Lecture08

CSE 417
Algorithms and Complexity

Greedy Algorithms
Winter 2023
Lecture 8

1/23/2023 CSE 417 1

Announcements

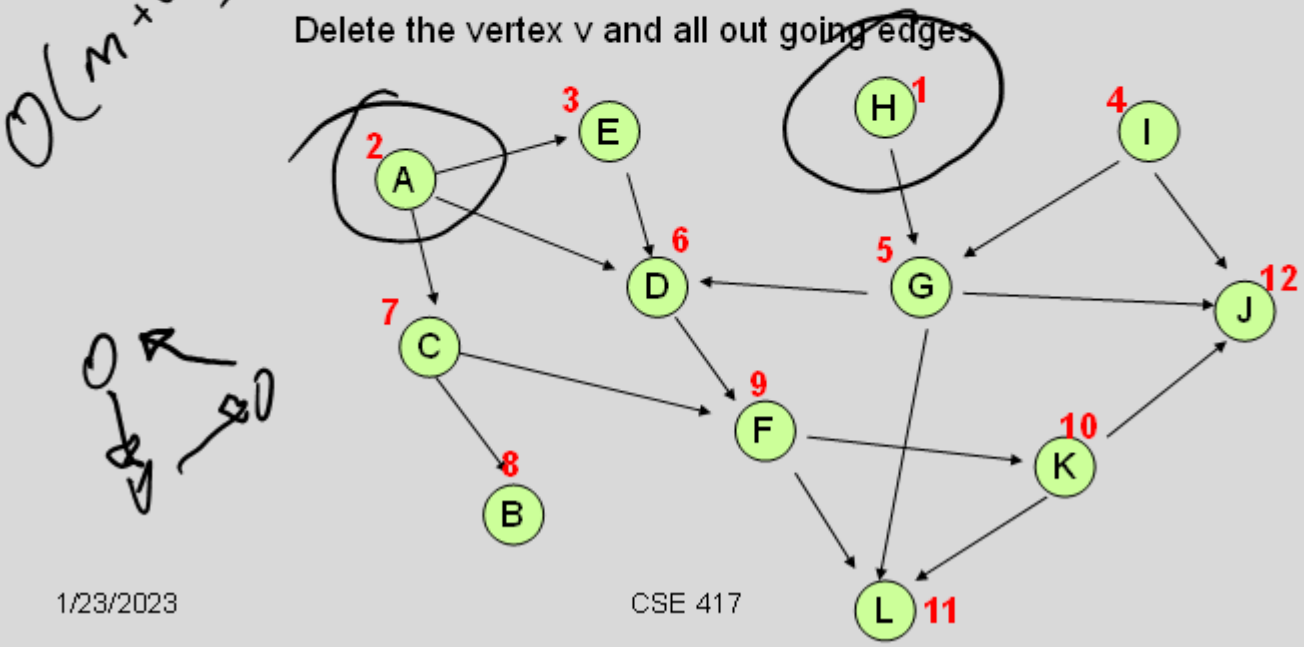
- Reading
  - For today, sections 4.1, 4.2,
  - For next week sections 4.4, 4.5, 4.7, 4.8
- Homework 3 is available
  - Random Graphs

Highlight from last lecture: Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

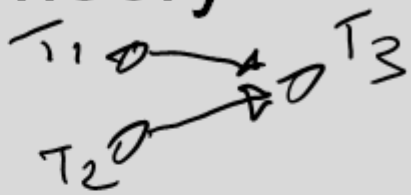
Delete the vertex v and all out going edges



Greedy Algorithms

- Solve problems with the simplest possible algorithm
- The hard part: showing that something simple actually works
- Pseudo-definition
  - An algorithm is Greedy if it builds its solution by adding elements one at a time using a simple rule

