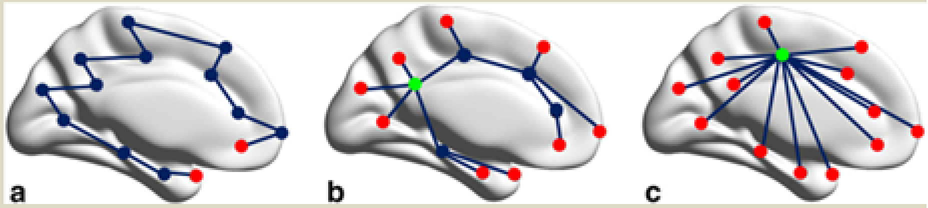
Lecture14



CSE 417
Algorithms and Complexity

Winter 2023
Lecture 14
Finishing Minimum Spanning Trees

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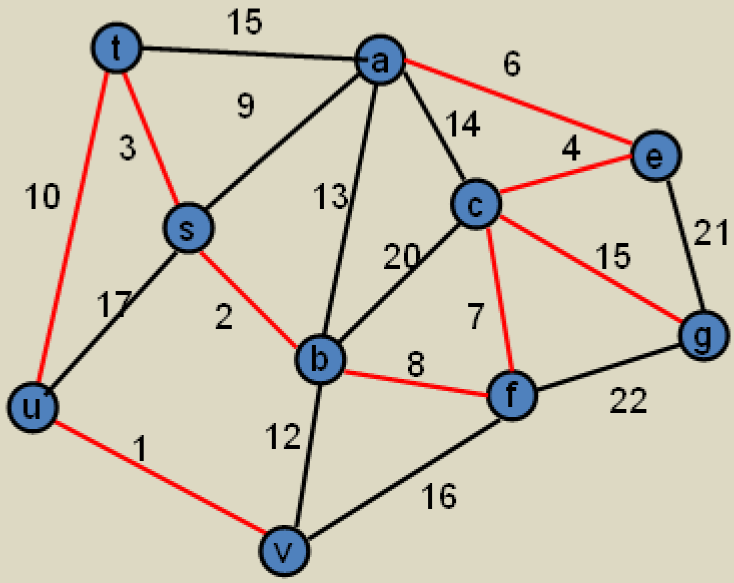
Announcements

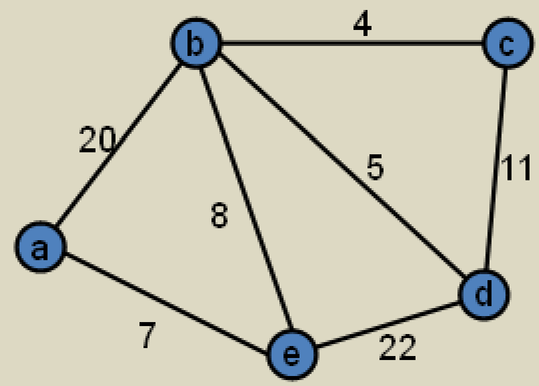
- Midterm, Wednesday, Feb 8
  - Closed book, closed notes, no calculators
  - Time limit: 50 minutes
  - Answer the problems on the exam paper.
  - If you need extra space use the back of a page
  - Problems are not of equal difficulty, if you get stuck on a problem, move on.
  - ``Justify your answer'' means give a short and convincing explanation. Depending on the situation, justifications can involve counter examples, or cite results established in the text or in lecture.

Minimum Spanning Tree



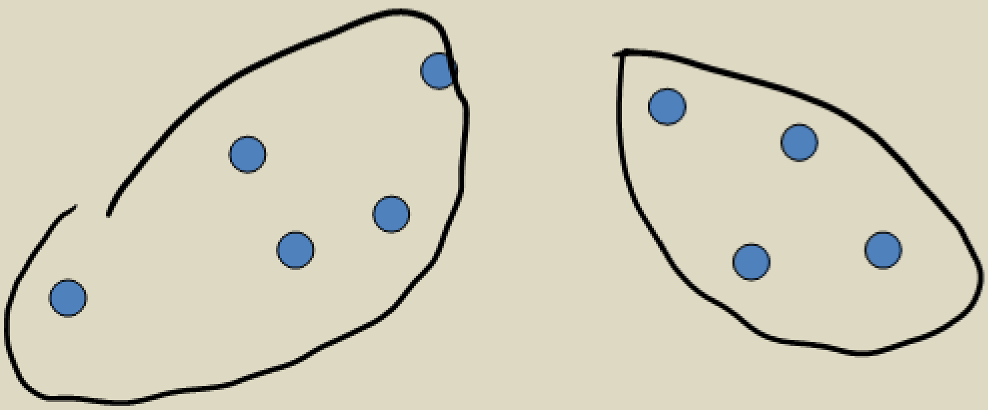
Greedy Algorithms for Minimum Spanning Tree

- Prim's Algorithm: Extend a tree by including the cheapest out going edge
- Kruskal's Algorithm: Add the cheapest edge that joins disjoint components



