CSE 421
Algorithms
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Lecture 12
Recurrences
Announcements

• Midterm
  – Monday, Nov 2, in class, closed book
  – Through section 5.2
Divide and Conquer

• Recurrences, Sections 5.1 and 5.2
• Algorithms
  – Counting Inversions (5.3)
  – Closest Pair (5.4)
  – Multiplication (5.5)
  – FFT (5.6)
Divide and Conquer

Array Mergesort(Array a) {
    n = a.Length;
    if (n <= 1)
        return a;
    b = Mergesort(a[0 .. n/2]);
    c = Mergesort(a[n/2+1 .. n-1]);
    return Merge(b, c);
}
Algorithm Analysis

• Cost of Merge
• Cost of Mergesort
\[ T(n) \leq 2T(n/2) + cn; \quad T(1) \leq c; \]
Recurrence Analysis

• Solution methods
  – Unrolling recurrence
  – Guess and verify
  – Plugging in to a “Master Theorem”
Unrolling the recurrence
Substitution

Prove $T(n) \leq cn \left( \log_2 n + 1 \right)$ for $n \geq 1$

Induction:
Base Case:

Induction Hypothesis:
A better mergesort (?)

• Divide into 3 subarrays and recursively sort
• Apply 3-way merge

What is the recurrence?
Unroll recurrence for
\[ T(n) = 3T(n/3) + dn \]
T(n) = aT(n/b) + f(n)
$T(n) = T(n/2) + cn$

Where does this recurrence arise?
Solving the recurrence exactly
\[ T(n) = 4T(n/2) + cn \]
\[ T(n) = 2T(n/2) + n^2 \]
\[ T(n) = 2T(n/2) + n^{1/2} \]
Recurrences

• Three basic behaviors
  – Dominated by initial case
  – Dominated by base case
  – All cases equal – we care about the depth