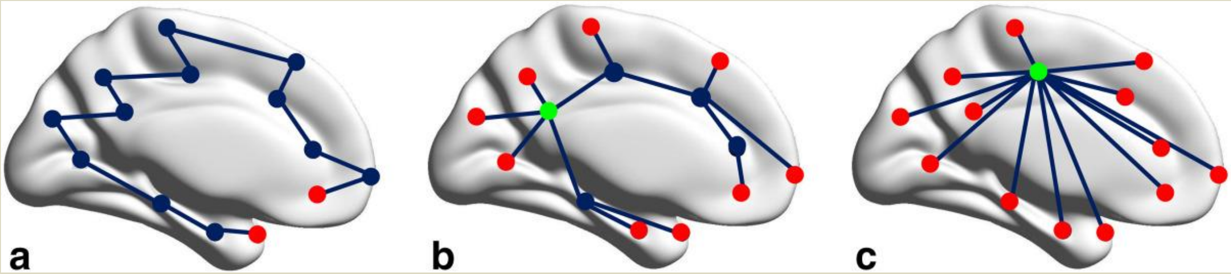


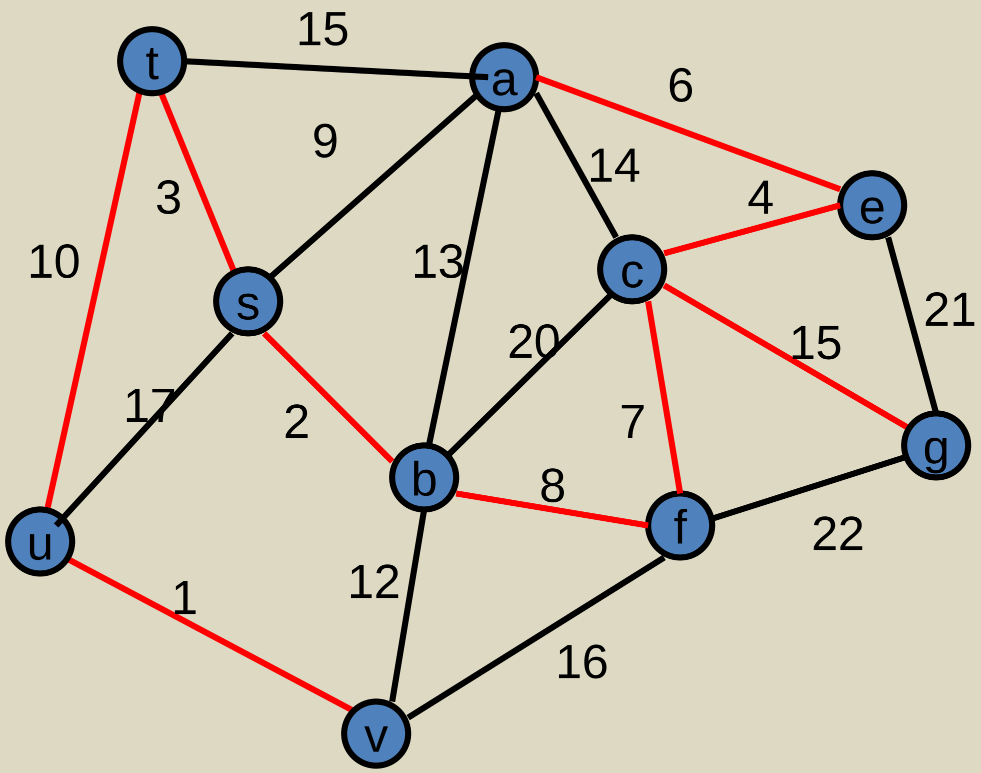
CSE 417
Algorithms and Complexity

Winter 2023
Lecture 14
Finishing Minimum Spanning Trees

Announcements

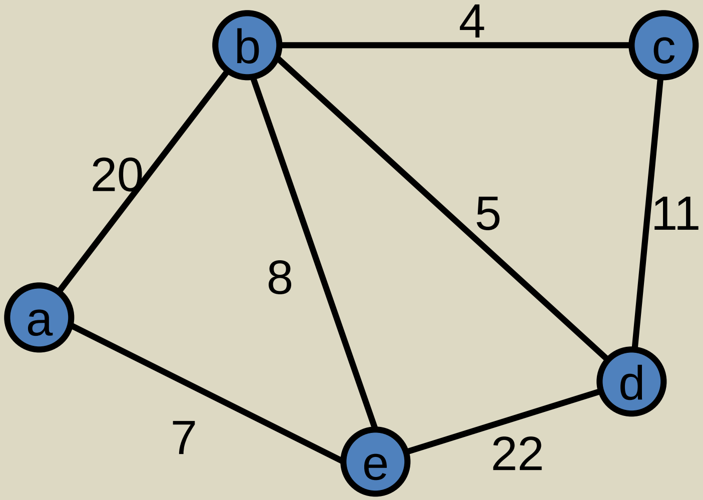
- Midterm, Wednesday, Feb 8
    - Closed book, closed notes, no calculators
    - Time limit: 50 minutes
    - Answer the problems on the exam paper.
    - If you need extra space use the back of a page
    - Problems are not of equal difficulty, if you get stuck on a problem, move on.
    - ``Justify your answer'' means give a short and convincing explanation. Depending on the situation, justifications can involve counter examples, or cite results established in the text or in lecture.

