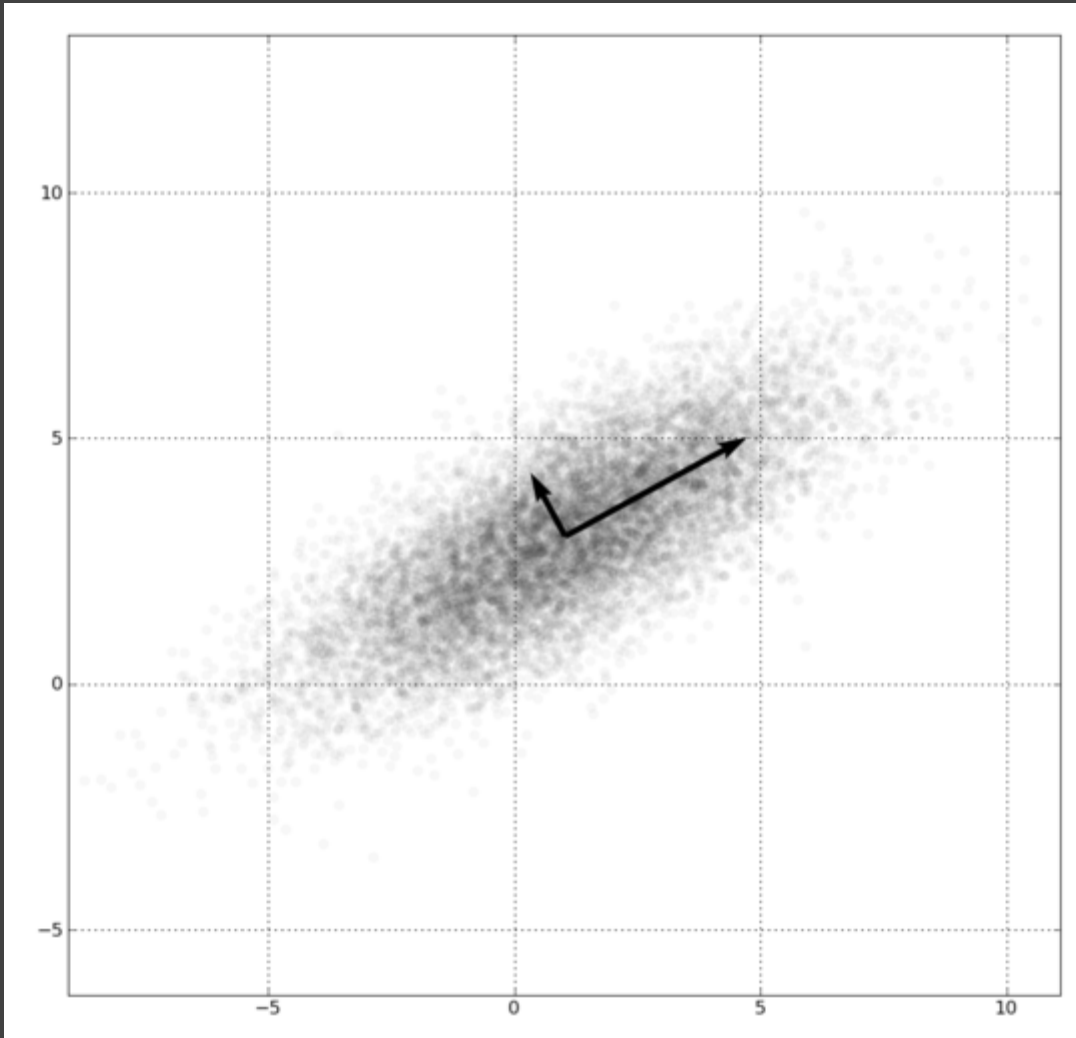
Spatial proximity  $\rightarrow$  data points are (approximately) more similar.