Scheduling Theory



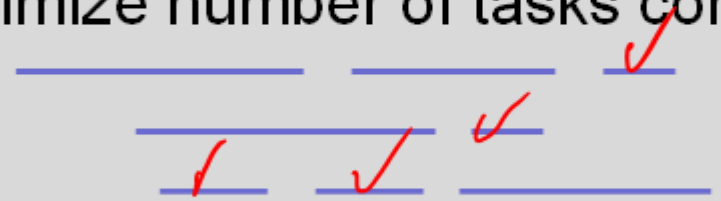
- Tasks
  - Processing requirements, release times, deadlines
- Processors
- Precedence constraints
- Objective function
  - Jobs scheduled, lateness, total execution time

1 - processor

K - processors

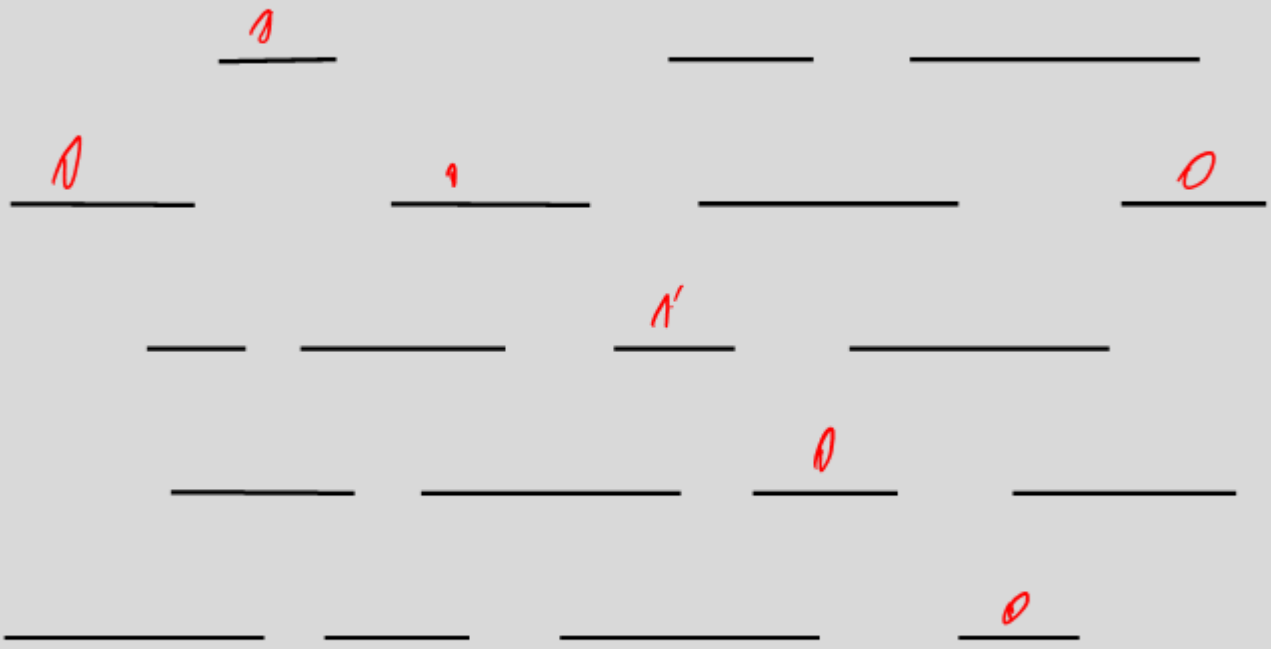
Interval Scheduling

- Tasks occur at fixed times
- Single processor
- Maximize number of tasks completed



- Tasks {1, 2, . . . N}
- Start and finish times: s(i), f(i)

What is the largest solution?



Greedy Algorithm for Scheduling

Template

Let T be the set of tasks, construct a set of independent tasks I, A is the rule determining the greedy algorithm

I = { }

While (T is not empty)

