CSE 431: Introduction to Theory of Computation Spring 2005

Problem Set 8

Due Friday, June 3, 2005, in class

Reading assignment: Sipser's book, Chapter 8.

Instructions: Same as for Problem set 1.

Each question is worth 10 points. Please be as clear and concise as possible in your arguments and answers. There is no optional problem on this problem set.

---

1. Problem 8.16, Sipser's book. (EQ_{REX} belongs to PSPACE.)

2. Problem 8.11, Sipser's book. (Properly nested parantheses is in Logspace)

3. We know that 3SAT is NP-complete, and 2SAT has a polynomial time algorithm. The precise complexity of 2SAT is in fact the class NL, i.e., 2SAT is NL-complete. In this problem, your task is to prove the weaker result that 2SAT is NL-hard.

   (Hint: Give a logspace reduction from PATH to $\overline{\mathrm{2SAT}}$, and then use NL = coNL. For the reduction, think of a directed edge connecting x to y as imposing the constraint $x \Rightarrow y$.)

4. In the last problem set, you showed that the language *3COLOR* is NP-complete. We now consider the language

   $$\mathit{2COLOR} = \{\langle G \rangle \mid G \text{ is an undirected graph that is 2-colorable}\} .$$

   Give a logspace reduction from *2COLOR* to the undirected connectivity problem, specifically the language $\overline{\mathit{UPATH}}$, where we define

   $$\mathit{UPATH} = \{\langle G, s, t \rangle \mid G \text{ is an undirected graph with a path between } s \text{ and } t\} .$$

   A very recent breakthrough result from Fall 2004 (and just published in a conference in May 2005) shows that $\mathit{UPATH} \in \mathrm{L}$. Using this together with your reduction, conclude that $\mathit{2COLOR} \in \mathrm{L}$.

   Hint: You can use the fact that a graph is 2-colorable iff it does not have any cycles of odd length.

   (Note that at the same time last year, **no one** *knew whether or not* $\mathit{2COLOR} \in \mathrm{L}$*!)*