

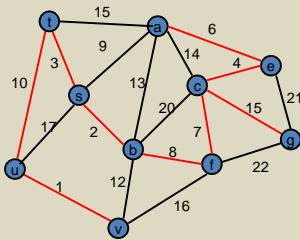
CSE 417 Algorithms and Complexity

Winter 2023
Lecture 14
Finishing Minimum Spanning Trees

Announcements

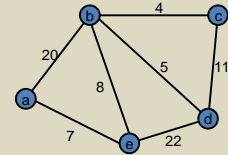
- Midterm, Wednesday, Feb 8
 - Closed book, closed notes, no calculators
 - Time limit: 50 minutes
 - Answer the problems on the exam paper.
 - If you need extra space use the back of a page
 - Problems are not of equal difficulty, if you get stuck on a problem, move on.
 - "Justify your answer" means give a short and convincing explanation. Depending on the situation, justifications can involve counter examples, or cite results established in the text or in lecture.

Minimum Spanning Tree



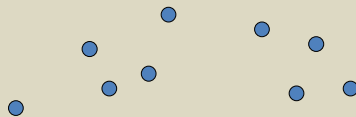
Greedy Algorithms for Minimum Spanning Tree

- Prim's Algorithm:
Extend a tree by including the cheapest outgoing edge
- Kruskal's Algorithm:
Add the cheapest edge that joins disjoint components



Application: Clustering

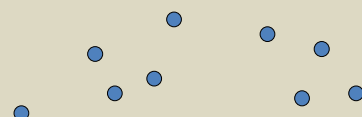
- Given a collection of points in an r -dimensional space and an integer K , divide the points into K sets that are closest together



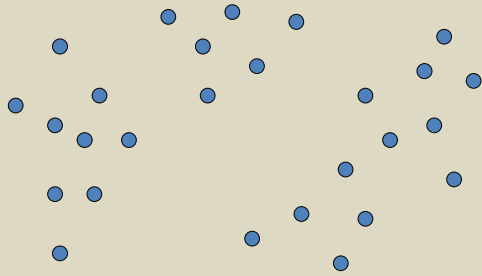
Distance clustering

- Divide the data set into K subsets to maximize the distance between any pair of sets

$$- \text{dist}(S_1, S_2) = \min \{ \text{dist}(x, y) \mid x \in S_1, y \in S_2 \}$$



Divide into 2 clusters

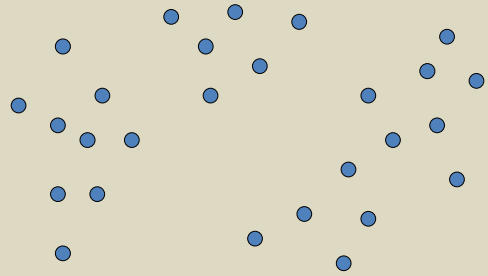


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Divide into 3 clusters

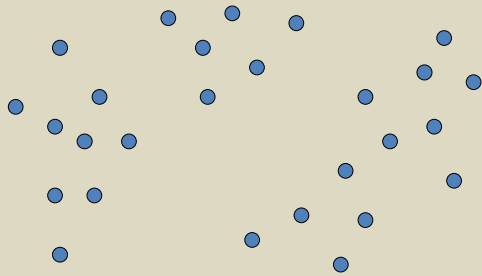


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Divide into 4 clusters



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Distance Clustering Algorithm

Let $C = \{\{v_1\}, \{v_2\}, \dots, \{v_n\}\}; T = \{\}$

while $|C| > K$

Let $e = (u, v)$ with u in C_i and v in C_j be the minimum cost edge joining distinct sets in C

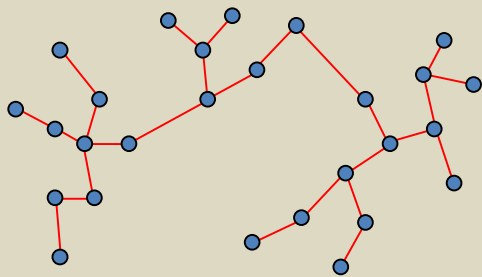
Replace C_i and C_j by $C_i \cup C_j$

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K-clustering

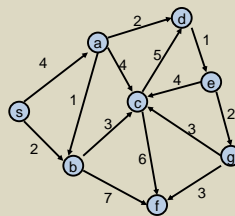


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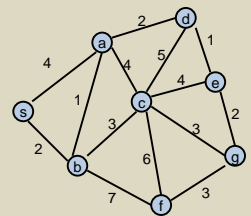
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Shortest paths in directed graphs vs undirected graphs



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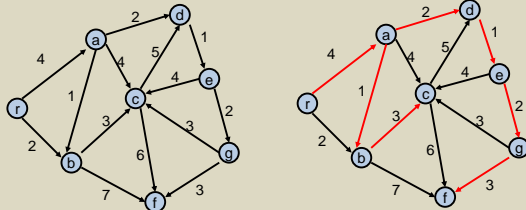
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What about the minimum spanning tree of a directed graph?

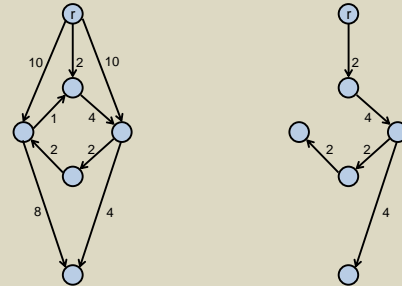
- Must specify the root r
- Branching: Out tree with root r



Assume all vertices reachable from r

Also called an arborescence

Finding a minimum branching



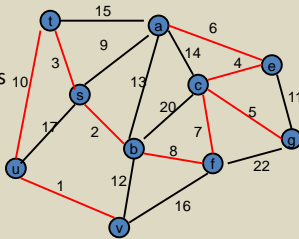
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Another MST Algorithm

- Choose minimum cost edge into each vertex
- Merge into components
- Repeat until done



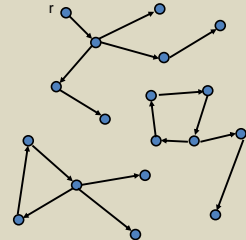
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Idea for branching algorithm

- Select minimum cost edge going into each vertex
- If graph is a branching then done
- Otherwise collapse cycles and repeat



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Midterm Questions????

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