Minimum Spanning Tree



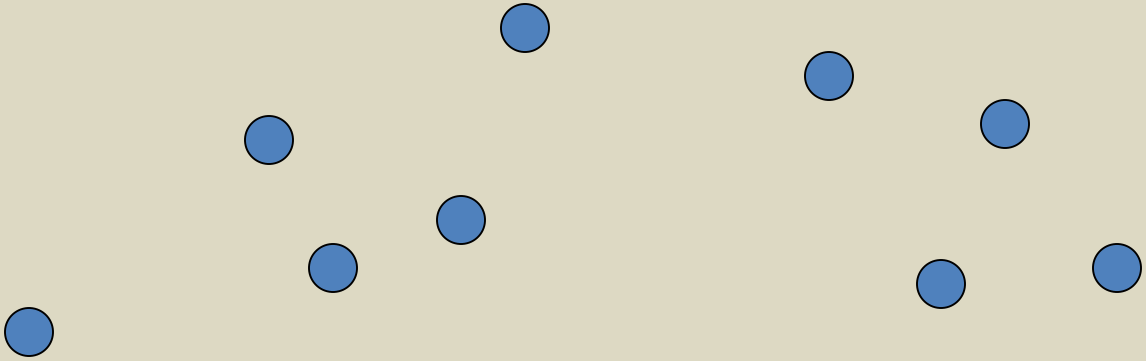
Greedy Algorithms for Minimum Spanning Tree

- Prim's Algorithm: Extend a tree by including the cheapest out going edge
- Kruskal's Algorithm: Add the cheapest edge that joins disjoint components



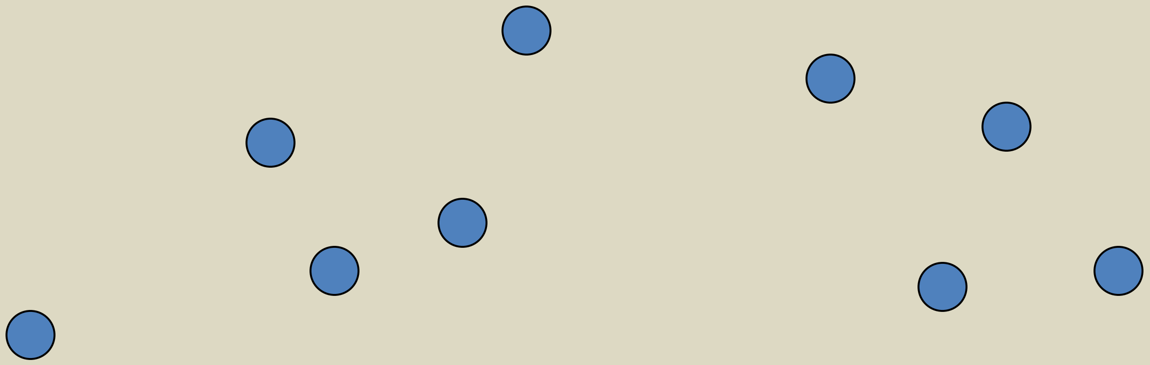
Application: Clustering

- Given a collection of points in an r-dimensional space and an integer K, divide the points into K sets that are closest together

