Who was Dijkstra?

- What were his major contributions?

Edsger Wybe Dijkstra
http://www.cs.utexas.edu/users/EWD/

Single Source Shortest Path Problem
- Given a graph and a start vertex s
  - Determine distance of every vertex from s
  - Identify shortest paths to each vertex
    - Express concisely as a "shortest paths tree"
    - Each vertex has a pointer to a predecessor on shortest path

Construct Shortest Path Tree from s

Warmup
- If P is a shortest path from s to v, and if t is on the path P, the segment from s to t is a shortest path between s and t
- WHY? (submit an answer)
Careful Proof

• Suppose s-v is a shortest path
• Suppose s-t is not a shortest path
• Therefore s-v is not a shortest path
• Therefore s-t is a shortest path

Prove if s-t not a shortest path then s-v is not a shortest path

Dijkstra’s Algorithm

\[ S = \emptyset; \quad d[s] = 0; \quad d[v] = \infty \text{ for } v \neq s \]

While \( S \neq V \)

Choose \( v \) in \( V - S \) with minimum \( d[v] \)

Add \( v \) to \( S \)

For each \( w \) in the neighborhood of \( v \)

\[ d[w] = \min(d[w], d[v] + c(v, w)) \]

Simulate Dijkstra's algorithm (starting from s) on the graph

Correctness Proof

• Elements in S have the correct label
• Key to proof: when \( v \) is added to \( S \), it has the correct distance label.
Proof

- Let $P_v$ be the path of length $d[v]$, with an edge $(u,v)$
- Let $P$ be some other path to $v$. Suppose $P$ first leaves $S$ on the edge $(x,y)$
  - $P = P_{sx} + c(x,y) + P_{yv}$
  - $\text{Len}(P_{sx}) + c(x,y) \geq d[y]$
  - $\text{Len}(P_{yv}) \geq 0$
  - $\text{Len}(P) \geq d[y] + 0 \geq d[v]$

Negative Cost Edges

- Draw a small example a negative cost edge and show that Dijkstra’s algorithm fails on this example

Bottleneck Shortest Path

- Define the bottleneck distance for a path to be the maximum cost edge along the path

How do you adapt Dijkstra’s algorithm to handle bottleneck distances
- Does the correctness proof still apply?