Instructions: The exam is on 80 points. You have 50 minutes to answer all the questions. You are allowed to use one typewritten (at least 11pt font) notes sheet.

1. Answer True or False for each of the following questions. You do not need to justify your answers. (3 × 5 = 15 points)
   (a) $A_{TM} \leq_m E_{TM}$
   (b) $A_{TM} \leq_m \overline{E_{TM}}$
   (c) Every language is either Turing recognizable or co-Turing-recognizable.
   (d) $\{\langle M \rangle \mid M$ is a Turing machine and $L(M)$ is Turing-recognizable$\}$ is undecidable.
   (e) $\text{Th}(\mathbb{N})$, the set of false number-theoretic sentences, is Turing-recognizable.

2. (20 points) Define the language $SELFACCEPT = \{\langle M \rangle \mid M$ is a Turing machine and $\langle M \rangle \in L(M)\}$. Prove that $SELFACCEPT$ is undecidable.

3. (20 points) Define a 2WAY-PDA to be a deterministic pushdown automaton (DPDA) that has a single stack but can move its input head in both directions on the input tape. Assume that the 2WAY-PDA can also detect when it has reached either end of the input tape. A 2WAY-PDA accepts its input by entering an accept state. Prove that the language $E_{2WAY-PDA} = \{\langle P \rangle \mid P$ is a 2WAY-PDA which accepts no string$\}$ is not Turing-recognizable.

4. (25 points) If language $L$ is defined over alphabet $\Sigma$ then the set of prefixes of strings in $L$ is defined formally as:
   $$\text{Prefix}(L) = \{x \mid \text{there exists a } y \in \Sigma^* \text{ such that } xy \in L\}.$$
   (a) Show that if $L$ is Turing-recognizable then $\text{Prefix}(L)$ is Turing-recognizable.
   (b) Intuitively, why doesn’t the method you used for part (a) also show that if $L$ is decidable then $\text{Prefix}(L)$ is decidable?
   (c) Actually come up with an example where you can prove that $L$ is decidable but $\text{Prefix}(L)$ is not decidable.