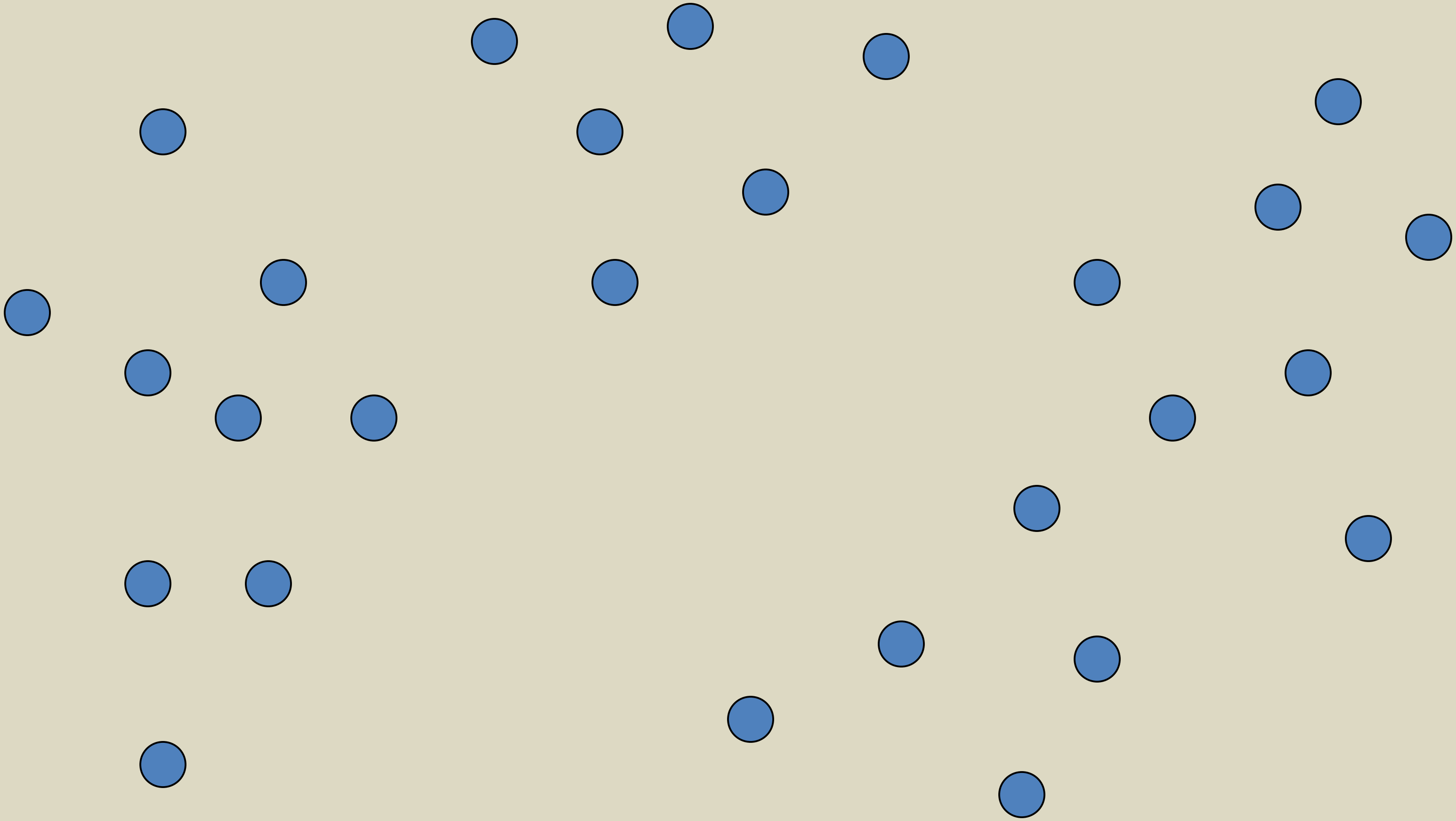


Distance clustering

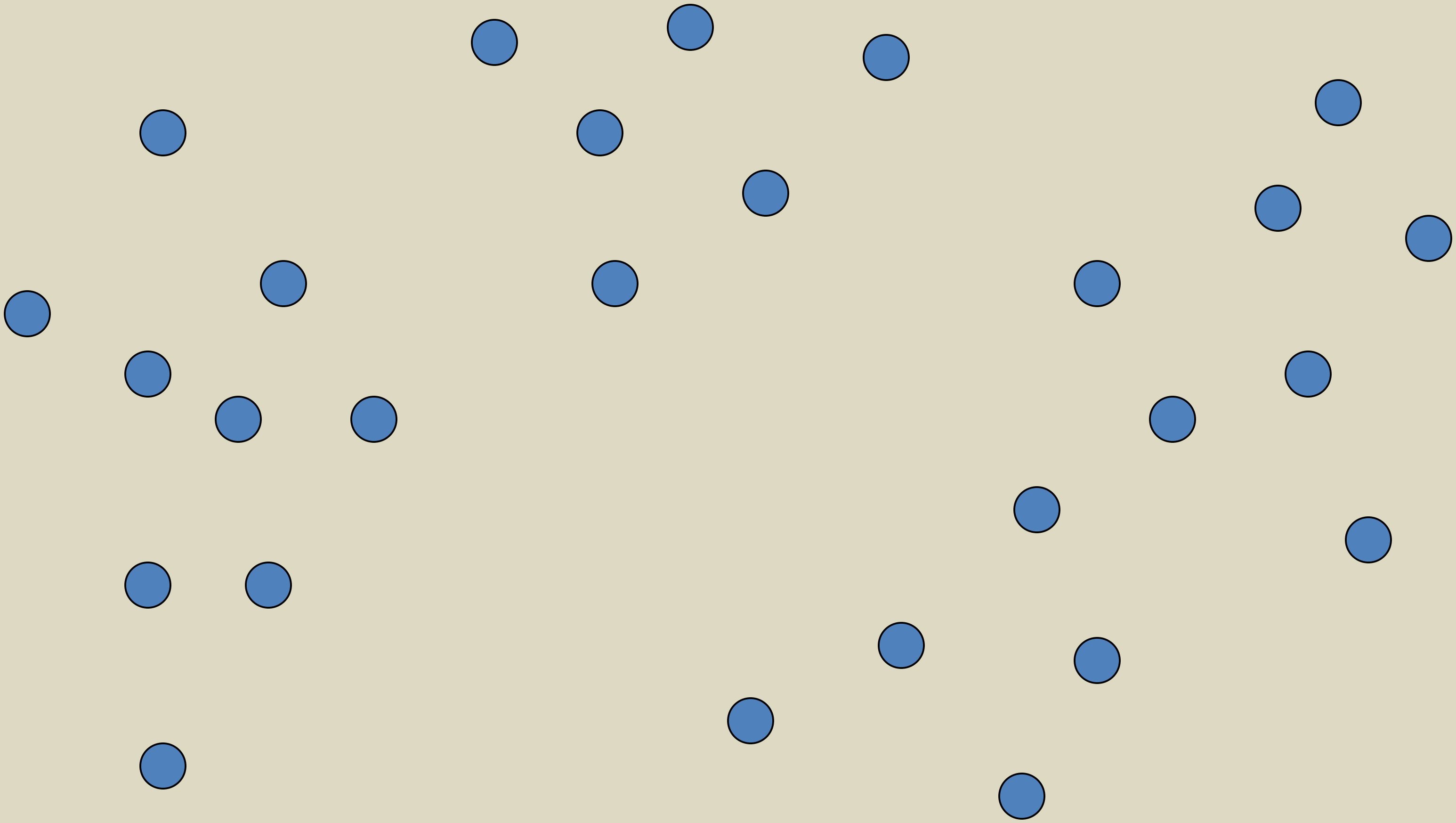
- Divide the data set into K subsets to maximize the distance between any pair of sets
  - dist $(S_1, S_2) = \min \{\mathrm{dist}(x, y) \mid x \text{ in } S_1, y \text{ in } S_2\}$