# Dimensionality Reduction



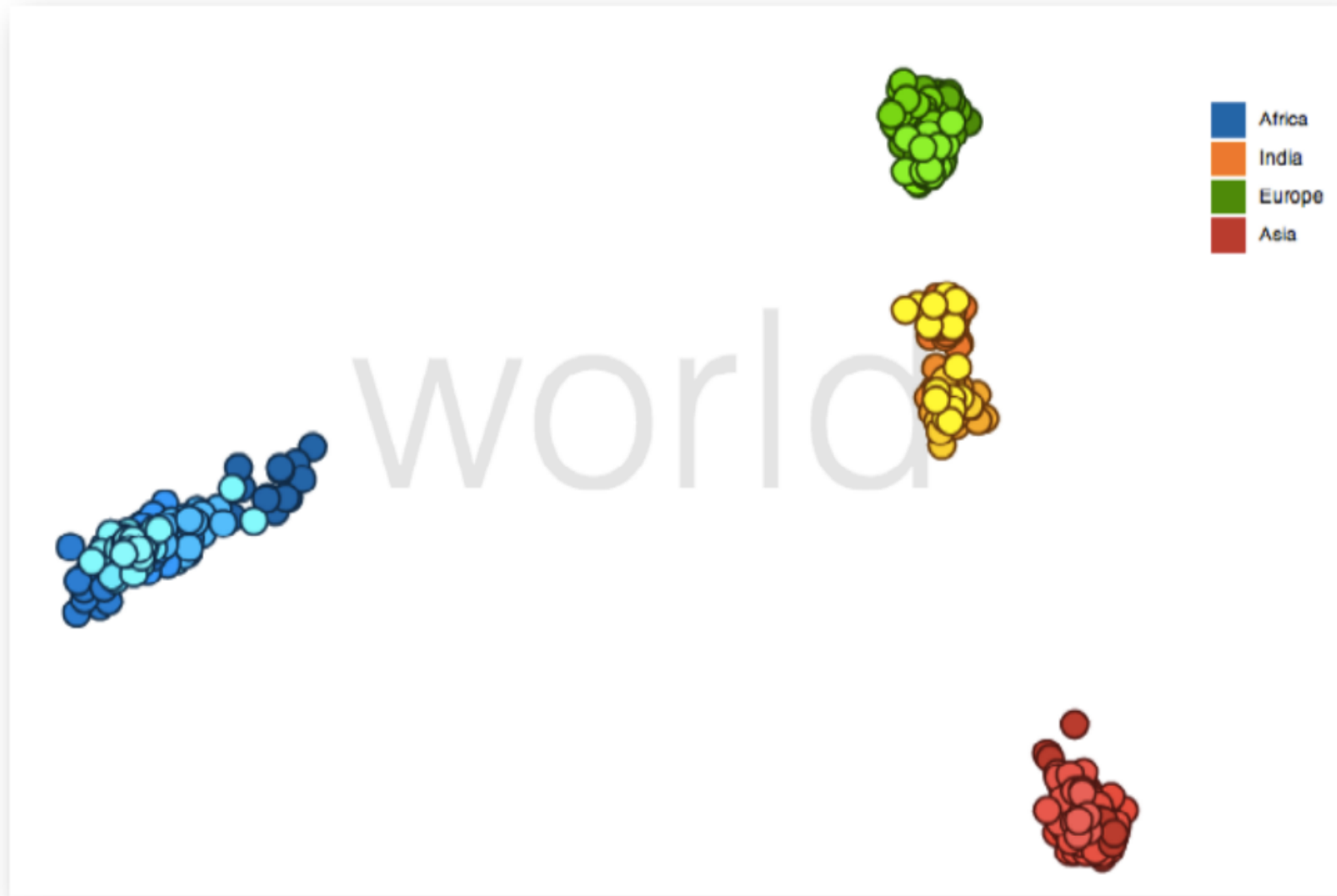
<http://www.ggobi.org/>

# Principal Components Analysis



1. Mean-center the data.
2. Find  $\perp$  basis vectors that maximize the data variance.
3. Plot the data using the top vectors.

# PCA of Genomes [Demiralp et al. '13]



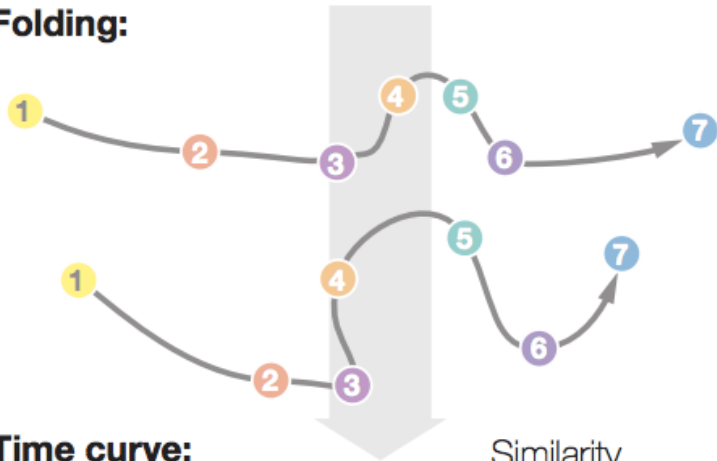
# Time Curves [Bach et al. '16]

## Timeline:



Circles are data cases with a time stamp.  
Similar colors indicate similar data cases.

## Folding:

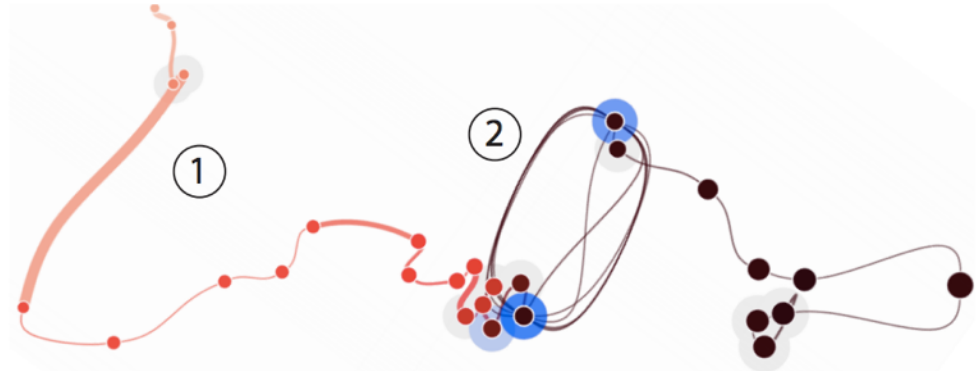


## Time curve:

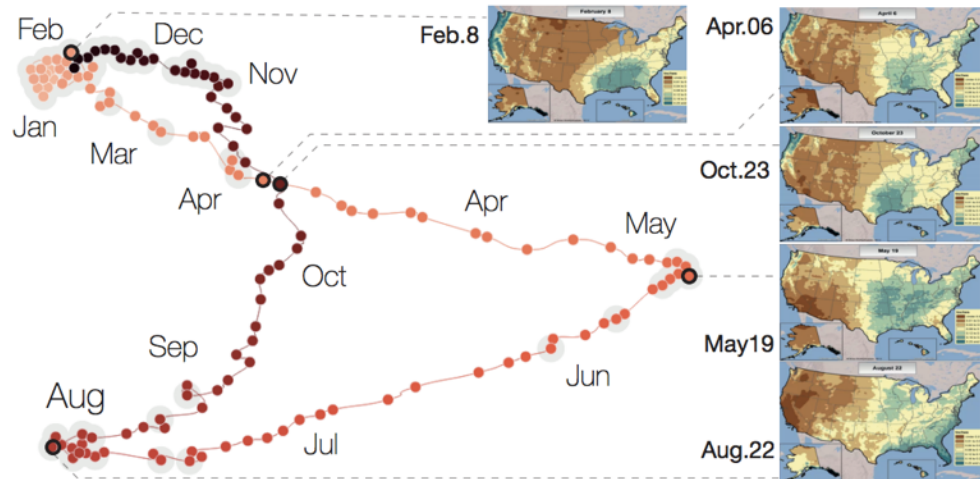


The temporal ordering of data cases is preserved.  
Spatial proximity now indicates similarity.