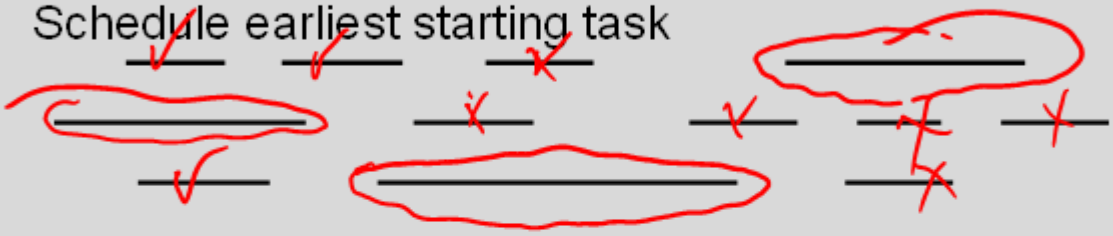
    Select a task t from T by a rule A

    Add t to I

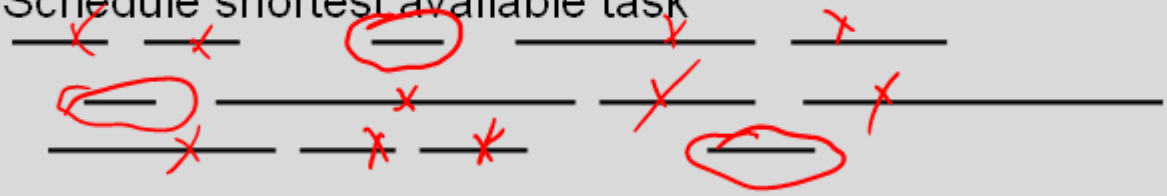
    Remove t and all tasks incompatible with t from T

Simulate the greedy algorithm for each of these heuristics

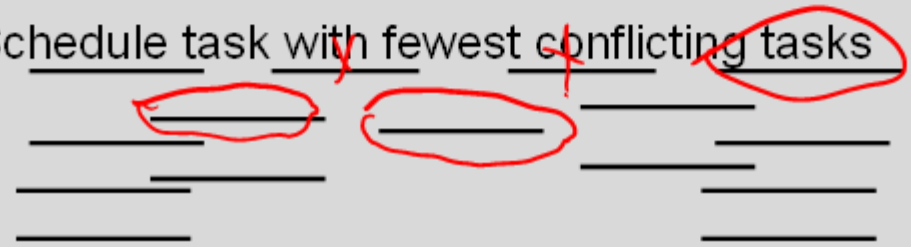
Schedule earliest starting task



Schedule shortest available task



Schedule task with fewest conflicting tasks



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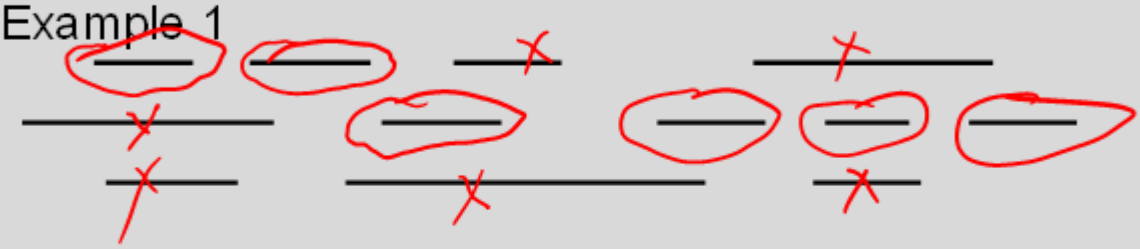
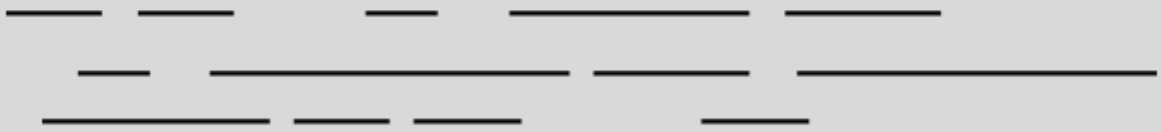
Greedy solution based on earliest finishing time

