1. (a) Show that the Post Correspondence Problem is decidable over a unary alphabet, i.e.,
   over the alphabet $\Sigma = \{1\}$.
   (b) Show that the Post Correspondence Problem is undecidable over a binary alphabet, i.e.,
   over the alphabet $\Sigma = \{0, 1\}$.

2. (a) Problem 5.19, Sipser’s book (Ambiguity of CFGs is undecidable)
   (b) Use the approach used in part (a) above to give a proof different from the one given in
   the previous problem set of the undecidability of $\text{DISJOINT}_{\text{CFG}}$ defined as:
   $$\text{DISJOINT}_{\text{CFG}} = \{\langle G_1, G_2 \rangle | G_1, G_2 \text{ are context-free grammars and } L(G_1) \cap L(G_2) = \emptyset \}.$$

3. Let
   $$\text{MODEXP} = \{\langle a, b, c, p \rangle | a, b, c, \text{ and } p \text{ are binary integers such that } a^b \equiv c \pmod{p} \}.$$
   Show that $\text{MODEXP} \in \text{P}$.

4. Show that $\text{P}$ is closed under the star operation. (Hint: Use dynamic programming.)

5. * (Optional Problem, Tricky!) Let $f : \mathbb{N} \to \mathbb{N}$ be any function where $f(n) = o(n \log n)$.
   Show that $\text{TIME}(f(n))$ contains only regular languages.
   (Suggestion: The key concept that aids showing the above is that of a crossing sequence.
   When a TM is run on an input, the crossing sequence at a given cell is the sequence of states
   that the machine enters at that cell as the computation progresses. Now, expand on the
   following two high-level ideas concerning crossing sequences. First, show that for a particular
   TM, if all crossing sequences on all inputs are of a fixed length $\ell$ or less, once can simulate the
   TM by an NFA. Second, show that a TM with unbounded crossing sequence length cannot
   run in $o(n \log n)$ time. For this, use a counting/pigeonholing argument to deduce repetition
   of crossing sequences at multiple places and then get a contradiction by “splicing” a minimal
   length input to a smaller string on which the TM has identical behavior.)