Reading assignment: Read Sections 7.1 and 7.2 of Sipser’s text. We will start Chapter 7 this week.

Problems:

1. A language \( B \) is called r.e.-complete if and only if (a) \( B \) is Turing-recognizable (equivalently, recursively enumerable) and (b) For all Turing-recognizable languages \( A \), \( A \leq_m B \). Prove that \( A_{TM} \) is r.e.-complete.

2. Show that \( A \) is decidable if and only if \( A \leq_m \{0^n1^n : n \geq 0\} \).

3. Let \( J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\} \). Show that neither \( J \) nor \( \overline{J} \) is Turing-recognizable.

4. Show that there is an undecidable language contained in \( 1^* \).

5. Which of the following problems are decidable? Justify each answer:
   
   (a) Given a Turing machine \( M \), does \( M \) accept 0101?
   
   (b) Given Turing machines \( M \) and \( N \), is \( L(N) \) the complement of \( L(M) \)?
   
   (c) Given a Turing machine \( M \), integers \( a \) and \( b \) and an input \( x \), does \( M \) run for more than \( a|x|^2 + b \) steps on input \( x \)?

6. (Bonus) Show that the following problem is undecidable: Given a Turing machine \( M \) and integers \( a \) and \( b \), does there exist an input \( x \) on which \( M \) runs for more than \( a|x|^2 + b \) steps on input \( x \)?

7. (Bonus) We showed previously that neither \( EQ_{TM} \) nor its complement is Turing-recognizable. Your problem is to show that, despite this, if you had a magic black box that decided \( A_{TM} \) that you could call repeatedly on different inputs, then you could decide \( EQ_{TM} \).