Lecture 11
Mapping Reducibility

Defn: A is *mapping reducible* to B (A \(\leq_m B\)) if there is computable function \(f\) such that \(w \in A \iff f(w) \in B\).

A special case of \(\leq_T\):
Call subr only once; its answer is *the* answer

Facts:
- \(A \leq_m B \& B \text{ decidable} \Rightarrow A\) is too
- \(A \leq_m B \& A \text{ undecidable} \Rightarrow B\) is too
- \(A \leq_m B \& B \leq_m C \Rightarrow A \leq_m C\)
Mapping Reducibility

Defn: A is *mapping reducible* to B (A ≤ₘ B) if there is computable function f such that w ∈ A ⇔ f(w) ∈ B

A special case of ≤ₜ:
Call subr only once; its answer is the answer

Theorem:
A ≤ₘ B & B decidable (recognizable) ⇒ A is too
A ≤ₘ B & A undecidable (unrecognizable) ⇒ B is too
A ≤ₘ B & B ≤ₘ C ⇒ A ≤ₘ C

*Most reductions we’ve seen were actually ≤ₘ reductions.*
\[ f(w) \in B \]

\[ w \in A \iff f(w) \in B \]
Mapping Reducibility

Defn: A is *mapping reducible* to B (A \(\leq_m B\)) if there is computable function \(f\) such that \(w \in A \iff f(w) \in B\)

Theorem:
- \(A \leq_m B \& B\) decidable \((\text{recognizable})\) \(\Rightarrow A\) is too
  - *pf:* To decide (recognize) \(w\) in \(A\) compute \(f(w)\), then use decider (recognizer, resp) for \(B\) on \(f(w)\).
- \(A \leq_m B \& A\) undecidable \((\text{unrecognizable})\) \(\Rightarrow B\) is too
  - *pf:* Contrapositive
- \(A \leq_m B \& B \leq_m C \Rightarrow A \leq_m C\)
  - *pf:* If \(f\) for \(A \rightarrow B\), \(g\) for \(B \rightarrow C\); then \(w \in A \iff g(f(w)) \in C\)
$\text{A}_{\text{TM}} \ (\leq_T \text{ vs } \leq_m) \text{ HALT}_{\text{TM}}$

\[
f(<M,w>) = <M',w>
\]

From Lecture 07
Other Examples of $\leq_m$

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\begin{align*}
A_{TM} \leq_m & \text{REGULAR}_{TM} & f(<M,w>) = <M_2> \\
\text{Build } M_2 \text{ so } L(M_2) = \Sigma^* / \{0^n | n\}, \text{ as } M \text{ accept/rejects } w \\
\text{EMPTY}_{TM} \leq_m & \text{EQ}_{TM} & f(<M>) = <M, M_{\text{reject}}> \\
L(M_{\text{reject}}) = \emptyset, \text{ so equiv to } M \text{ iff } L(M) = \emptyset \\
A_{TM} \leq_m & \text{MPCP} \\
\text{MPCP} \leq_m & \text{PCP} \\
\end{align*}
\]

5.2

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\begin{align*}
A_{TM} \leq_m & \text{EMPTY}_{TM} & f(<M,w>) = <M_1> \\
\text{Build } M_1 \text{ so } L(M_1) = \{w\} / \emptyset, \text{ as } M \text{ accept/rejects } w
\end{align*}
\]