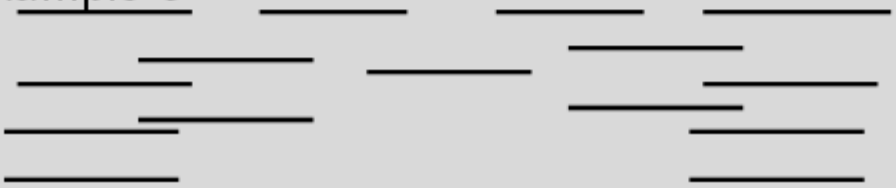
Example 1



Example 2



Example 3



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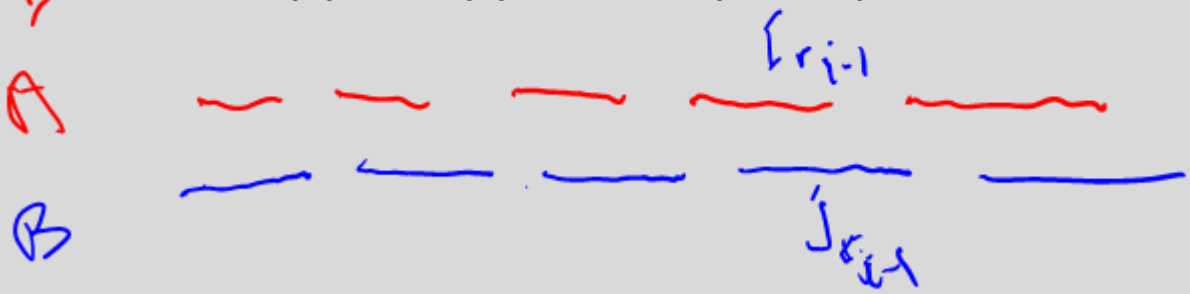
Theorem: Earliest Finish Algorithm is Optimal

- Key idea: Earliest Finish Algorithm stays ahead
- Let A = $\{i_1, \ldots, i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let B = $\{j_1, \ldots, j_m\}$ be the set of tasks found by a different algorithm in increasing order of finish times
- Show that for $r \leq \min(k, m)$, $f(i_r) \leq f(j_r)$

Stay ahead lemma

- A always stays ahead of B, $f(i_r) \leq f(j_r)$
- Induction argument
  - $f(i_1) \leq f(j_1)$
  - If $f(i_{r-1}) \leq f(j_{r-1})$ then $f(i_r) \leq f(j_r)$