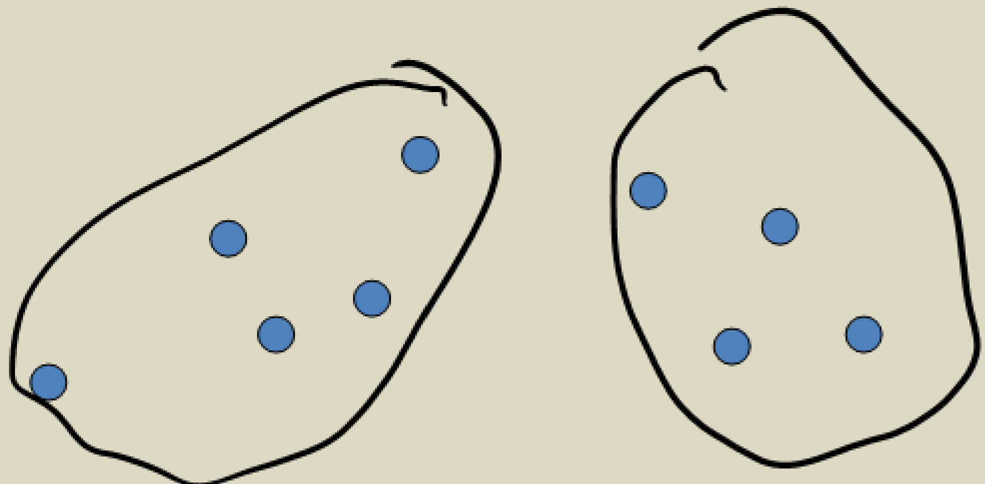
Application: Clustering

- Given a collection of points in an r-dimensional space and an integer K, divide the points into K sets that are closest together

