Reading assignment: Read Chapter 5 of Sipser’s text. We will cover section 5.3 before we cover computation histories in section 5.1.

Problems:

1. Suppose that $A \subseteq \{\langle M \rangle \mid M$ is a decider TM$\}$ and that $A$ is Turing-recognizable. Prove that there is a decidable language $D$ such that $D \neq L(M)$ for any $M$ with $\langle M \rangle \in A$. (Hint: You may find it helpful to consider an enumerator for $A$.)

   (In general it seems hard to tell if a TM is a decider but one might guess that there could be some easy-to-recognize special format for a restricted class of TMs such that (1) any TM in the format must be a decider, and (2) for every decider there is an equivalent TM in this format. The answer to this question rules this out.)

2. Let $L = \{\langle M, w \rangle \mid M$ attempts to move left while on the left end of its tape during its computation on input $w\}$. Prove that $L$ is undecidable.

3. Let $R = \{\langle M, w \rangle \mid M$ attempts to move left at some step of its computation on input $w\}$. Prove that $R$ is decidable.

4. For a string $w \in \{0, 1\}^*$, let the $1$’s-complement of $w$, $\overline{w}$, be the string obtained by replacing each 0 of $w$ by a 1 and each 1 of $w$ by a 0. Let $C = \{\langle M \rangle \mid M$ is a TM with input alphabet $\{0, 1\}$ such that, for every $w \in \{0, 1\}^*$, $M$ accepts $w$ if and only if $M$ accepts $\overline{w}\}$. Show that $C$ is undecidable.

5. Show that $A$ is Turing-recognizable if and only if $A \leq_m A_{TM}$.

6. Show that $A$ is decidable if and only if $A \leq_m 0^*1^*$.

7. (Extra credit) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the busy beaver function $BB : \mathbb{N} \rightarrow \mathbb{N}$ as follows: For each value of $k$, consider all $k$-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that $BB$ is not a computable function.