ASSIGNMENT 7. Due Tuesday, June 3rd, in class (or via email to cse431-staff@cs before class starts)

There are three problems + one extra credit.

1. The Japanese game Gomoku is played on a 19-by-19 board. There are two players: One has black stones and the other has white stones. The players alternately place stones until the board is full or someone wins. A player wins when they get five of their stones in a row, either horizontally, vertically, or diagonally. Consider the game Asym-Gomoku which is played on an $n \times n$ board according to the same rules.

Define the language

$$GM = \{ \langle P \rangle : P \text{ is a position in Asym-Gomoku where White has a winning strategy} \}$$

A position is simply a state of the game board, together with which player moves next. A winning strategy means that there is a way for White to force a win.

Prove that $GM \in \text{PSPACE}$.

2. Recall that $\text{EXPTIME} = \bigcup_k \text{TIME}(2^{n^k})$. Let $\text{NEXPTIME} = \bigcup_k \text{NTIME}(2^{n^k})$ be the class of languages decidable on a non-deterministic TM with exponential time. Your goal in this problem is to show that if $\text{EXPTIME} \neq \text{NEXPTIME}$, then $P \neq NP$.

To accomplish this, it will help to use the function

$$\text{pad} : \Sigma^* \times \mathbb{N} \rightarrow \Sigma^*\#^*$$

defined by $\text{pad}(s, l) = s\#^j$ where $j = \max(0, l - \text{length}(s))$. In other words, the function $\text{pad}$ adds enough $\#$ characters to the end of $s$ so that it has length exactly $l$ (and it just returns $s$ if $\text{length}(s) > l$).

For a language $A$ and a function $f : \mathbb{N} \rightarrow \mathbb{N}$, we define a new language

$$\text{pad}(A, f(n)) = \{ \text{pad}(s, f(\text{length}(s))) : s \in A \}$$

a) Prove that if $A \in \text{TIME}(n^{10})$, then $\text{pad}(A, n^2) \in \text{TIME}(n^5)$.

b) Prove that if $\text{EXPTIME} \neq \text{NEXPTIME}$, then $P \neq NP$. 

3. Let $A$ be the language of properly nested parentheses. For example, $(()$ and $()()()()$ are in $A$, but $)$ is not. Show that $A \in L$.

(Note that $L$ is the class of languages that can be decided in $O(\log n)$ space. We will see the definition on Tuesday, 5/27.)

**OPTIONAL PROBLEM (You may do this problem for extra credit, OR you can do it instead of the first three problems!)**

Consider the language

$$\text{ACYCLIC} = \{ (G) : \text{ } G \text{ is an undirected graph with no cycles } \}$$

(Note that $G$ could be disconnected.) Prove that $\text{ACYCLIC} \in L$. 