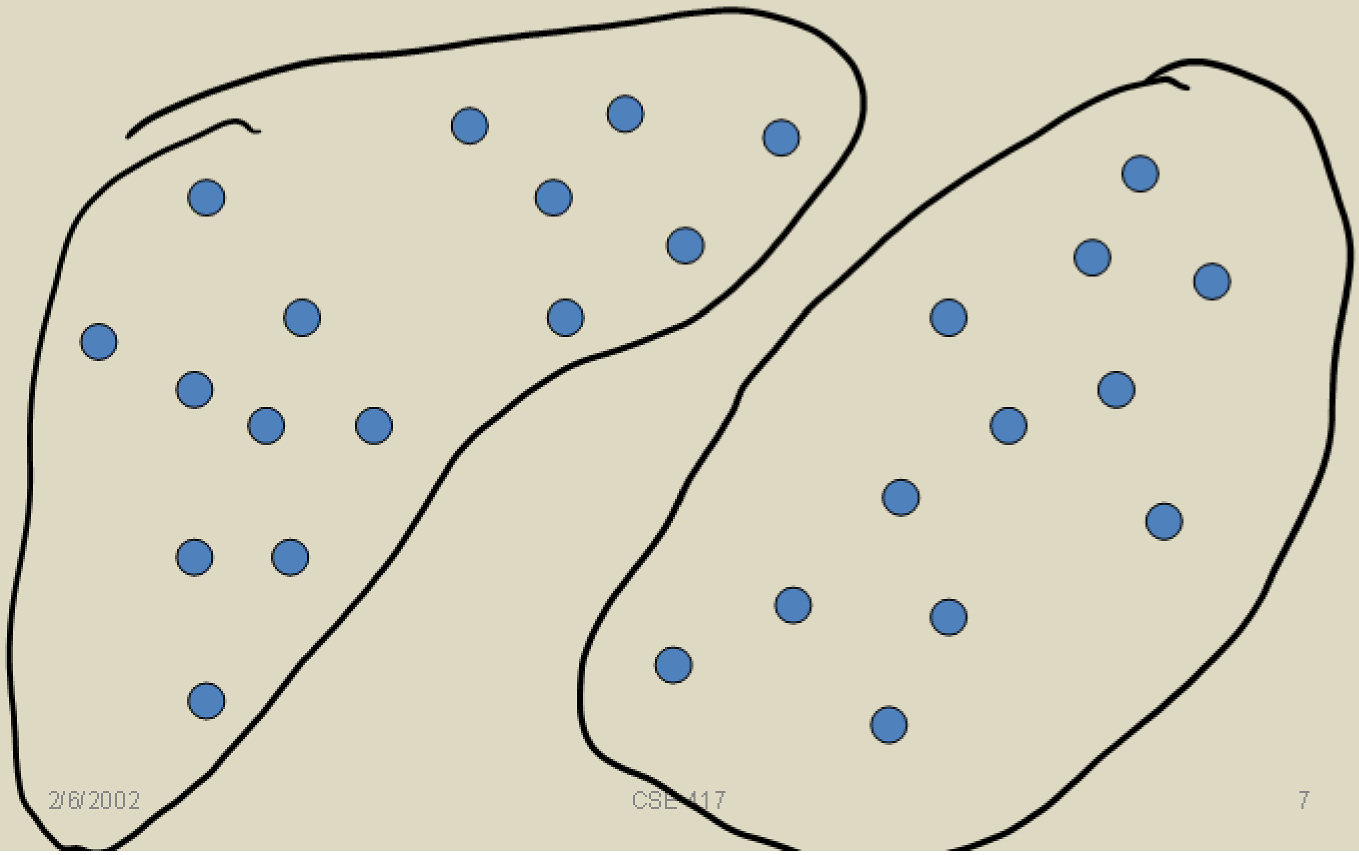


Distance clustering

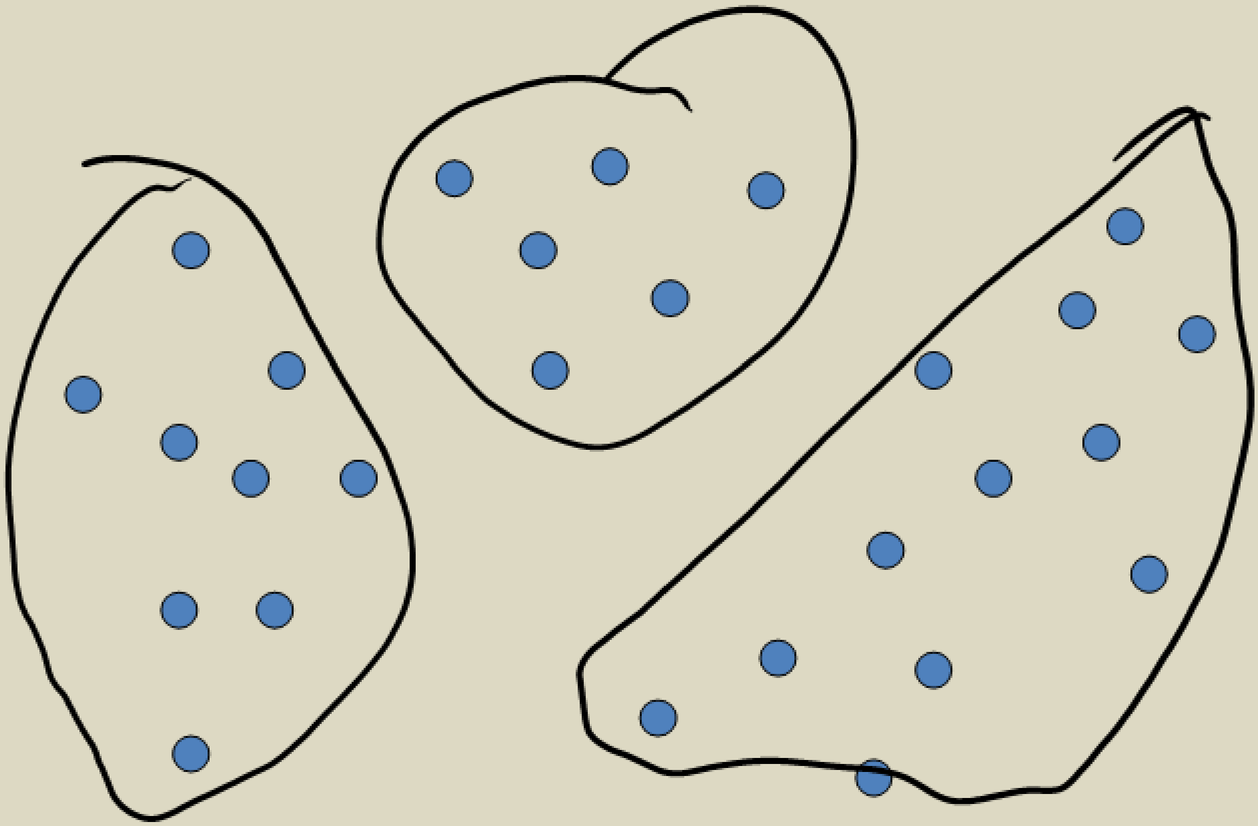
- Divide the data set into K subsets to maximize the distance between any pair of sets
  - dist $(S_1, S_2) = \min \{dist(x, y) \mid x \text{ in } S_1, y \text{ in } S_2\}$



