1. Which of the following problems about Turing machines are decidable and which are not? Briefly justify your answers.
   (a) To determine, given a Turing machine $M$ and a string $w$, whether $M$ ever moves its head to the left when it is run on input $w$.
   (b) To determine, given a Turing machine $M$, whether the tape ever contains four consecutive 1’s during the course of $M$’s computation when it is run on input 01.

2. Problem 4.18, Sipser’s book.

3. (a) Prove that a language $A$ is Turing recognizable if and only if $A$ is mapping reducible to $A_{TM}$.
   (b) Prove that a language $B$ is decidable if and only if $B$ is mapping reducible to $\{0^n1^n | n \geq 1\}$.

4. Let
   $$f(x) = \begin{cases} 
   3x + 1 & \text{for odd } x \\ 
   x/2 & \text{for even } x 
   \end{cases}$$
   for any natural number $x$. If you start with an integer $x$ and iterate $f$, you obtain a sequence: $x, f(x), f(f(x)), \ldots$ Stop if you ever hit 1. Extensive computer tests have shown that every starting point $x$ between 1 and a large positive integer gives a sequence that ends in 1. The question of whether this happens for all starting points is unsolved, and is called the $3x + 1$ problem.
   
   Suppose that $A_{TM}$ were decidable by a TM $H$. Use $H$ to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

5. * (Optional Problem) Say that an NFA is ambiguous if it accepts some string along two different computation branches. Let $\text{AMBIG}_{NFA} = \{ \langle N \rangle \mid N \text{ is an ambiguous NFA} \}$. Show that $\text{AMBIG}_{NFA}$ is decidable.