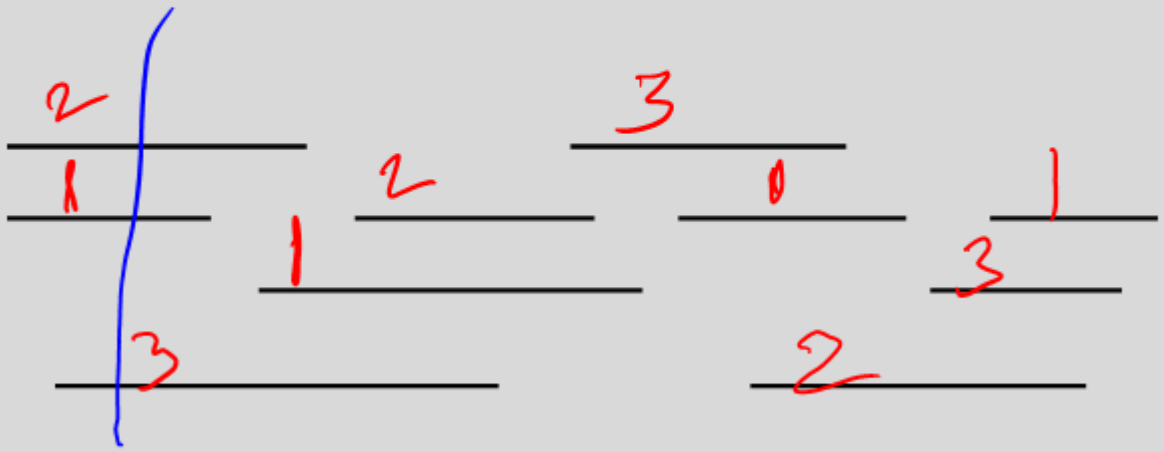


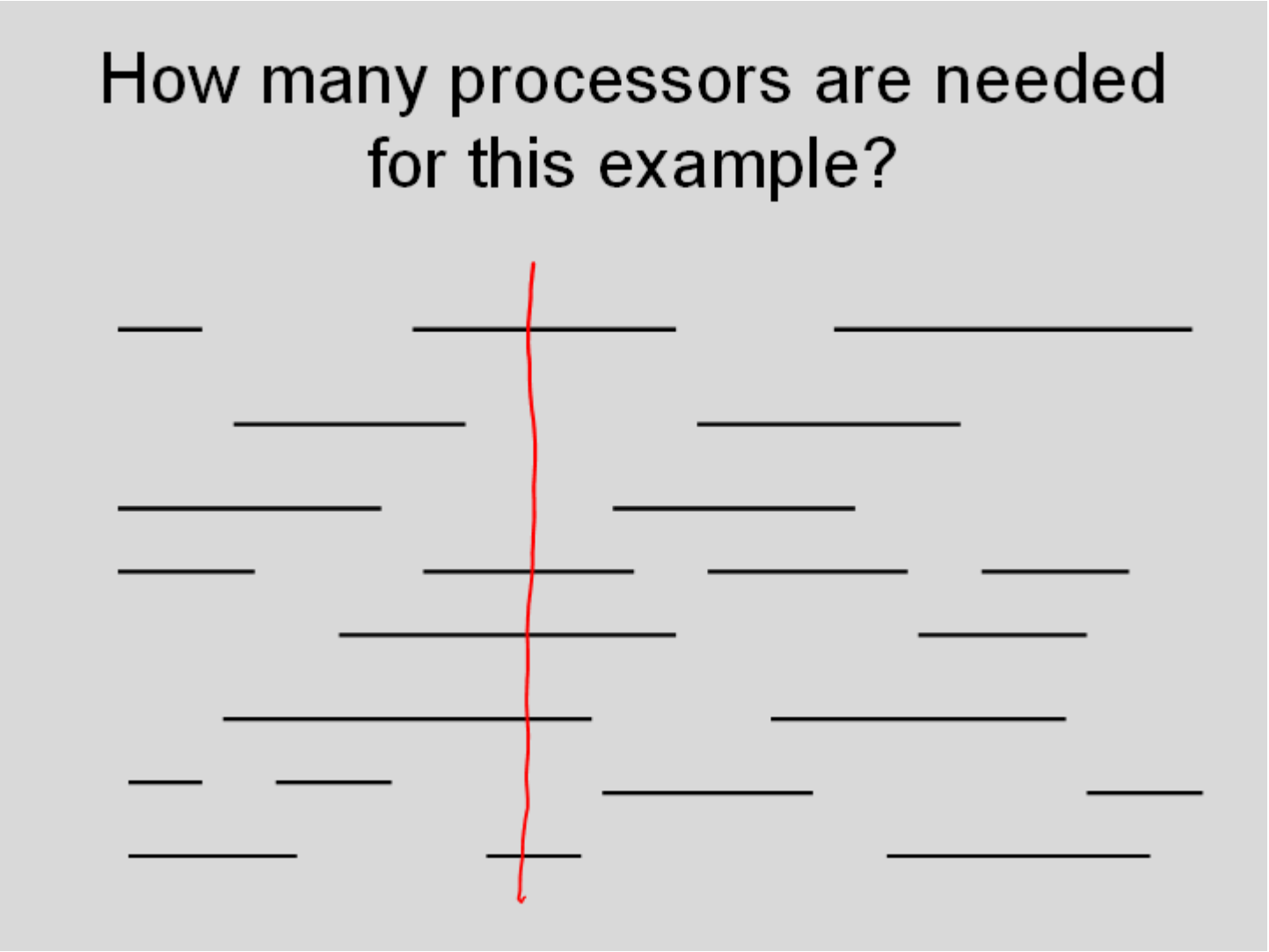
Completing the proof

- Let A = $\{i_1, \ldots, i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let O = $\{j_1, \ldots, j_m\}$ be the set of tasks found by an optimal algorithm in increasing order of finish times
- If k < m, then the Earliest Finish Algorithm stopped before it ran out of tasks