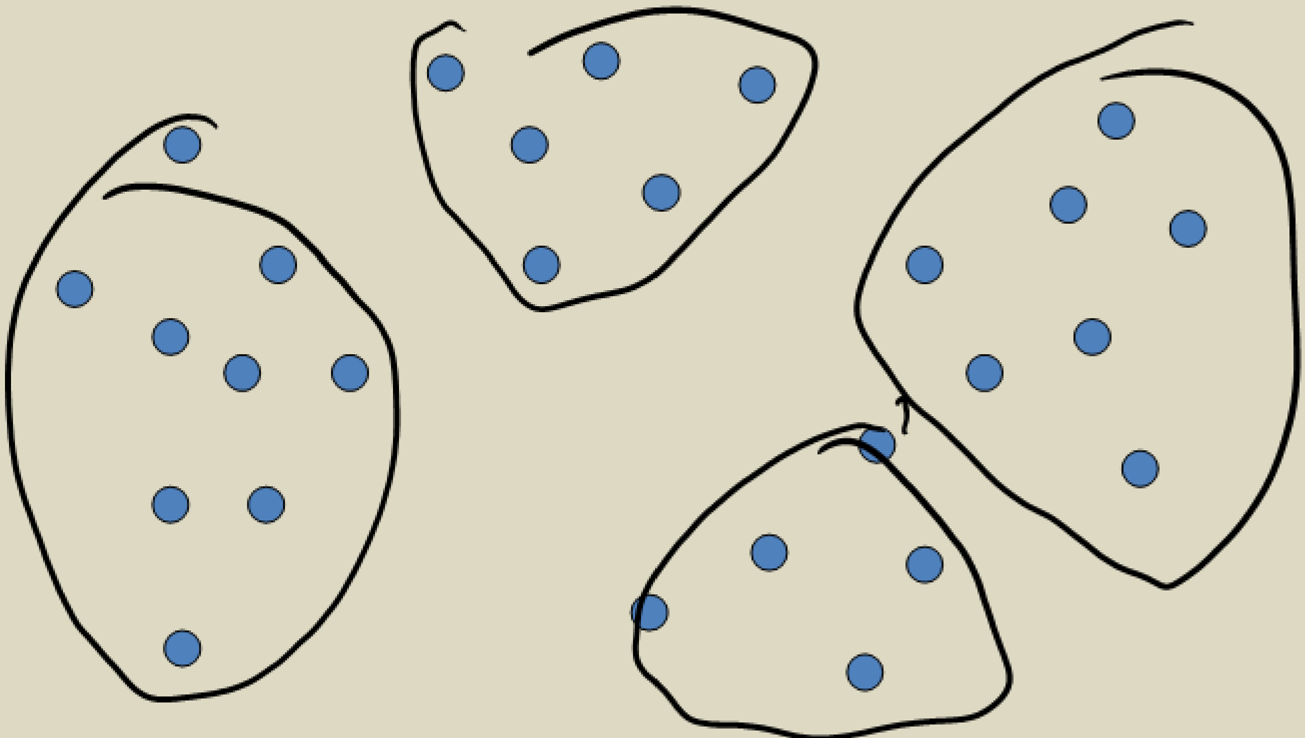
Divide into 2 clusters



Divide into 3 clusters



Divide into 4 clusters



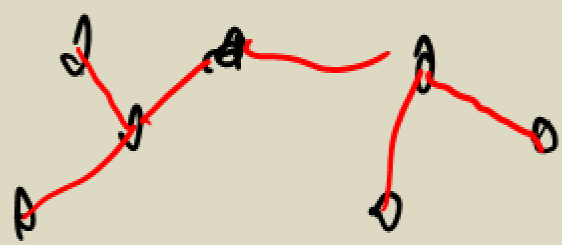
Distance Clustering Algorithm

Let $C = \{\{v_1\}, \{v_2\}, \ldots, \{v_n\}\}$; $T = \{\ \}$

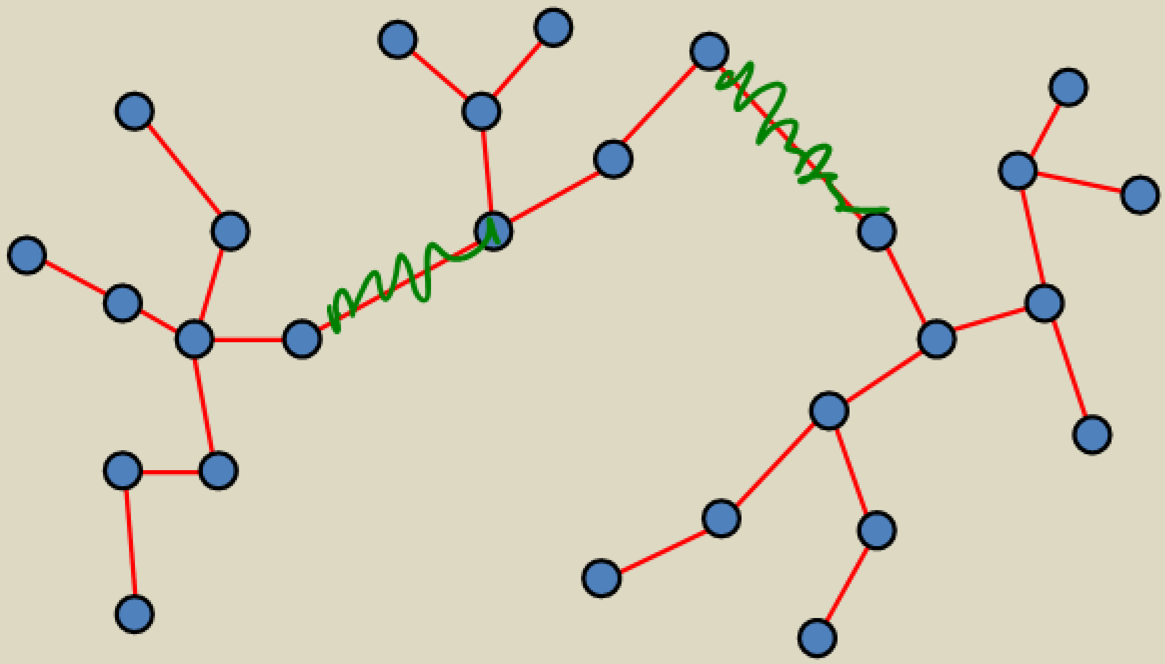
while $|C| > K$

Let $e = (u, v)$ with u in C_i and v in C_j be the minimum cost edge joining distinct sets in C

