Announcements

- Midterm
  - Feb 9, In class, closed book
  - Through section 5.2

Divide and Conquer

- Recurrences, Sections 5.1 and 5.2
- Algorithms
  - Counting Inversions (5.3)
  - Closest Pair (5.4)
  - Multiplication (5.5)
  - FFT (5.6)

Algorithm Analysis

- Cost of Merge
- Cost of Mergesort

```
Array Mergesort(Array a){
    n = a.Length;
    if (n <= 1)
        return a;
    b = Mergesort(a[0 .. n/2]);
    c = Mergesort(a[n/2+1 .. n-1]);
    return Merge(b, c);
}
```

T(n) <= 2T(n/2) + cn; T(1) <= c;
Recurrence Analysis

- Solution methods
  - Unrolling recurrence
  - Guess and verify
  - Plugging in to a “Master Theorem”

Substitution

Prove $T(n) \leq cn (\log_2 n + 1)$ for $n \geq 1$

Induction:
- Base Case:

  Induction Hypothesis:

Unrolling the recurrence

A better mergesort (?)

- Divide into 3 subarrays and recursively sort
- Apply 3-way merge

What is the recurrence?

Unroll recurrence for $T(n) = 3T(n/3) + dn$

$T(n) = aT(n/b) + f(n)$
Recurrences

- Three basic behaviors
  - Dominated by initial case
  - Dominated by base case
  - All cases equal – we care about the depth

T(n) = T(n/2) + cn

Where does this recurrence arise?

Solving the recurrence exactly

T(n) = 4T(n/2) + cn

T(n) = 2T(n/2) + \( n^2 \)

T(n) = 2T(n/2) + n^{1/2}