Outline

- Network flow definitions
- Flow examples
- Augmenting Paths
- Residual Graph
- Ford Fulkerson Algorithm
- Cuts
- Maxflow-MinCut Theorem

Network Flow Definitions

- Capacity
- Source, Sink
- Capacity Condition
- Conservation Condition
- Value of a flow

Flow Example

Flow assignment and the residual graph
Network Flow Definitions

• Flowgraph: Directed graph with distinguished vertices s (source) and t (sink)
• Capacities on the edges, $c(e) \geq 0$
• Problem, assign flows $f(e)$ to the edges such that:
  – $0 \leq f(e) \leq c(e)$
  – Flow is conserved at vertices other than s and t
    • Flow conservation: flow going into a vertex equals the flow going out
  – The flow leaving the source is as large as possible

Flow Example

Find a maximum flow

Augmenting Path Algorithm

• Augmenting path
  – Vertices $v_1, v_2, \ldots, v_k$
  • $v_1 = s, v_k = t$
  • Possible to add $b$ units of flow between $v_j$ and $v_{j+1}$ for $j = 1 \ldots k-1$
Residual Graph

• Flow graph showing the remaining capacity
• Flow graph $G$, Residual Graph $G_R$
  – $G$: edge $e$ from $u$ to $v$ with capacity $c$ and flow $f$
  – $G_R$: edge $e'$ from $u$ to $v$ with capacity $c - f$
  – $G_R$: edge $e''$ from $v$ to $u$ with capacity $f$

Build the residual graph

Augmenting Path Lemma

• Let $P = v_1, v_2, \ldots, v_k$ be a path from $s$ to $t$ with minimum capacity $b$ in the residual graph.
• $b$ units of flow can be added along the path $P$ in the flow graph.