Replace C_i and C_j by $C_i \cup C_j$



K-clustering



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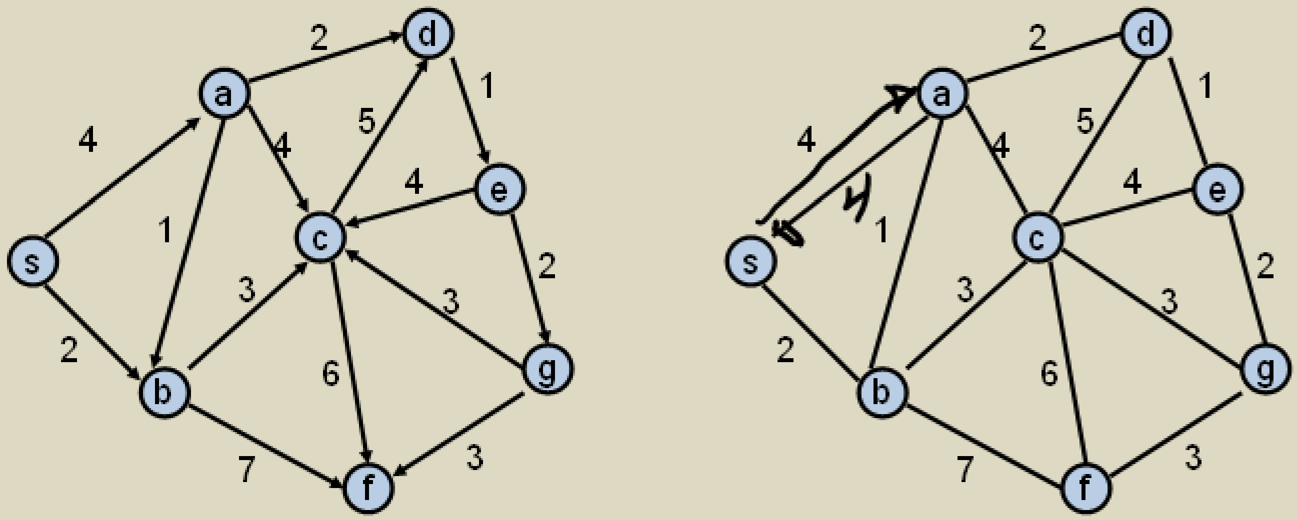
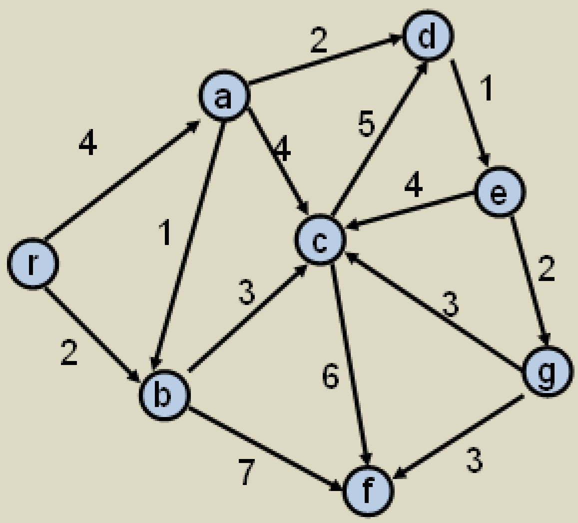
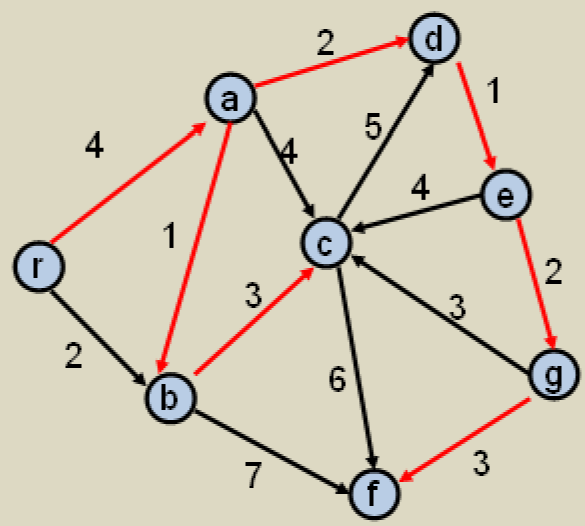
Shortest paths in directed graphs vs undirected graphs



What about the minimum spanning tree of a directed graph?