(a) Folding time



Wikipedia "Chocolate" Article



U.S. Precipitation over 1 Year

# Many Reduction Techniques!

Principal Components Analysis (PCA)

Multidimensional Scaling (MDS)

Locally Linear Embedding (LLE)

t-Dist. Stochastic Neighbor Embedding (t-SNE)

Isomap

Auto-Encoder Neural Networks

Topological methods

...

# Many Reduction Techniques!

*Principal Components Analysis (PCA)*

Multidimensional Scaling (MDS)

Locally Linear Embedding (LLE)

*t-Dist. Stochastic Neighbor Embedding (t-SNE)*

Isomap

Auto-Encoder Neural Networks

Topological methods

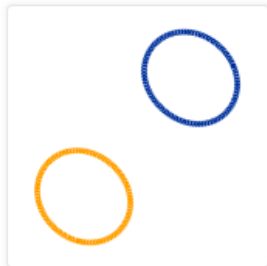
...



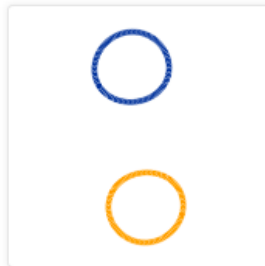
# Visualizing t-SNE [Wattenberg et al. '16]



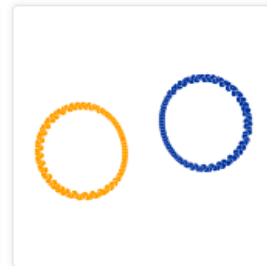
*Original*



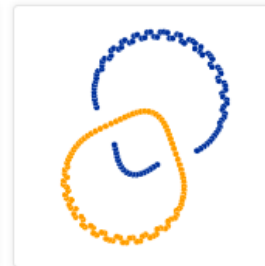
Perplexity: 2  
Step: 5,000



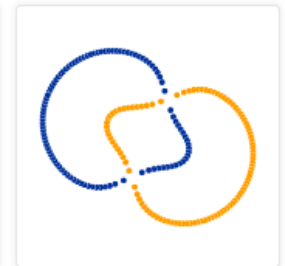
