CSE 421
Algorithms
Lecture 21
Network Flow, Part 1
Network Flow
Outline

• Network flow definitions
• Flow examples
• Augmenting Paths
• Residual Graph
• Ford Fulkerson Algorithm
• Cuts
• Maxflow-MinCut Theorem
Network Flow Definitions

- Capacity
- Source, Sink
- Capacity Condition
- Conservation Condition
- Value of a flow
Flow Example
Flow assignment and the residual graph
Network Flow Definitions

• Flowgraph: Directed graph with distinguished vertices s (source) and t (sink)
• Capacities on the edges, $c(e) \geq 0$
• Problem, assign flows $f(e)$ to the edges such that:
  – $0 \leq f(e) \leq c(e)$
  – Flow is conserved at vertices other than s and t
    • Flow conservation: flow going into a vertex equals the flow going out
  – The flow leaving the source is as large as possible
Flow Example
Find a maximum flow
Flow Example
Residual Graph

- Flow graph showing the remaining capacity
- Flow graph $G$, Residual Graph $G_R$
  - $G$: edge $e$ from $u$ to $v$ with capacity $c$ and flow $f$
  - $G_R$: edge $e'$ from $u$ to $v$ with capacity $c - f$
  - $G_R$: edge $e''$ from $v$ to $u$ with capacity $f$
Flow assignment and the residual graph
Augmenting Path Algorithm

• Augmenting path
  – Vertices $v_1, v_2, \ldots, v_k$
    • $v_1 = s$, $v_k = t$
    • Possible to add $b$ units of flow between $v_j$ and $v_{j+1}$ for $j = 1 \ldots k-1$

![Diagram of network flow](image-url)
Build the residual graph

Residual graph:
Find two augmenting paths
Augmenting Path Lemma

- Let $P = v_1, v_2, \ldots, v_k$ be a path from $s$ to $t$ with minimum capacity $b$ in the residual graph.
- $b$ units of flow can be added along the path $P$ in the flow graph.
Proof

• Add $b$ units of flow along the path $P$
• What do we need to verify to show we have a valid flow after we do this?
Ford-Fulkerson Algorithm (1956)

while not done

    Construct residual graph $G_R$
    Find an s-t path $P$ in $G_R$ with capacity $b > 0$
    Add $b$ units along in $G$

If the sum of the capacities of edges leaving S is at most $C$, then the algorithm takes at most $C$ iterations