CSE 431 Spring 2015
Assignment #3

Due: Friday, April 24, 2015

Reading assignment: Read Chapter 5 of Sipser’s text. We will cover section 5.3 before we cover computation histories in section 5.1. Before Friday, begin reading Chapter 7.

Problems:

1. Suppose that \( A \subseteq \{ \langle M \rangle \mid M \text{ is a decider TM} \} \) and that \( A \) is Turing-recognizable. (That is, \( A \) only contains descriptions of TMs that are deciders but it might not contain all such descriptions.)
   Prove that there is a decidable language \( D \) such that \( L(M) \neq D \) for any \( M \) with \( \langle M \rangle \in A \). (Intuitively, this means that one couldn’t come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for \( A \).)

2. Let \( ODD_{TM} = \{ \langle M \rangle \mid M \text{ is a TM that accepts an odd number of strings} \} \). Show that \( ODD_{TM} \) is undecidable.

3. A useless state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

4. Show that for all Turing-recognizable problems \( A \), \( A \leq_{m} A_{TM} \).

5. Show that \( A \) is decidable if and only if \( A \leq_{m} \{0^n 1^n : n \geq 0 \} \).

6. (Bonus) Let \( \Gamma = \{0, 1, \text{blank}\} \) be the tape alphabet for all TMs in this problem. Define the busy beaver function \( BB : \mathbb{N} \rightarrow \mathbb{N} \) as follows: For each value of \( k \), consider all \( k \)-state TMs that halt when started with a blank tape. Let \( BB(k) \) be the maximum number of 1s that remain on the tape among all of these machines. Show that \( BB \) is not a computable function.