Perplexity: 5  
Step: 5,000



Perplexity: 30  
Step: 5,000



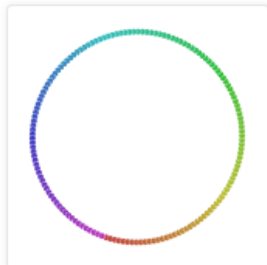
Perplexity: 50  
Step: 5,000



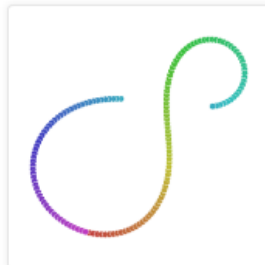
Perplexity: 100  
Step: 5,000



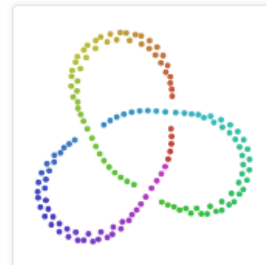
*Original*



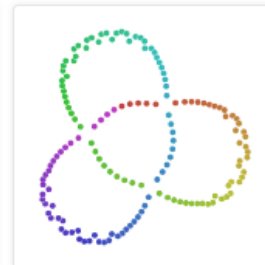
Perplexity: 2  
Step: 5,000



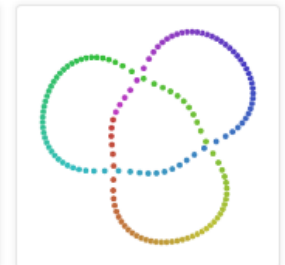
Perplexity: 5  
Step: 5,000



Perplexity: 30  
Step: 5,000



Perplexity: 50  
Step: 5,000



Perplexity: 100  
Step: 5,000

# Summary

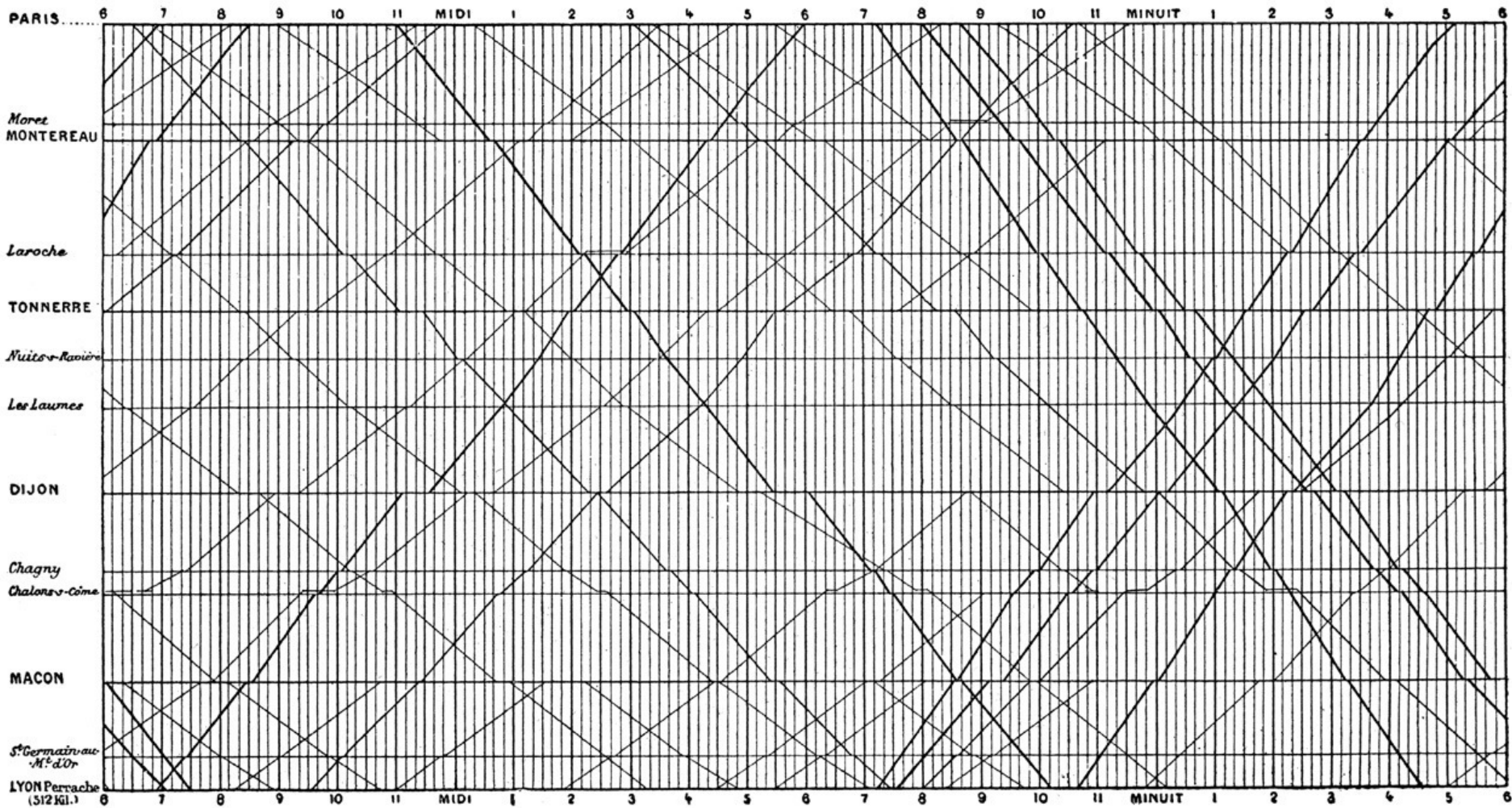
Spatial layout is the most important encoding  
... but you need to be in the right space.

Geometric properties of spatial transforms  
support geometric reasoning

Emphasize important information

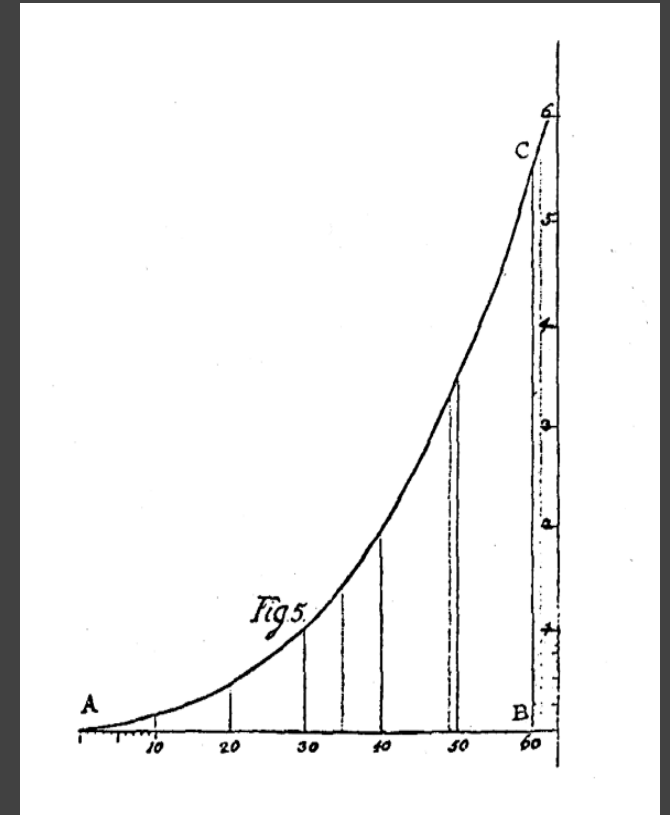
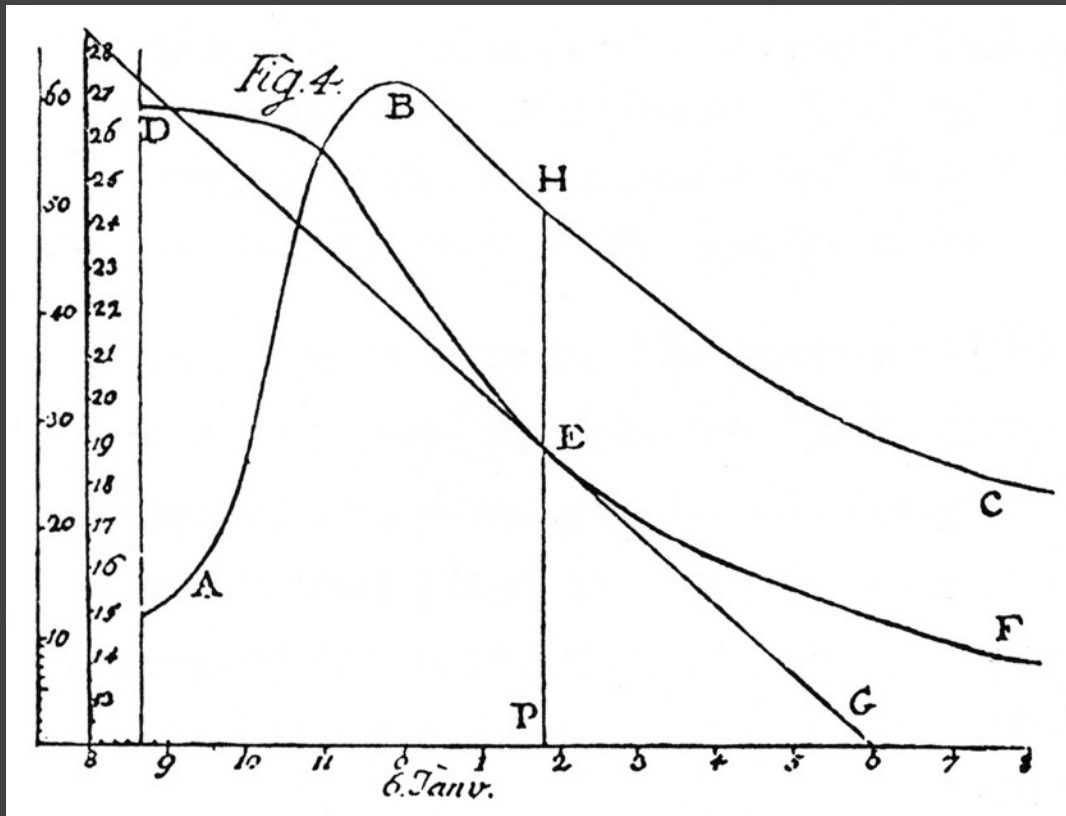
Consider *what* to show, not just *how*

