CSE 431: Introduction to Theory of Computation Spring 2005

Sample Midterm

Instructions: The exam is on 80 points. You have 50 minutes to answer all the questions. You are allowed to use one typewritten (at least 11pt font) notes sheet.

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1. Answer True of False for each of the following questions. You do not need to justify your answers. ($3 \times 5 = 15$ points)

   (a) $A_{\text{TM}} \leq_m E_{\text{TM}}$

   (b) $A_{\text{TM}} \leq_m \overline{E_{\text{TM}}}$

   (c) Every language is either Turing recognizable or co-Turing-recognizable.

   (d) $\{\langle M \rangle \mid M$ is a Turing machine and $L(M)$ is Turing-recognizable$\}$ is undecidable.

   (e) $\overline{\text{Th}(\mathbb{N})}$, the set of false number-theoretic sentences, is Turing-recognizable.

2. (20 points) Define the language $SELFACCEPT = \{\langle M \rangle \mid M$ is a Turing machine and $\langle M \rangle \in L(M)\}$. Prove that $SELFACCEPT$ is undecidable.

3. (20 points) Define a 2WAY-PDA to be a **deterministic** pushdown automaton (DPDA) that has a single stack but can move its input head in *both* directions on the input tape. Assume that the 2WAY-PDA can also detect when it has reached either end of the input tape. A 2WAY-PDA accepts its input by entering an accept state. Prove that the language $E_{\text{2WAY-PDA}} = \{\langle P \rangle \mid P$ is a 2WAY-PDA which accepts no string$\}$ is not Turing-recognizable.

4. (25 points) If language L is defined over alphabet Σ then the set of prefixes of strings in L is defined formally as:

$$\text{Prefix}(L) = \{x | \text{ there exists a } y \in \Sigma^* \text{ such that } xy \in L\} .$$

   (a) Show that if L is Turing-recognizable then Prefix(L) is Turing-recognizable.

   (b) Intuitively, why doesn't the method you used for part (a) also show that if L is decidable then Prefix(L) is decidable?

   (c) Actually come up with an example where you can prove that L is decidable but Prefix(L) is not decidable.