CSE 421
Algorithms
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Lecture 12
Recurrences

Announcements

- Midterm
  - Monday, Nov 2, in class, closed book
  - Through section 5.2

Divide and Conquer

- Recurrences, Sections 5.1 and 5.2
- Algorithms
  - Counting Inversions (5.3)
  - Closest Pair (5.4)
  - Multiplication (5.5)
  - FFT (5.6)

Array Mergesort(Array a) {
    n = a.Length;
    if (n <= 1)
        return a;
    b = Mergesort(a[0 .. n/2]);
    c = Mergesort(a[n/2+1 .. n-1]);
    return Merge(b, c);
}

Algorithm Analysis

- Cost of Merge
- Cost of Mergesort

T(n) <= 2T(n/2) + cn; T(1) <= c;
Recurrence Analysis

- Solution methods
  - Unrolling recurrence
  - Guess and verify
  - Plugging in to a "Master Theorem"

Unrolling the recurrence

Substitution

Prove \( T(n) \leq cn (\log_2 n + 1) \) for \( n \geq 1 \)

Induction:
Base Case:

Induction Hypothesis:

A better mergesort (?)

- Divide into 3 subarrays and recursively sort
- Apply 3-way merge

What is the recurrence?

Unroll recurrence for \( T(n) = 3T(n/3) + dn \)

\[ T(n) = aT(n/b) + f(n) \]
Where does this recurrence arise?

Solving the recurrence exactly

\[ T(n) = 4T(n/2) + cn \]

\[ T(n) = 2T(n/2) + n \]

\[ T(n) = 2T(n/2) + n^{1/2} \]

Recurrences

• Three basic behaviors
  – Dominated by initial case
  – Dominated by base case
  – All cases equal – we care about the depth