1. Let $S$ be an infinite, Turing-recognizable language. Prove that $S$ has an infinite, decidable subset. (Hint: You might use problem 4 of the first homework.)

2. Define the language

$$A = \left\{ \langle M \rangle : M \text{ is an NFA that only accepts palindromes in } \{0,1\}^* \right\}.$$  

(Note that for $\langle M \rangle$ to be in $A$, it does not need to accept all palindromes, it just cannot accept any non-palindromes.)

Prove that $A$ is decidable.

3. (Problem 4.21 in Sipser.) Say that an NFA is ambiguous if its accepts some string along two different computation branches. Let $\text{AMBIG}_{\text{NFA}} = \{ \langle N \rangle : N \text{ is an ambiguous NFA} \}$. Show that $\text{AMBIG}_{\text{NFA}}$ is decidable.

[Hint: One elegant way to solve this problem is to construct a suitable DFA and then run $E_{\text{DFA}}$ on it ($E_{\text{DFA}}$ is defined on page 168 in Chapter 4).]

4. A complex number is algebraic if it is the root of a non-zero polynomial with integer coefficients. Show that the set of algebraic numbers countable.

5. (Extra credit) Show that single-tape Turing machines that cannot write on the portion of the tape containing the input can only recognize regular languages.