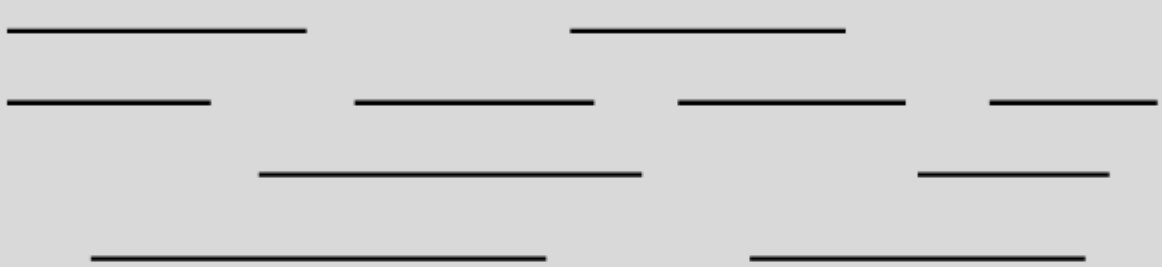
Scheduling all intervals

- Minimize number of processors to schedule all intervals

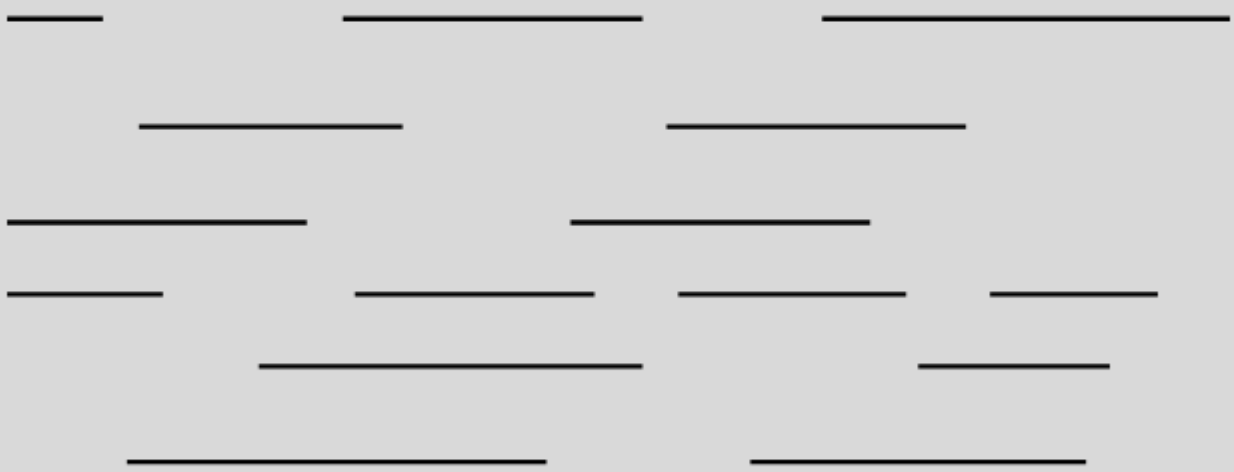



How many processors are needed for this example?

Prove that you cannot schedule this set of intervals with two processors



Depth: maximum number of intervals active



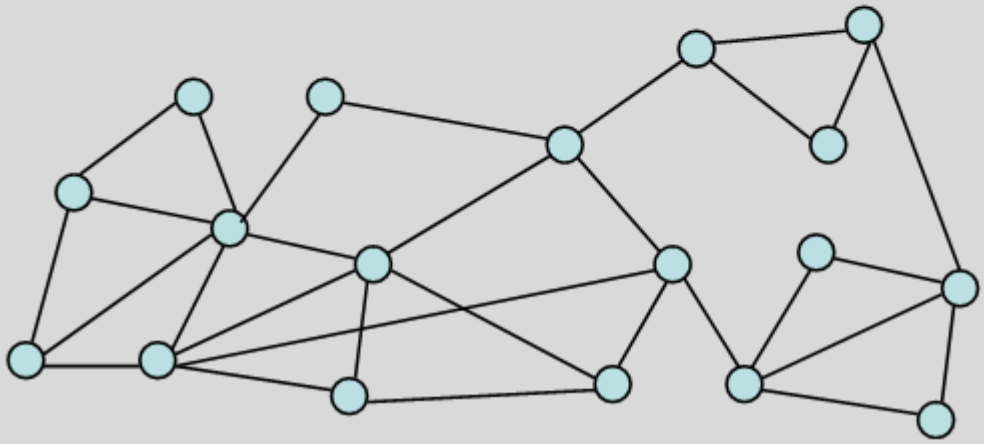
Algorithm

- Sort by start times
- Suppose maximum depth is d, create d slots
- Schedule items in increasing order, assign each item to an open slot