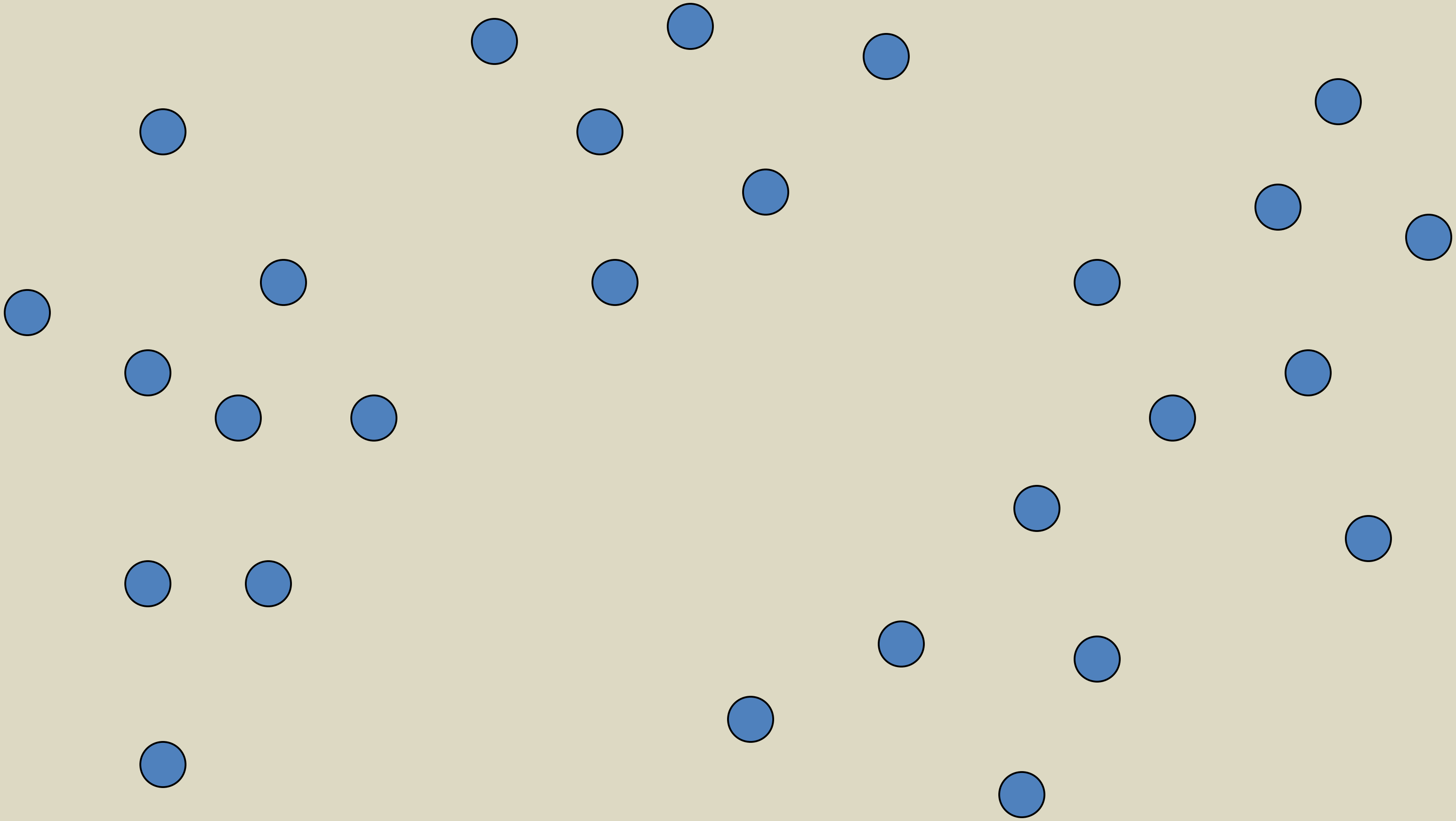
Divide into 2 clusters



Divide into 3 clusters



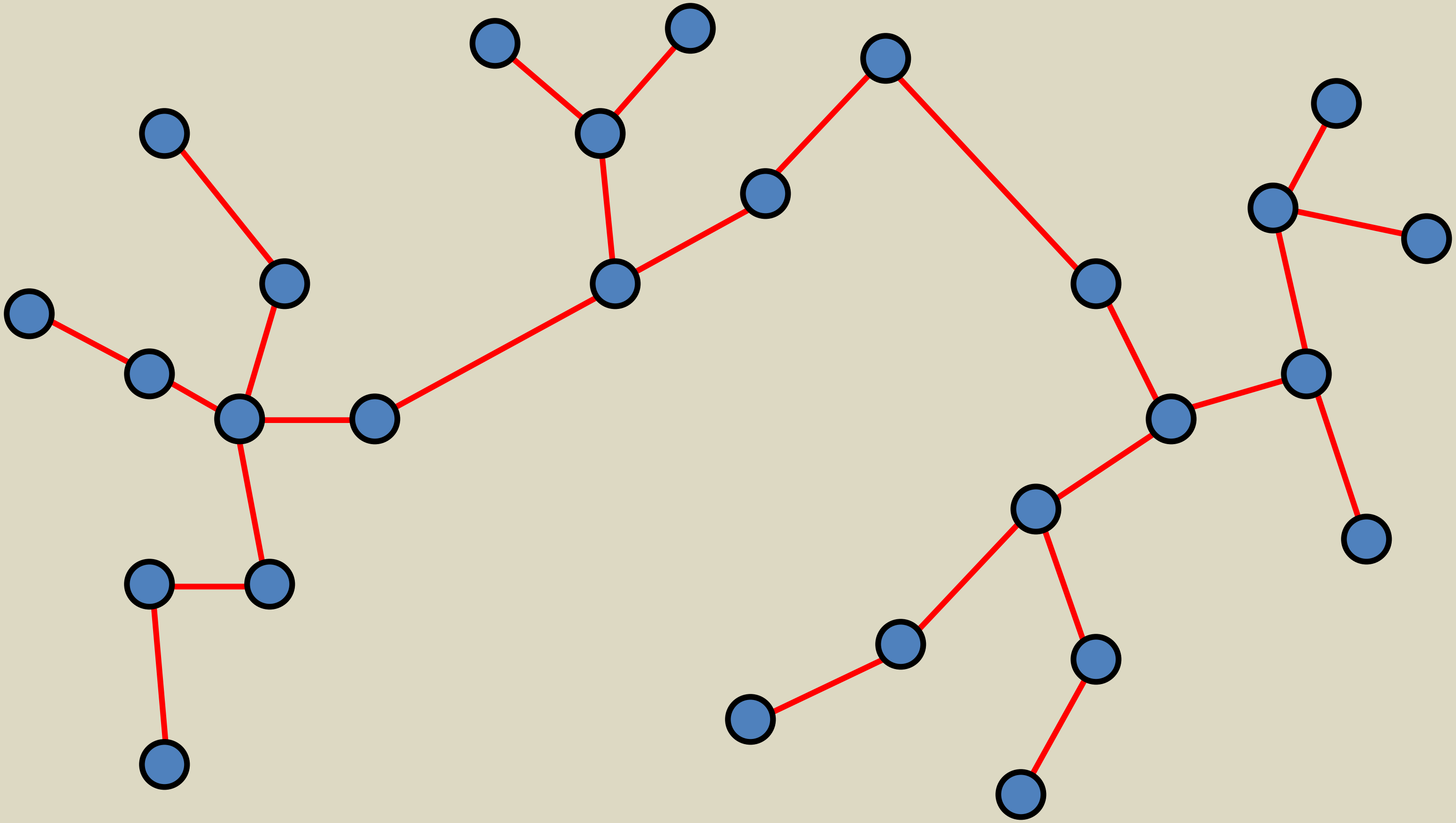
Divide into 4 clusters



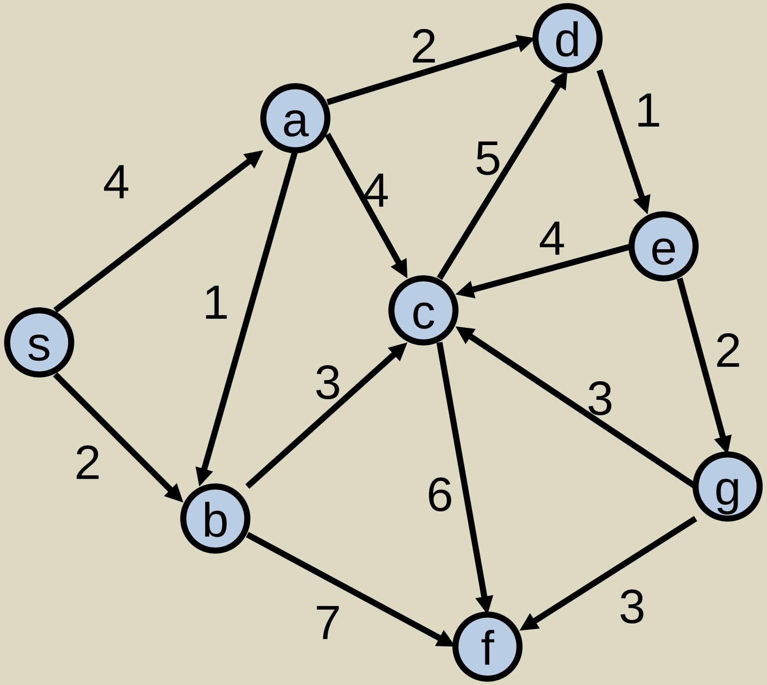
