Reading assignment: Finish reading Chapter 5 of Sipser’s text. (You may also want to skim section 6.3 of the text.)

Problems:

1. (10 points) Show that there is a undecidable language contained in $1^*$. 

2. (10 points) Let $S = \{ \langle M \rangle \mid L(M) = \{ \langle M \rangle \} \}$ and $M$ is a Turing machine. Prove that neither $S$ nor $\overline{S}$ is Turing-recognizable.

3. Which of the following problems are decidable? Justify each answer:
   
   (a) (10 points) Given Turing machines $M$ and $N$, is $L(N)$ the complement of $L(M)$?
   
   (b) (10 points) Given a Turing machine $M$, integers $a$ and $b$, and input $x$, does $M$ run for more than $a|x|^2 + b$ steps on input $x$?
   
   (c) (20 points) Given a program $P$ written in Java, or C, or (insert your favorite programming language) that does not read any input but is executed with no bound on the size of integers, does $P$ ever attempt to index an array outside its allocated array bounds.


5. (Extra Credit) Show that the following problem is undecidable: Given a Turing machine $M$ and integers $a$ and $b$, does there exist an input $x$ on which $M$ runs for more than $a|x|^2 + b$ steps on input $x$?

6. (Extra Credit) Rice’s Theorem shows that for every ‘non-trivial’ property $\mathcal{P}$ of languages,

$$\mathcal{P}_{TM} = \{ \langle M \rangle \mid L(M) \text{ has property } \mathcal{P} \}$$

is undecidable where by ‘$\mathcal{P}$ is non-trivial’ we mean that $\mathcal{P}$ contains some but not all Turing-recognizable languages. Some of these $\mathcal{P}_{TM}$ are not only undecidable, they are also not Turing-recognizable:

Show that if there is some infinite Turing-recognizable language $L$ that has property $\mathcal{P}$ but none of the finite subsets of $L$ have property $\mathcal{P}$ then $\mathcal{P}_{TM}$ is not Turing recognizable.