# Graphical Calculation



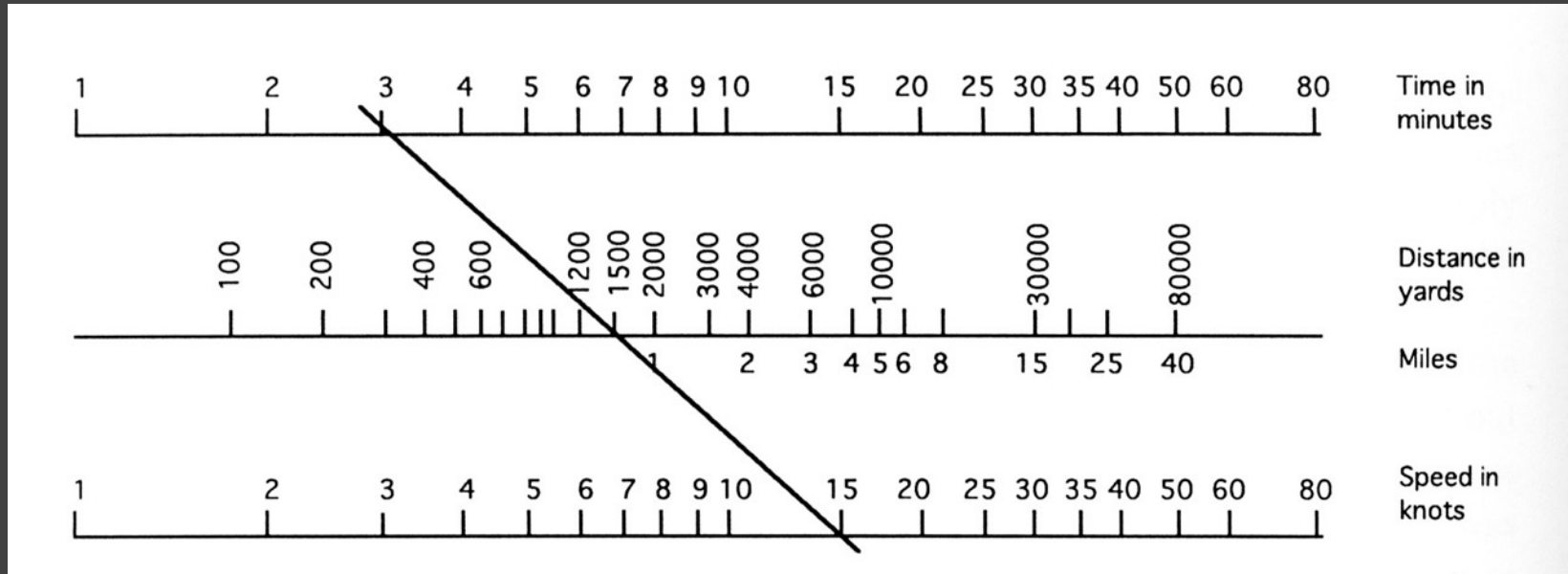
E. J. Marey, *La Méthode Graphique* (Paris, 1885), p. 20. The method is attributed to the French engineer, Ibry.