Distance Clustering Algorithm

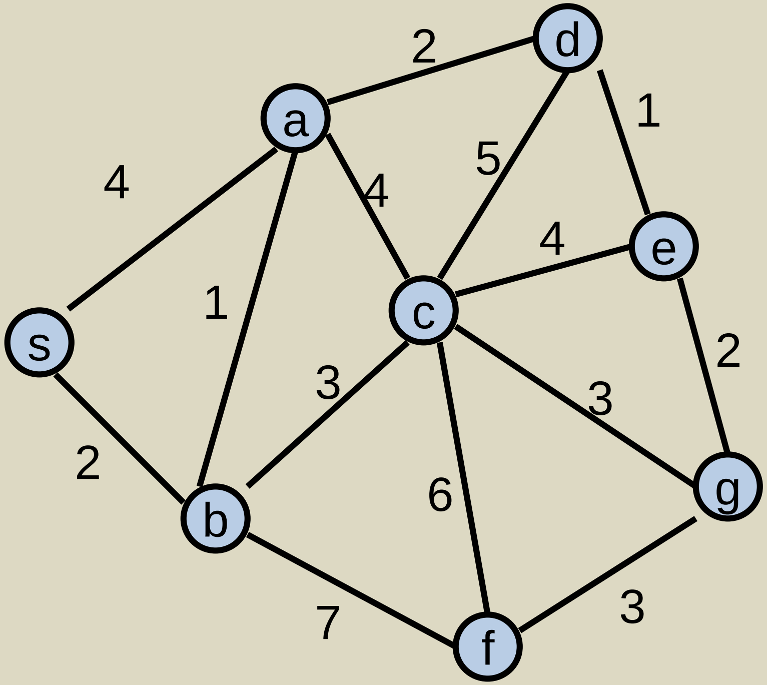
```
Let $C = \{\{v_1\}, \{v_2\},. . ., \{v_n\}\}$;  $T = \{ \}$
while $|C| > K$
        Let $e = (u, v)$ with $u$ in $C_i$ and $v$ in $C_j$ be the
