CSE 431 Spring 2006
Assignment #5
Due: Friday, May 12, 2006

Reading assignment: Read Sections 7.1-7.3 of Sipser’s text.

Problems:

1. Sipser’s text: Problem 7.6 (both editions).


3. Sipser’s text: Problem 7.7 (both editions).

4. Sipser’s text: Problem 7.11 (both editions).

5. All the computational problems we have described are defined as languages, i.e. yes/no questions. This problem gives an idea as to why that gives us enough information.

   Given a function \( f : \{0, 1\}^* \rightarrow \{0, 1\}^* \) we say that \( f \) is computable in polynomial time iff there is some TM computing \( f \) whose running time is \( O(n^k) \) for some \( k \). We say that \( f \) is length-preserving if \( |f(x)| = |x| \) for every input \( x \). Define the language \( L_f = \{ \langle x, i \rangle \mid \text{the } i\text{-th bit of } f(x) \text{ is 1} \} \).

   (a) Show that if \( f \) is polynomial-time computable then \( L_f \in P \).

   (b) Show that if \( f \) is length-preserving and \( L_f \in P \) then \( f \) is polynomial-time computable.

6. (Bonus*) In this question you will show that if an ordinary 1-tape TM \( M \) has running time \( o(n \log n) \) then \( L(M) \) must be regular.

   A crossing-sequence is the sequence of states on which, and directions from which, a fixed cell is entered during the course of a computation.

   (a) Show that if the lengths of all the crossing sequences for a TM are bounded by some constant \( k \) (independent of the input length) then \( L(M) \) is regular by building an NFA to recognize \( L(M) \).

   (b) Use a pigeonhole argument to argue that any TM running in \( o(n \log n) \) time on any sufficiently long input must have a repeated crossing sequence on two cells that contain the input.

   (c) Show that if a 1-tape TM has crossing sequences of arbitrarily large size then it cannot take run in \( o(n \log n) \) time. To do this, consider a minimal-length string that produces a long crossing sequence and use part (b) to derive a contradiction by splicing out a piece of the input string using the repeated crossing sequence.

   (d) Finally, put the pieces together to produce the claimed result.