# Lambert's Graphical Construction



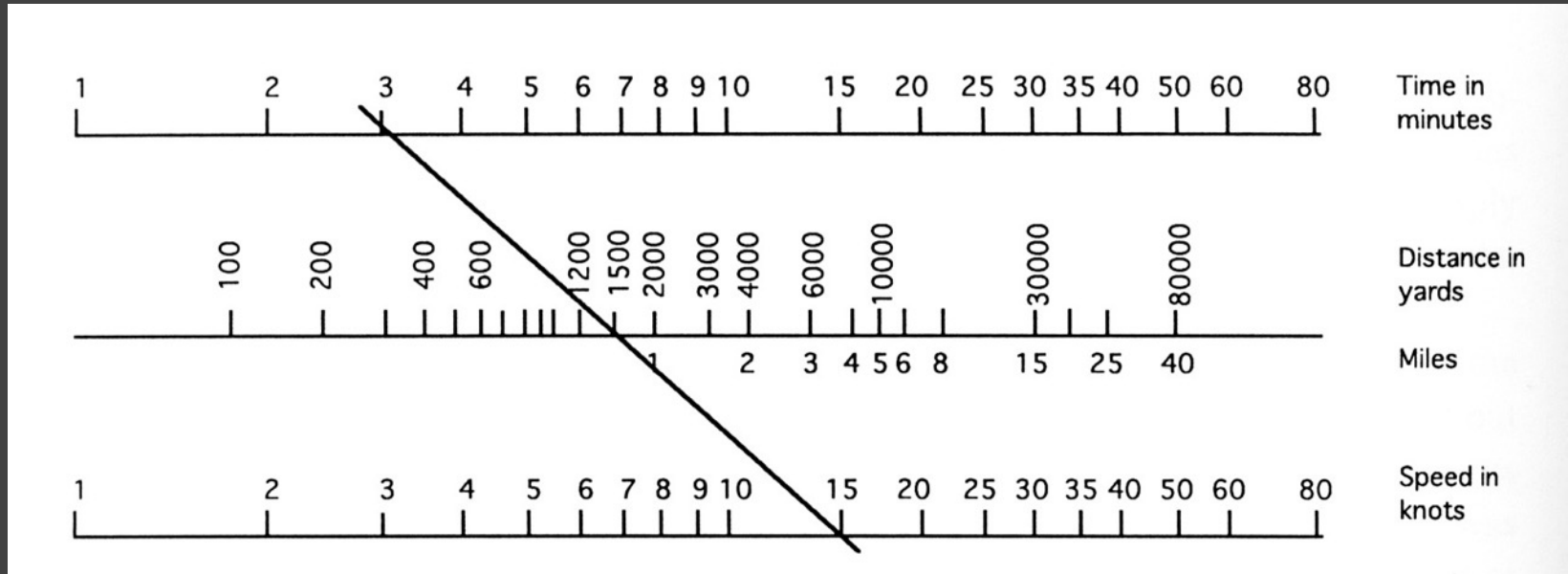
Johannes Lambert used graphs to study the rate of water evaporation as function of temperature [from Tufte 83].

# Nomograms



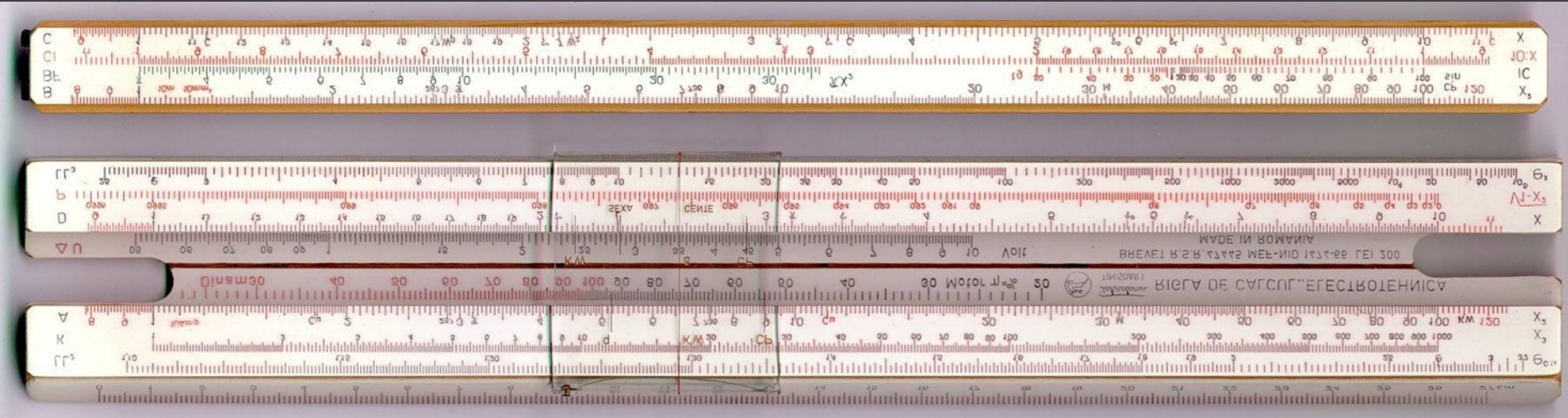
Sailing: The Rule of Three

# Nomograms



1. Compute in any direction; fix  $n-1$  params and read  $n$ th param
2. Illustrate sensitivity to perturbation of inputs
3. Clearly show domain of validity of computation

# Slide Rule



Model 1474-66 Electrotechnica 18 Scales

Tehnolemn Timisoara Slide Rule Archive  
<http://pubpages.unh.edu/~jwc/tehnolemn/>