        minimum cost edge joining distinct sets in $C$
        Replace $C_i$ and $C_j$ by $C_i$ U $C_j$
```

K-clustering



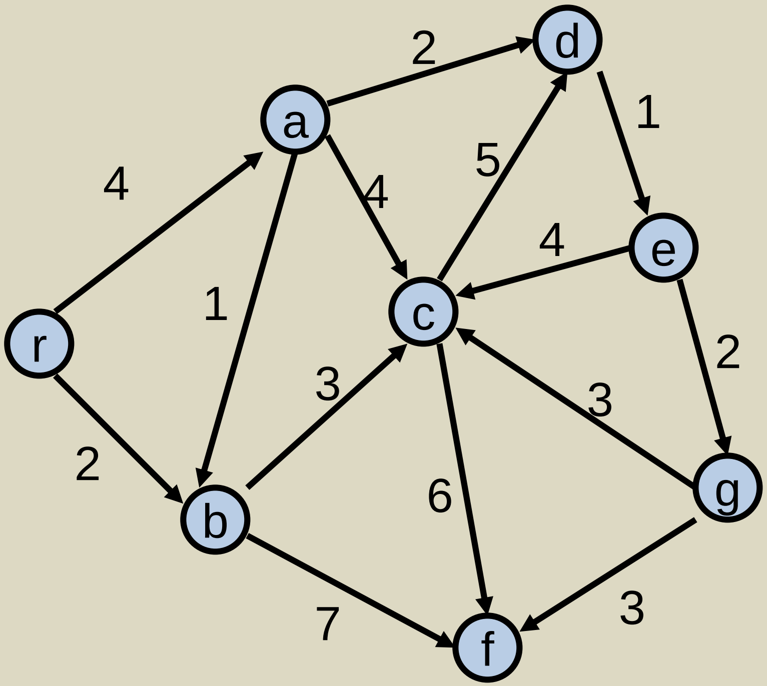
Shortest paths in directed graphs vs undirected graphs

