Optimal linear interpolation

$$\text{Error} = \sum(y_i - ax_i - b)^2$$

Determine set of $K$ lines to minimize error

Optimal sub-solution property

Optimal multi-segment interpolation

Compute $\text{Opt}[k, j]$ for $0 < k < j < n$

for $j := 1$ to $n$
  \( \text{Opt}[1, j] = E_{1j} \);
for $k := 2$ to $n-1$
  for $j := 2$ to $n$
    $t := E_{1j}$
    for $i := 1$ to $j - 1$
      $t = \min(t, \text{Opt}[k-1, i] + E_{ij})$
    $\text{Opt}[k, j] = t$
Determining the solution

- When Opt[k, j] is computed, record the value of i that minimized the sum
- Store this value in an auxiliary array
- Use to reconstruct solution

Variable number of segments

- Segments not specified in advance
- Penalty function associated with segments
- Cost = Interpolation error + C x #Segments

Penalty cost measure

- Opt[j] = min(E1,j, min(Opt[i] + Ei,j)) + P

Subset Sum Problem

- Let w1,...,wn = {6, 8, 9, 11, 13, 16, 18, 24}
- Find a subset that has as large a sum as possible, without exceeding 50

Adding a variable for Weight

- Opt[j, K] the largest subset of {w1, ..., wj} that sums to at most K
- {2, 4, 7, 10}
  - Opt[2, 7] =
  - Opt[3, 7] =
  - Opt[3,12] =
  - Opt[4,12] =

Subset Sum Recurrence

- Opt[j, K] the largest subset of {w1, ..., wj} that sums to at most K
Subset Sum Grid

\[ \text{Opt}[j, K] = \max(\text{Opt}[j - 1, K], \text{Opt}[j - 1, K - w_j] + w_j) \]

\{2, 4, 7, 10\}

Knapsack Problem

- Items have weights and values
- The problem is to maximize total value subject to a bound on weight
- Items \( \{i_1, i_2, \ldots, i_n\} \)
  - Weights \( \{w_1, w_2, \ldots, w_n\} \)
  - Values \( \{v_1, v_2, \ldots, v_n\} \)
  - Bound \( K \)
- Find set \( S \) of indices to:
  - Maximize \( \sum_{i \in S} v_i \) such that \( \sum_{i \in S} w_i \leq K \)

Knapsack Grid

\[ \text{Opt}[j, K] = \max(\text{Opt}[j - 1, K], \text{Opt}[j - 1, K - w_j] + v_j) \]

Weights \( \{2, 4, 7, 10\} \)  Values: \( \{3, 5, 9, 16\} \)