Reading assignment: Read Chapter 5 of Sipser’s text. We will cover section 5.3 before we cover computation histories in section 5.1.

Problems:

1. Suppose that $A \subseteq \{\langle M \rangle \mid M$ is a decider TM$\}$ and that $A$ is Turing-recognizable. (That is, $A$ only contains descriptions of TMs that are deciders but it might not accept all such descriptions.)
   Prove that there is a decidable language $D$ such that $L(M) \neq D$ for any $M$ with $\langle M \rangle \in A$.
   (Intuitively, this means that one couldn’t come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for $A$.)

2. Let $T = \{\langle M \rangle \mid M$ is a TM that accepts $w^R$ whenever it accepts $w\}$. Show that $T$ is undecidable.


4. Show that for all Turing-recognizable problems $A$, $A \leq_m A_{TM}$.


6. (Bonus) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the busy beaver function $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of $k$, consider all $k$-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that $BB$ is not a computable function.