**CSE 421 Algorithms**

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Lecture 29  
Complexity Theory

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**Announcements**

- **Final exam,**  
  - Monday, December 14, 2:30-4:20 pm  
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
- **Review session,**  
  - Today, 2:30-4:30 pm. Lowe 101  
- **Online course evaluations available**

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**NP Complete Problems**

1. Circuit Satisfiability  
2. Formula Satisfiability  
   a. 3-SAT  
3. Graph Problems  
   a. Independent Set  
   b. Vertex Cover  
   c. Clique  
4. Path Problems  
   a. Hamiltonian cycle  
   b. Hamiltonian path  
5. Partition Problems  
   a. Three dimensional matching  
   b. Exact cover  
6. Graph Coloring  
7. Number problems  
   a. Subset sum  
8. Integer linear programming  
9. Scheduling with release times and deadlines

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**Karp’s 21 NP Complete Problems**

- Circuit Satisfiability
- Formula Satisfiability
- Graph Problems (Independent Set, Vertex Cover, Clique)
- Path Problems (Hamiltonian cycle, Hamiltonian path)
- Partition Problems (3D matching, Exact cover)
- Graph Coloring
- Number problems (Subset sum)
- Integer linear programming
- Scheduling with release times and deadlines

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**What we don’t know**

- **P vs. NP**

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**If P != NP, is there anything in between**

- Yes, Ladner [1975]
- Problems not known to be in P or NP Complete  
  - Factorization  
  - Discrete Log  
  - Graph Isomorphism

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Solve $a^g = b$ over a finite group
Coping with NP Completeness

- Approximation Algorithms
  - Christofides algorithm for TSP (Undirected graphs satisfying triangle inequality)
- Solution guarantees on greedy algorithms
  - Bin packing

Christofides Algorithm (simplified)

Coping with NP-Completeness

- Branch and Bound
  - Euclidean TSP

Coping with NP-Completeness

- Local Search
  - Modify solution until a local minimum is reached
  - Interchange algorithm for TSP
  - Recoloring algorithms
    - Simulated annealing

Complexity Theory

- Computational requirements to recognize languages
- Models of Computation
- Resources
- Hierarchies

Time complexity

- P: (Deterministic) Polynomial Time
- NP: Non-deterministic Polynomial Time
- EXP: Exponential Time
Space Complexity

- Amount of Space (Exclusive of Input)
- L: Logspace, problems that can be solved in $O(\log n)$ space for input of size $n$
  - Related to Parallel Complexity
- PSPACE, problems that can be required in a polynomial amount of space

So what is beyond NP?

NP vs. Co-NP

- Given a Boolean formula, is it true for some choice of inputs
- Given a Boolean formula, is it true for all choices of inputs

Problems beyond NP

- Exact TSP, given a graph with edge lengths and an integer $K$, does the minimum tour have length $K$
- Minimum circuit, given a circuit $C$, is it true that there is no smaller circuit that computes the same function as $C$

Polynomial Hierarchy

- Level 1
  - $\exists X_1 \Phi(X_1), \forall X_1 \Phi(X_1)$
- Level 2
  - $\forall X_2 \exists X_3 \Phi(X_1, X_3), \exists X_1 \forall X_2 \Phi(X_1, X_2)$
- Level 3
  - $\forall X_3 \exists X_2 \forall X_3 \Phi(X_1, X_2, X_3), \exists X_1 \forall X_2 \exists X_3 \Phi(X_1, X_2, X_3)$

Polynomial Space

- Quantified Boolean Expressions
  - $\exists X_1 \forall X_2 \exists X_3 ... \exists X_n \forall X_1 \forall X_2 \exists X_3 ... \exists X_n \forall X_n$\Phi(X_1, X_2, X_3 ... X_n, X_n)$
- Space bounded games
  - Competitive Facility Location Problem
- Counting problems
  - The number of Hamiltonian Circuits in a graph