- Must specify the root r
- Branching: Out tree with root r

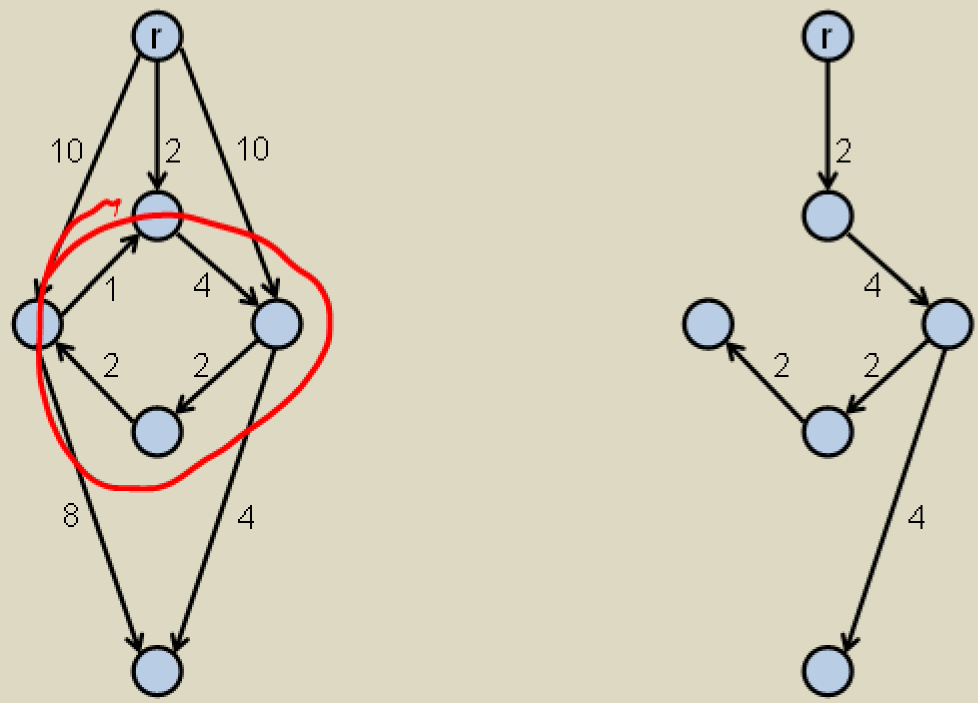




Assume all vertices reachable from r

Also called an arborescence

Finding a minimum branching



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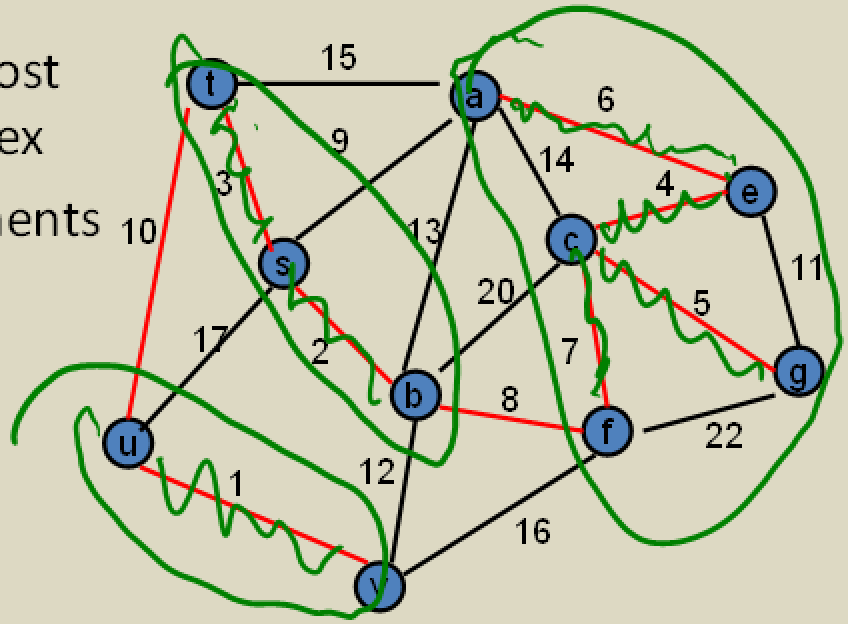
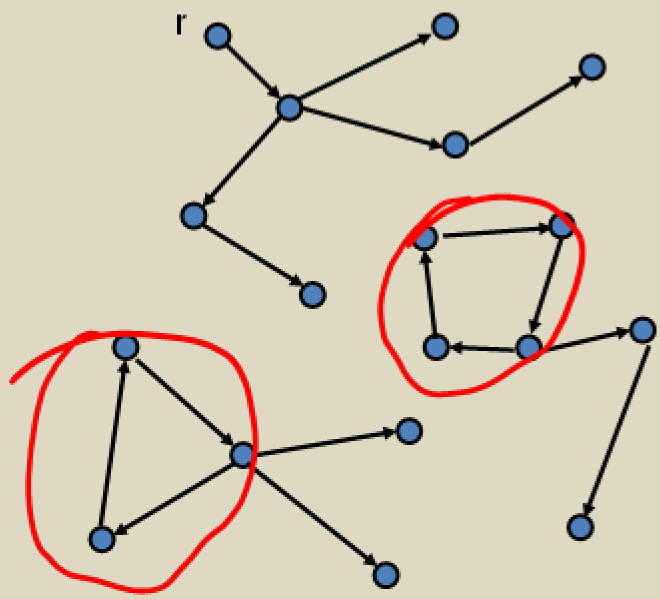
Another MST Algorithm

