CSE 431 Spring 2007
Assignment #2

Due: Friday, April 13, 2006

Reading assignment: Read Chapter 4 of Sipser’s text.

Problems:

1. Show that both the decidable languages and the Turing recognizable languages are closed under the concatenation, intersection, and star operations.

2. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)

3. Use the result of question 2 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

4. Let $INFINITE_{PDA} = \{ \langle M \rangle \mid M$ is a PDA and $L(M)$ is an infinite language}. Show that $INFINITE_{PDA}$ is decidable.

5. Show that the set of complex numbers,

$$QUADRATIC-ROOT = \{ x \in \mathbb{C} \mid \text{there are integers } a \neq 0, b, \text{ and } c \text{ such that } ax^2 + bx + c = 0 \}$$

is countable.

6. (Bonus) Let $C$ be a language. Prove that $C$ is Turing-recognizable iff there is a decidable language $D$ such that $C = \{ x \mid \exists y \text{ such that } \langle x, y \rangle \in D \}$.