CSE 421 - Lecture 11

Monday, January 29, 2018 2:24 PM

Thm: Greed is OPT.

Pf: Suppose Greedy allocate d classroom.

Goal: Show depth $\geq d$.

$$OPT \geq depth \geq d = Greedy$$

by Observation

Say at time t greedy allocate d-th classroom for lecture j.

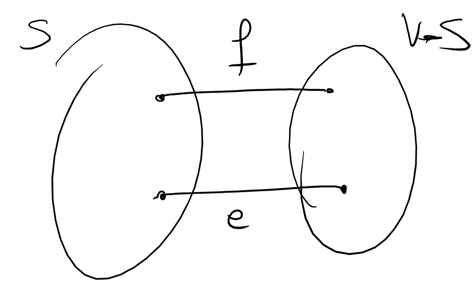
$t+\varepsilon$ we have d-conflicting lecture

$\Rightarrow$ depth $\geq d$.

Cut Property:

Claim: Let e be the smallest edge in a cut $(S, V-S)$.

T^* be the MST. Then $e \in T^*$.



Pf. (By contradiction).

Suppose $e \notin T^*$.

$\exists f \in T^*$ s.t. $f \in (S, V-S)$.

$c_e < c_f$ by def of e.

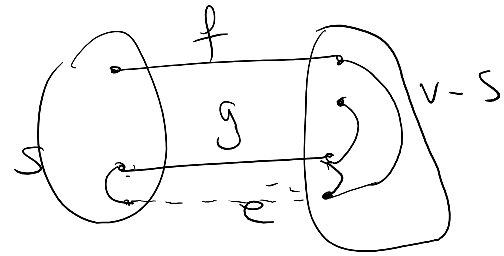
Substitute e with f. I get a smaller tree. A contradiction

$$T' = T^* - \{f\} \cup \{e\}.$$

Incorrect

Add e to T^*.

get a cycle C.



C crosses $(S, V-S)$ even # times

One of which is e.

But then must be another edge $g \in C$

s.t. $g \in (S, V-S)$.

$c_e < c_g$

$T = T^* - \{g\} \cup \{e\}$ exchange

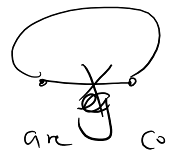
Tree is conn...

Tree no cycle

Tree has n-1 edges

I claim T is a tree.

T has n-1 edges.



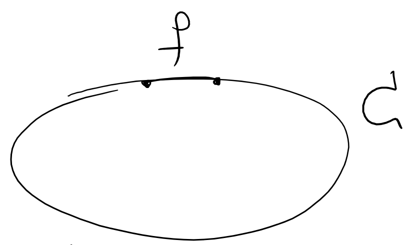
T is connected, BC endpoints of g are connected by the rest of cycle C.

$c(T) < c(T^*)$ BC $c_e < c_g$

contradiction!

Cycle Property: Let f be the largest edge in cycle C.

Then $f \notin T^*$.

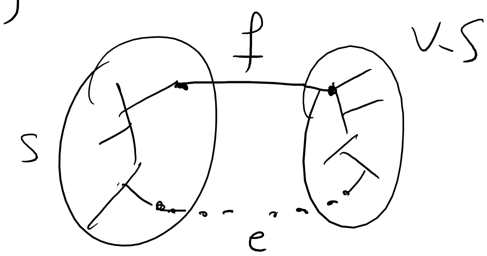


Pf. (By contradiction)

Suppose $f \in T^*$.

Rem f from T^*. That gives a cut $(S, V-S)$

S, V-S are connected comp of T^* when I remove f.



$\exists\ e \in C$ s.t. $e \in (S, V-S)$.

$c_e < c_f$.

$T = T^* - \{f\} \cup \{e\}$. exchange

claim T is a tree.

n-1 edges ✓

T is connected ✓

$c(T) < c(T^*)$ contradiction