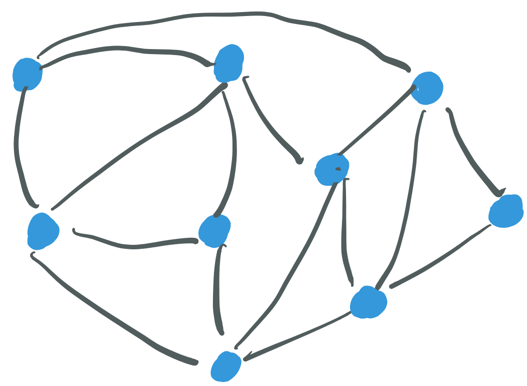
Homework #4 CSE 431 Winter 2018

(1) A graph $G=(V,E)$ is called *planar* if it can be drawn in the plane without edge crossings:



Define the language:

NONPLANAR

$= \{\langle G\rangle :$ G is an undirected graph that is *not* planar$\}$

Use Kuratowski's theorem (see Wikipedia) to prove that NONPLANAR $\in$ NP

Extra credit: Can you show that

PLANAR $= \{\langle G\rangle :$ G is a planar graph$\}$

is in NP? Use as simple a certificate as possible.
[Make sure the length of the certificate is polynomial in the input size!]

(2) Recall from class the language

$3COL = \{\langle G \rangle : G \text{ is a 3-colorable graph}\}$

(a) Prove that $3COL \leq_p 4COL$

(b) Consider the language:

TEN-3COL =
$\{\langle G \rangle :$ G is an undirected graph and has at least ten distinct proper 3-colorings$\}$

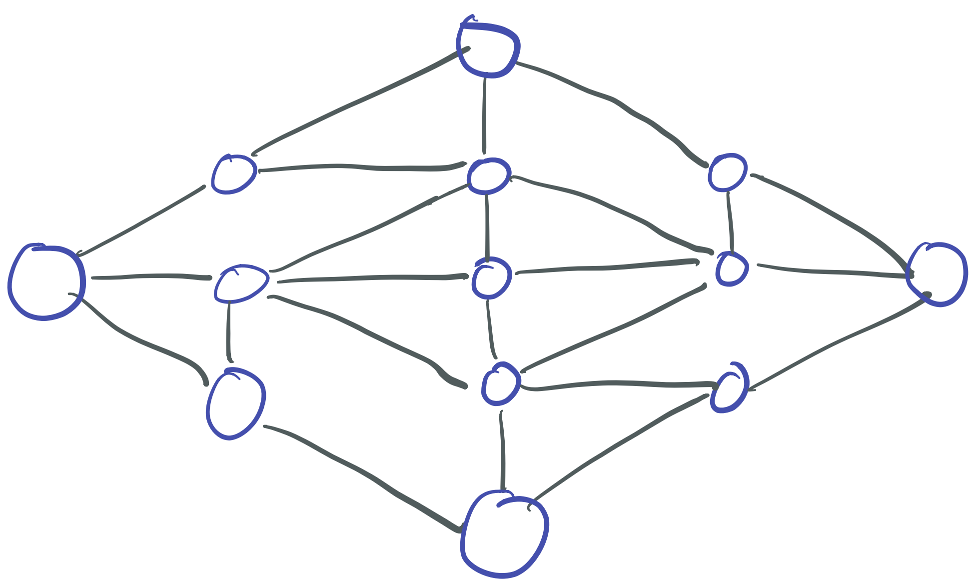
Prove that $3COL \leq_p$ TEN-3COL

Extra credit: Consider the language:

$$\text{PLANAR-3COL} = \{\langle G \rangle : G \text{ is a planar, 3-colorable graph}\}$$

Show that $3COL \leq_p$ PLANAR-3COL.

Hint: Use the following gadget to replace edge crossings:



(3) Show that if 2COL is NP-complete, then $P = NP$.

Extra credit: Two undirected graphs H and G are isomorphic if the vertices of H can be reordered to obtain G.

Show that if $P = NP$, there is a polynomial-time algorithm that, given isomorphic graphs H and G, outputs the ordering.