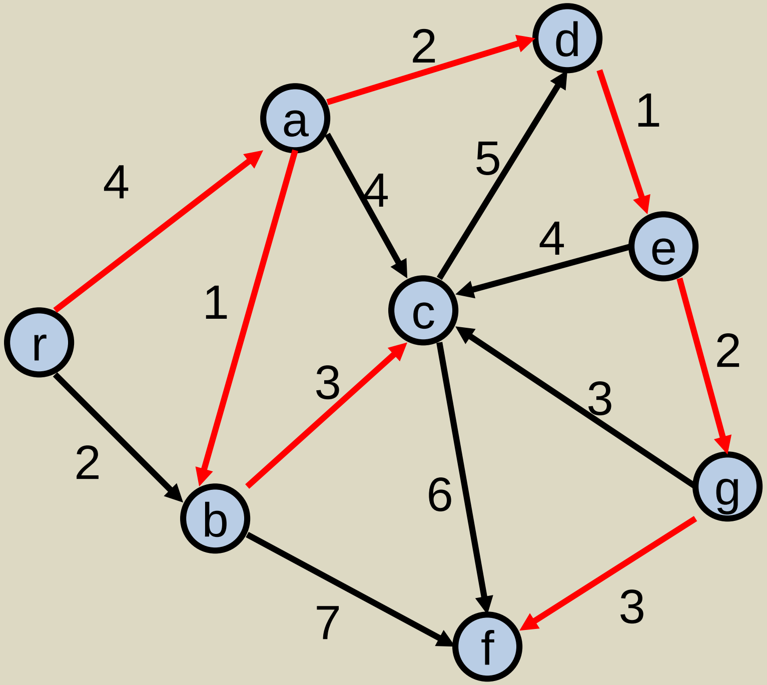




What about the minimum spanning tree of a directed graph?