# Chi-Square Test

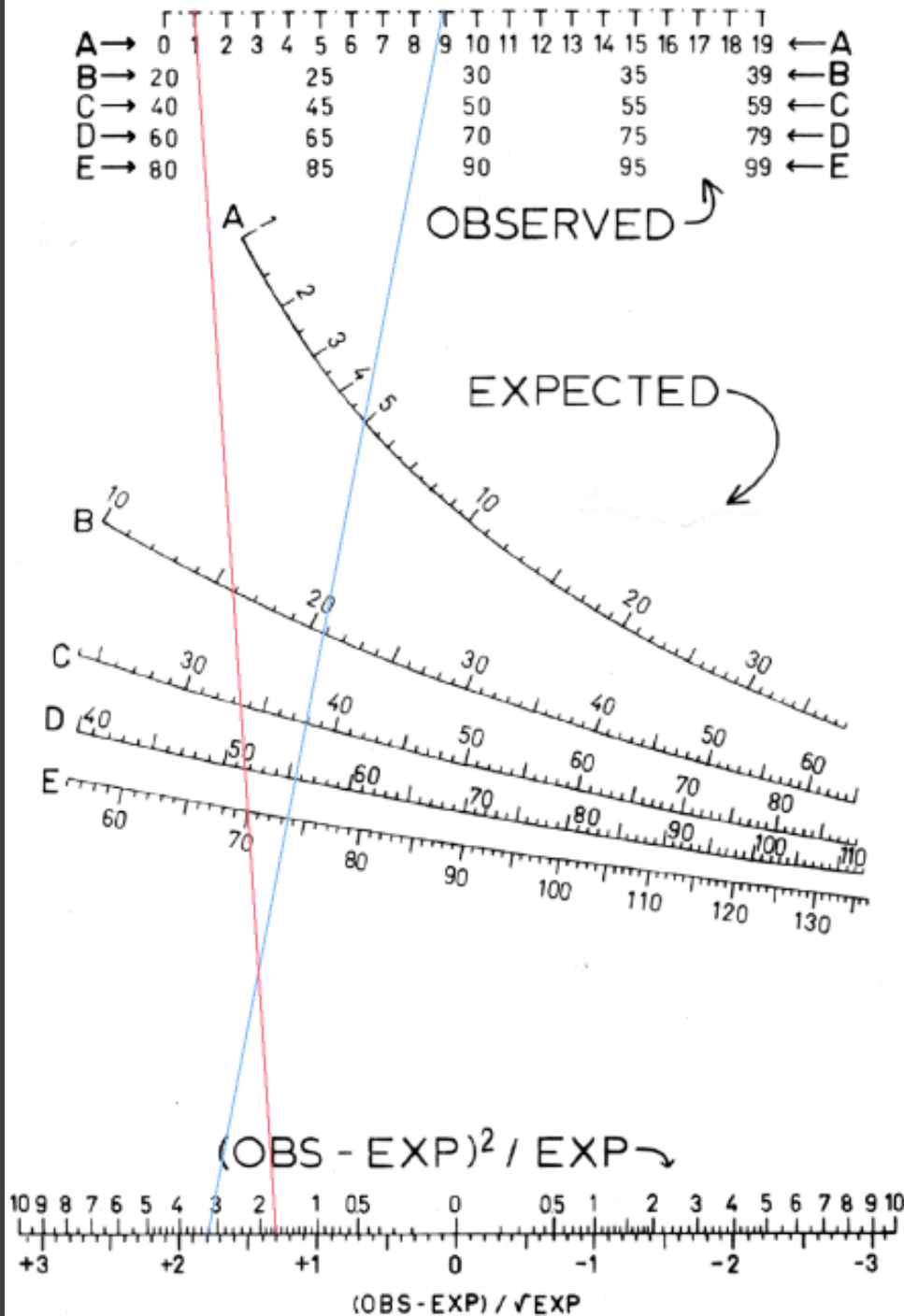
$$(Obs - Exp)^2 / Exp$$

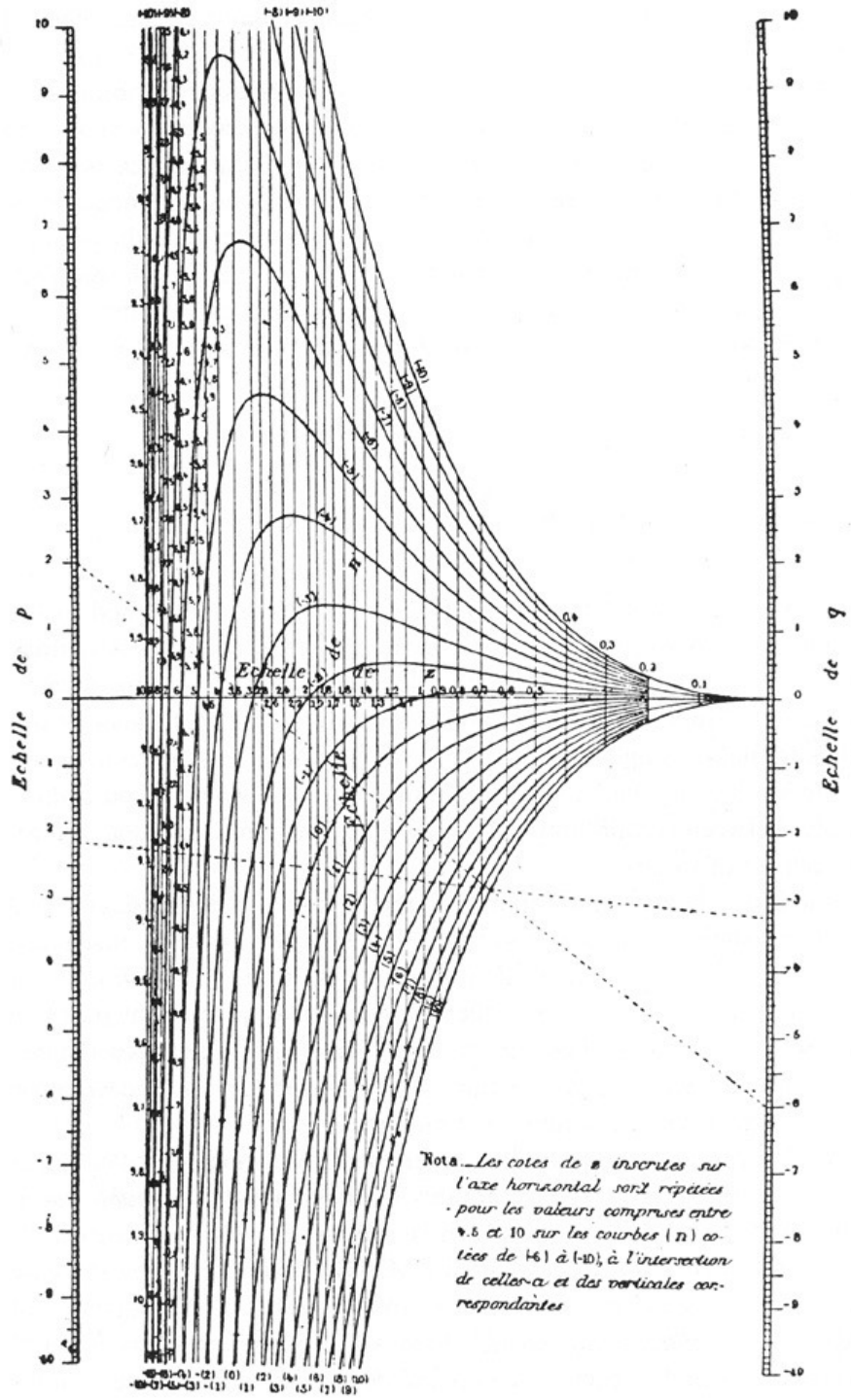
Blue line:

$$(9 - 5)^2 / 5 = 3.2$$

Red line:

