1. 3.15(b)

2. 3.16(b) Prove it two different ways: first using an ordinary (deterministic) TM, then using a nondeterministic TM. [You may do 3.15(b) using either model.]

3. 4.7

4. Let $\mathcal{F} = \{f : \mathbb{N} \rightarrow \mathbb{N}\}$, and $\mathcal{F}_2 = \{b : \mathbb{N} \rightarrow \{0, 1\}\}$, i.e., the set of all functions mapping natural numbers to natural numbers and the set of all $\{0, 1\}$-valued functions on $\mathbb{N}$, resp. Show that both sets are uncountably infinite.

Extra credit: Show that both have the same cardinality as the reals.

5. Let $L$ be a language. Prove
   (a) $L$ is recognizable if and only if there is a decidable language $D$ such that
   $$L = \{x \mid \exists y \text{ s.t. } \langle x, y \rangle \in D\}.$$ 
   (b) $L$ is co-recognizable if and only if there is a decidable language $D$ such that
   $$L = \{x \mid \forall y \text{ s.t. } \langle x, y \rangle \in D\}.$$ 

6. (a) 4.28
   (b) Read definition 7.1 (“time complexity”). Suppose the set $A$ in 4.28 included TMs deciding every language decidable in time $n^2$, say. What can you say about the time complexity of the decidable language $D$ built from that $A$?
   (c) Extra Credit: Show that such a set $A$ is Turing enumerable, i.e., it is possible to enumerate a series of TMs, each of which is a decider, and every language decidable in time $n^2$ will be decided by at least one of the machines in the list.