- Must specify the root r
- Branching: Out tree with root r

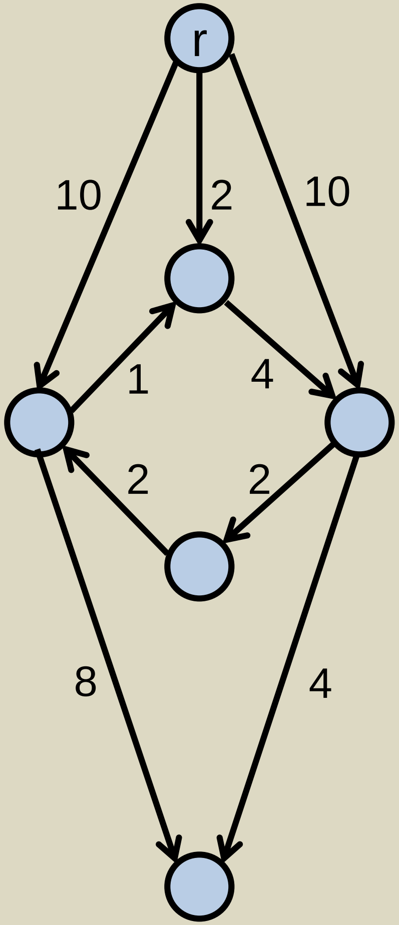


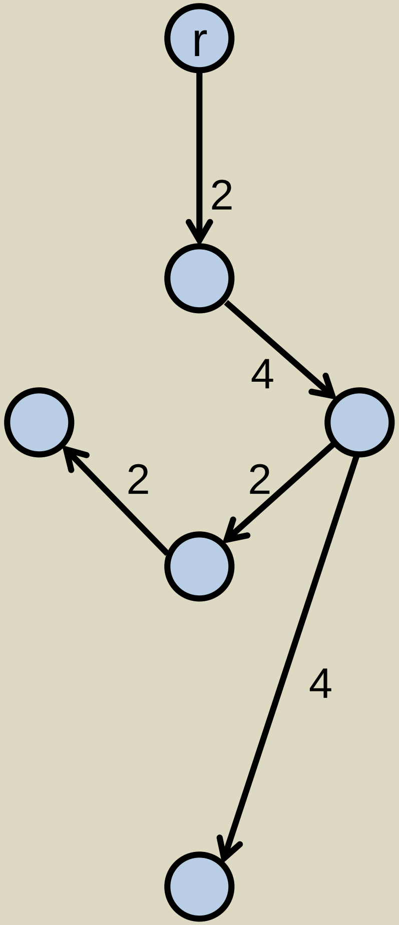


Assume all vertices reachable from r

Also called an arborescence

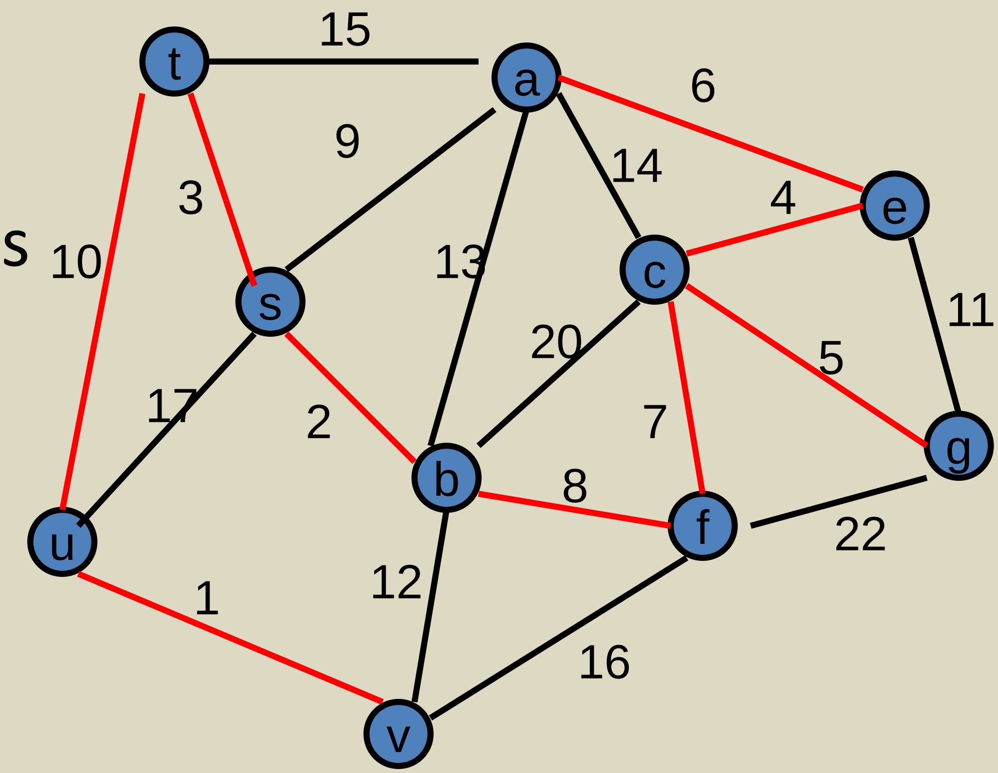
Finding a minimum branching





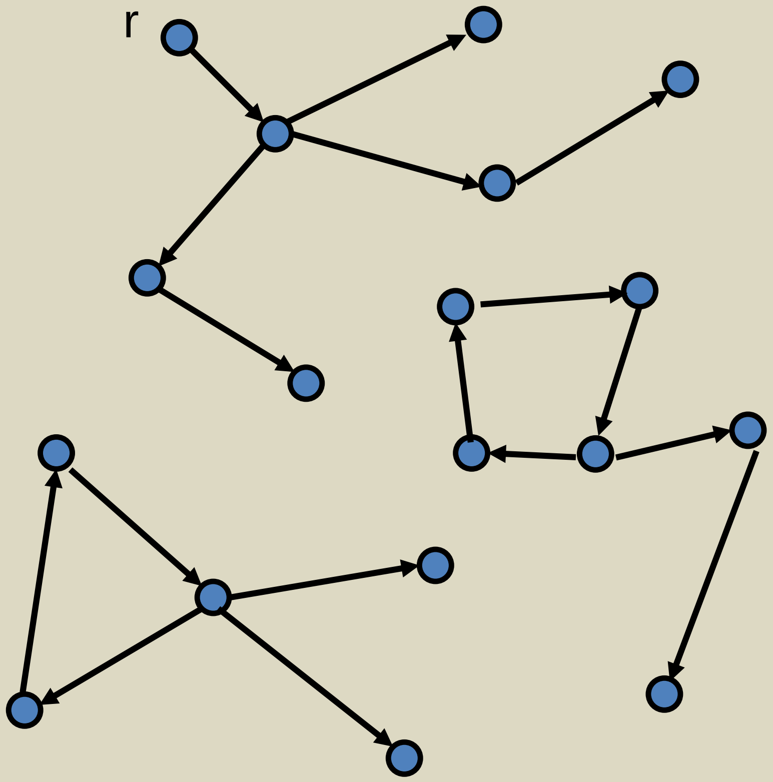
Another MST Algorithm

- Choose minimum cost edge into each vertex
- Merge into components
- Repeat until done



Idea for branching algorithm

- Select minimum cost edge going into each vertex
- If graph is a branching then done
- Otherwise collapse cycles and repeat



Midterm Questions????