All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Use a diagonal argument to show that
   \[ \text{halts on all inputs} \]
   is not Turing recognizable. Hint: Your proof by contradiction will assume that there is a Turing enumerater of \( AH_{TM} \). The enumerator outputs the members of \( AH_{TM} \) in the order \( \langle M_1 \rangle, \langle M_2 \rangle, \ldots \). The possible inputs are members of \( \{0,1\}^* \) which can also be indexed \( s_1, s_2, \ldots \). Define a Turing machine decider \( M \) whose encoding cannot appear in \( AH_{TM} \).

2. (10 points) Let \( E \) be an enumerator with the property that if \( E \) enumerates an infinite language in the order \( w_1, w_2, \ldots \) (the eventual output of \( E \) is \( w_1 \# w_2 \# w_3 \# \cdots \)) and \( |w_i| < |w_{i+1}| \) for all \( i \). Show that the language enumerated is Turing decidable.

3. (10 points) Show that every infinite Turing recognizable language has an infinite Turing decidable subset. Hint: Use 2 above.