Assignment 1
Due Friday, April 6

All solutions should be neatly written or typeset. All major steps in proofs and algorithms must be justified.

1. (10 points) This problem is designed to give you practice in coding Turing machines. Use a state diagram to define a Turing machine that decides the language

\[ L = \{ w \in \{0, 1\}^* : w \text{ has an equal number of 0’s and 1’s} \}. \]

Use an example to demonstrate how your machine operates.

2. (10 points) This is another problem designed to give you practice in coding Turing machines. Define \( x \) is a substring of \( y \) if there are strings \( u \) and \( v \) such that \( y = uxv \). Note that the empty string is always a substring of any string and that a string is always a substring of itself. Use a state diagram to define a Turing machine that decides the language

\[ L = \{ \#x\#y : x, y \in \{0, 1\}^* \text{ and } x \text{ is a substring of } y \}. \]

The alphabet of \( L \) is \( \{0, 1, \#\} \). Use an example to demonstrate how your machine operates.

3. (10 points) An alternative definition of the storage for a Turing machine is a two-way infinite tape. That is, the tape does not have a beginning or an end. For such a machine we assume that the input is written some place on the tape, with blanks to the left and to the right. The head starts on the first symbol of the input. Given a Turing machine \( M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_f) \) with a two-way infinite tape, design a Turing machine \( M' \) with a one-way infinite tape such that \( L(M') = L(M) \).