- Choose minimum cost edge into each vertex
- Merge into components
- Repeat until done



Idea for branching algorithm

- Select minimum cost edge going into each vertex
- If graph is a branching then done
- Otherwise collapse cycles and repeat



Midterm Questions????

prove $9n^2 + rn + s$ is $O(n^2)$

$3n^2 + 2$ is $O(n^2)$

$f(n)$ is $O(g(n))$

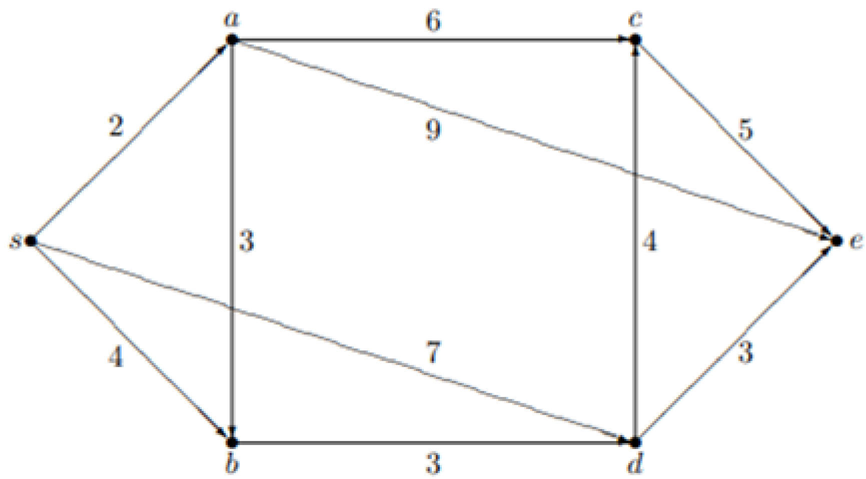
if $\exists\, c > 0, N_0$, s.th.

$f(n) \leq c\,g(n)$

for $n > n_0$

Problem 1. Dijkstra's (10 points):

Use the following graph to simulate versions of Dijkstra's algorithm in parts a) and c) starting from the vertex s.



a) Simulate Dijkstra's shortest path algorithm on the graph above by filling in the table. The entries should contain the preliminary distance values.

Problem 14. Fun with Big-Oh (10 points):

a) Order the following functions in increasing order by their growth rate:

1. $(\log n)^{\log n}$
2. n^4
3. $n^3 + n^5$
4. $2^{\sqrt{\log n}}$
5. $(0.01)^n$
6. $2^{n/10}$

