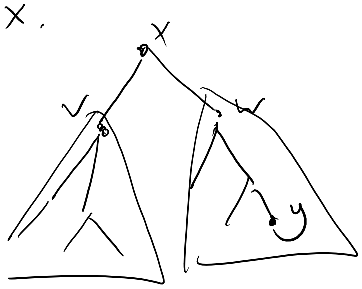
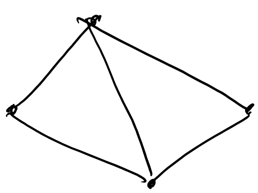
Claim: Suppose y is discovered during DFS(x). Then y is a descendant of x.



Because all undiscovered nodes that we visit during DFS(x) will be
child of x
or child of a child of x
or ...

Lemma: Say T is DFS Tree (s). Suppose $\{x,y\} \notin T$. ($\{x,y\} \in E$).
Then x is a ancestor of y or vice versa.

Pf. W.L.O.G. assume x is discovered first
We call DFS(x). (at this point y is undiscovered).
By prev. Claim enough to show y is discovered during DFS(x).
By time the For loop of DFS(x) gets y, y must be discovered
(o.w. edge $\{x,y\} \in T$ and it is a contradiction).
So y is discovered during DFS(x).



For all planar graphs
$m \leq 3n-4$
Hint: Any planar has a vertex v $\deg(v) \leq 5$.
Then, Use induction.

Claim: Any planar graph has a vertex v: $\deg(v) \leq 5$.

$\sum_v \deg(v) = 2m \leq 6n-8$.

$\Rightarrow \exists v: \deg(v) < 6$.

IH: $\forall$ planar graph G with $n-1$ vertices, color with 6 colors.
IS. Give planar graph G with n vertices.

$\exists v$ of degree ≤ 5 remove v. $G - \{v\}$ is plan becau you can draw on plan

So by IH we can color $G - \{v\}$.

We color v with a color not appear in its neighbors.

We can do that BC $deg(v) \leq 5$.

Claim If G has a topological order then it is a DAG.

pf.