CSE 431: Introduction to Theory of Computation Spring 2005

PROBLEM SET 3

Due Friday, April 22, 2005, in class

Reading assignment: Sipser's book, Chapters 4 and 5.

Instructions: Same as for Problem set 1.

Each question is worth 10 points. Please be as clear and concise as possible in your arguments and answers. The optional problem is for extra credit.

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1. Which of the following problems about Turing machines are decidable and which are not? Briefly justify your answers.

   (a) To determine, given a Turing machine M and a string w, whether M ever moves it head to the left when it is run on input w.

   (b) To determine, given a Turing machine M, whether the tape ever contains four consecutive 1's during the course of M's computation when it is run on input 01.

2. Problem 4.18, Sipser's book.

3. (a) Prove that a language A is Turing recognizable if and only if A is mapping reducible to A_{TM}.

   (b) Prove that a language B is decidable if and only if B is mapping reducible to $\{0^n1^n \mid n \geq 1\}$.

4. Let

$$f(x) = \begin{cases} 3x+1 & \text{for odd } x \\ x/2 & \text{for even } x \end{cases}$$

   for any natural number x. If you start with an integer x and iterate f, you obtain a sequence: x, $f(x)$, $f(f(x))$, ... Stop if you ever hit 1. Extensive computer tests have shown that every starting point x between 1 and a large positive integer gives a sequence that ends in 1. The question of whether this happens for **all** starting points is unsolved, and is called the $3x+1$ problem.

   Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3x+1$ problem.

5. * **(Optional Problem)** Say that an NFA is ambiguous if it accepts some string along two different computation branches. Let $\mathsf{AMBIG}_{\mathsf{NFA}} = \{\langle N \rangle \mid N \text{ is an ambiguous NFA}\}$. Show that $\mathsf{AMBIG}_{\mathsf{NFA}}$ is decidable.