- Take logs

$\sqrt{\log n}$

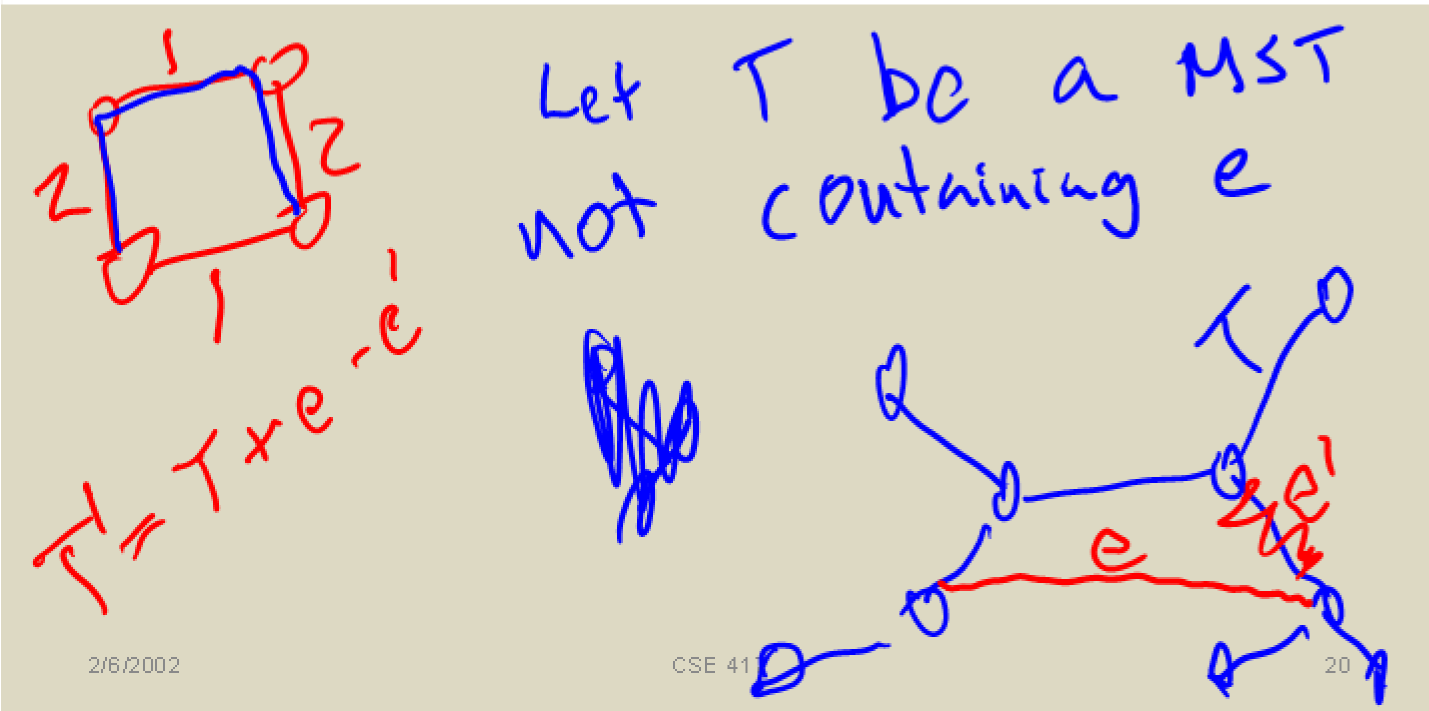
b) Is $n^2 \in \Theta(3n^3 + 2n)$? Explain.

c) Is an algorithm with run time $O(n!)$ ever preferable to an algorithm with runtime $O(n)$? Explain.

Problem 15. Minimum Spanning Tree (10 points):
Let $G = (V, E)$ be a connected, undirected graph with edge weights. The edge weights are not necessarily distinct (e.g., the graph may have two or more edges of the same weight).

a) Let e be a minimum weight edge in E. Prove that e is not necessarily in every minimum spanning tree for G.

b) Let e be a minimum weight edge in E. Prove that e is in some minimum spanning tree for G.



Problem 16. Starbucks Placement (10 points):

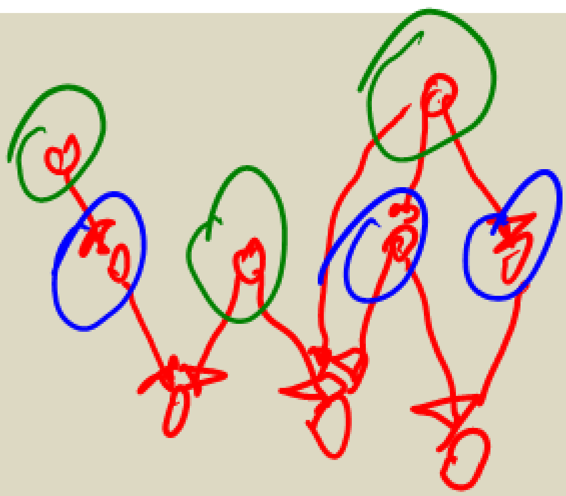
Washington State law requires that there is a Starbucks within 15 miles when driving on the freeway.[1] The purpose of this problem is to design an algorithm that Starbucks can use to place it's stores along a freeway to ensure that all points are within a fixed distance of a store. The stores can only be placed at off ramps, so there is a list of possible locations given as an increasing sequence of integers. The different directions of the freeway are considered separately, and no back tracking is allowed to reach a store. The problem is: Given a set of integers $A = \{a_1, a_2, \ldots a_n\}$ in increasing order, find a subset S of A which is as small as possible, such that for every $x \in [0, K]$, there is an $a \in S$ with $a - L \leq x \leq a$ where K is the length of the freeway, and L is the maximum allowed distance from a Starbucks.

a) Give a greedy algorithm that finds a minimum sized set of locations that guarantees every point is with distance L of a Starbucks.

b) Prove that the first item that your greedy algorithm selects is a member of some optimal solution to the problem.

Problem 17. Scheduling (10 points):

Let G be the precedence graph (prerequisite graph) for the courses in the Computer Science major. Describe an algorithm for determining the minimum number of quarters to complete the major. (Design the algorithm for over achievers with no bound on the number of courses that can be taken per quarter[2]. Also, assume that every course is offered every quarter.) Justify that your algorithm is correct. You do not need to give the runtime for your algorithm (but it should be a polynomial time algorithm.)



K-pocessor scheduling with precidence constraints