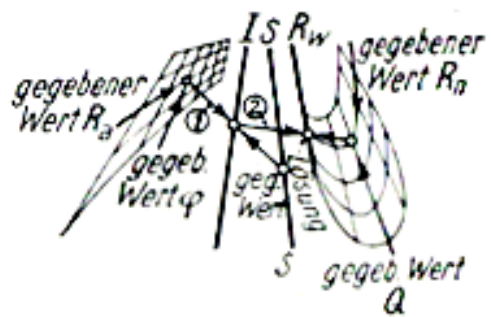
$$(81 - 70)^2 / 70 = 1.7$$



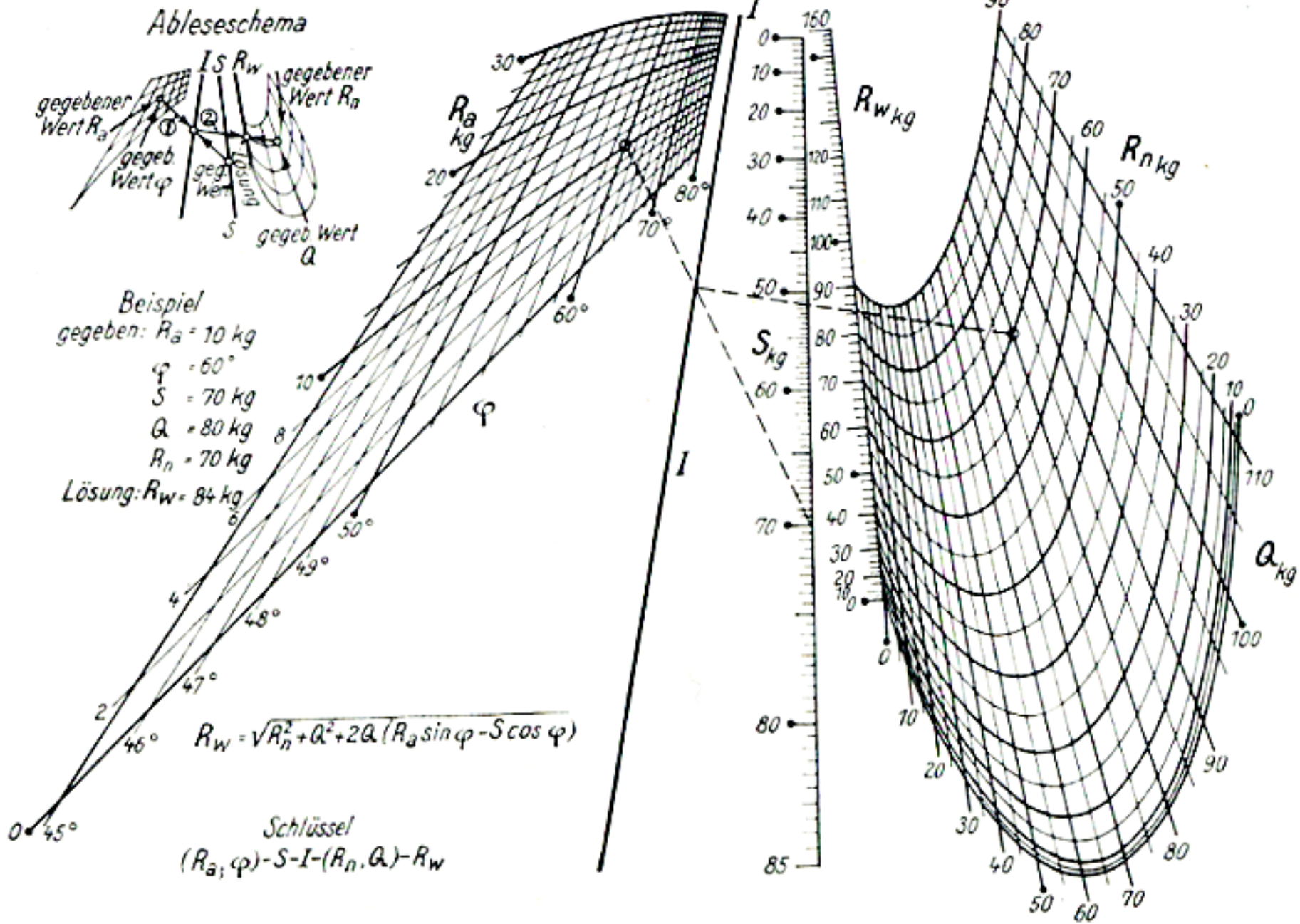


*Nota. — Les cotes de  $n$  inscrites sur l'axe horizontal sont répétées pour les valeurs comprises entre 4.5 et 10 sur les courbes (n) cotées de (4) à (10), à l'intersection de celles-ci et des verticales correspondantes*

### Ableseschema



Beispiel  
 gegeben:  $R_a = 10 \text{ kg}$   
 $\varphi = 60^\circ$   
 $S = 70 \text{ kg}$   
 $Q = 80 \text{ kg}$   
 $R_n = 70 \text{ kg}$   
 Lösung:  $R_w = 84 \text{ kg}$



$$R_w = \sqrt{R_n^2 + Q^2 + 2Q(R_a \sin \varphi - S \cos \varphi)}$$

Schlüssel  
 $(R_a, \varphi) - S - I - (R_n, Q) - R_w$