Open Pit Mining
• Each unit of earth has a profit (possibly negative)
• Getting to the ore below the surface requires removing the dirt above
• Test drilling gives reasonable estimates of costs
• Plan an optimal mining operation

Determine an optimal mine

Generalization
• Precedence graph G=(V,E)
• Each v in V has a profit p(v)
• A set F is feasible if when w in F, and (v,w) in E, then v in F.
• Find a feasible set to maximize the profit

Min cut algorithm for profit maximization
• Construct a flow graph where the minimum cut identifies a feasible set that maximizes profit
Precedence graph construction

- Precedence graph \( G=(V,E) \)
- Each edge in \( E \) has infinite capacity
- Add vertices \( s, t \)
- Each vertex in \( V \) is attached to \( s \) and \( t \) with finite capacity edges

Show a finite value cut with at least two vertices on each side of the cut

The sink side of the cut is a feasible set

- No edges permitted from \( S \) to \( T \)
- If a vertex is in \( T \), all of its ancestors are in \( T \)

Setting the costs

- If \( p(v) > 0 \)
  - \( \text{cap}(v,t) = p(v) \)
  - \( \text{cap}(s,v) = 0 \)
- If \( p(v) < 0 \)
  - \( \text{cap}(s,v) = -p(v) \)
  - \( \text{cap}(v,t) = 0 \)
- If \( p(v) = 0 \)
  - \( \text{cap}(s,v) = 0 \)
  - \( \text{cap}(v,t) = 0 \)

Enumerate all finite \( s,t \) cuts and show their capacities

Minimum cut gives optimal solution

Why?
Computing the Profit

- Cost(W) = $\sum_{w \in W; p(w) < 0} p(w)$
- Benefit(W) = $\sum_{w \in W; p(w) > 0} p(w)$
- Profit(W) = Benefit(W) – Cost(W)

- Maximum cost and benefit
  - C = Cost(V)
  - B = Benefit(V)

Express Cap(S,T) in terms of B, C, Cost(T), Benefit(T), and Profit(T)

Cap(S,T) = B – Profit(T)

Summary

- Construct flow graph
  - Infinite capacity for precedence edges
  - Capacities to source/sink based on cost/benefit
- Finite cut gives a feasible set of tasks
- Minimizing the cut corresponds to maximizing the profit
- Find minimum cut with a network flow algorithm