Wednesday, January 31, 2018 2:25 PM

Claim: At any point in time, for all trees, At least $2^{L(root)}$ nodes in tree.

Pf. By Induction on time.

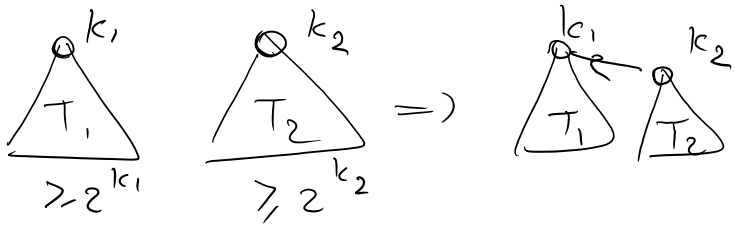
Base Case: At time 0 All comp ↑ tree $L(root) = 0$, # nodes in tree $2^{L(root)} = 1$ node in any comp

IH: Claim holds by time t.

IS. Show holds by time t+1.

Find ✓

Suppose Merge

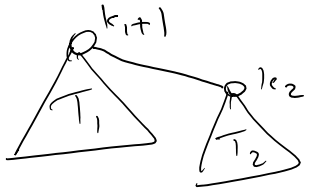


If $k_1 > k_2$ ✓

$k_1 < k_2$ ✓

If $k_1 = k_2 = k$

$2^k + 2^k \geq 2^{k+1}$



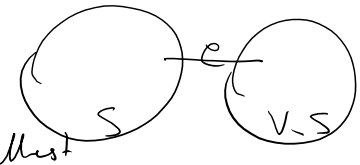
If edge weights are distinct => unique MST.

By Contradiction

Suppose two MST. T_1, T_2.

$\Rightarrow \exists e \in T_1 : e \notin T_2$

Remove e from T_1

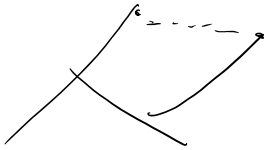


By cut prop e is smallest

e has to be in T_2 contradiction!

2nd pf Add e to T_2

Creates a cycle



Largest edge is not in MST

But it is in $T_1 \cup T_2$ contradiction

pf By induction. That we have found the shortest path to all vertices in S. (at any point of ALG).