- Correctness proof: When we reach an item, we always have an open slot

Greedy Graph Coloring

Theorem: An undirected graph with maximum degree K can be colored with K+1 colors

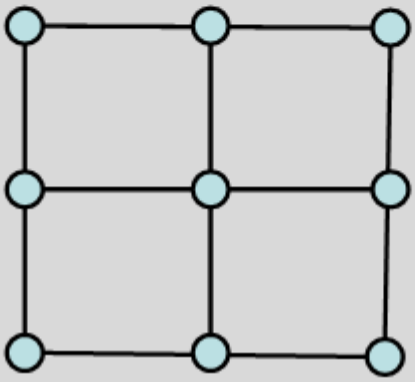


Coloring Algorithm, Version 1

```
Let k be the largest vertex degree
Choose k+1 colors

for each vertex v
      Color[v] = uncolored

for each vertex v
      Let c be a color not used in N[v]
      Color[v] = c
```



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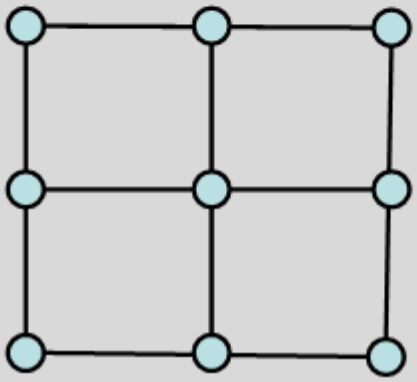
Coloring Algorithm, Version 2

```
for each vertex v
      Color[v] = uncolored

for each vertex v
      Let c be the smallest color not used in N[v]
      Color[v] = c
```

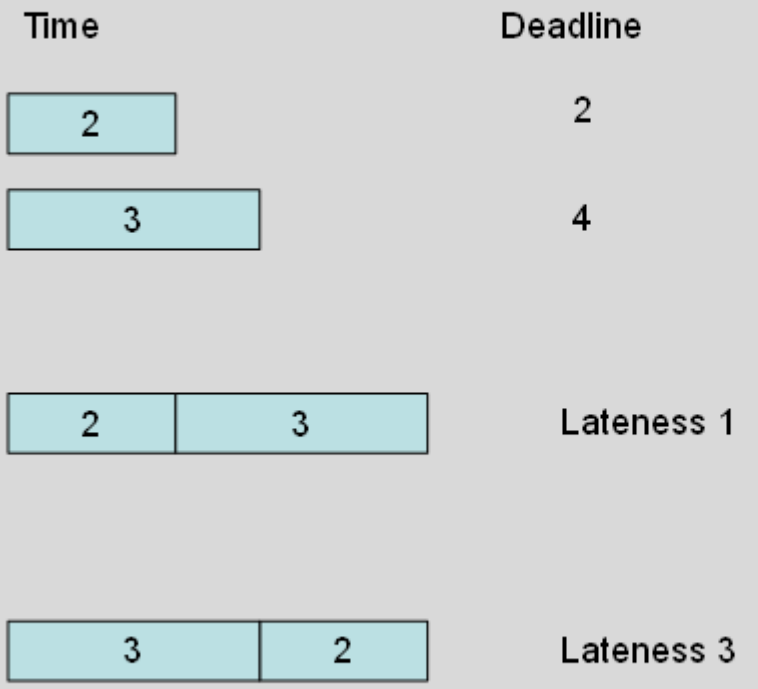


Scheduling tasks

- Each task has a length t_i and a deadline d_i
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

- Goal minimize maximum lateness
  - Lateness = $f_i - d_i$ if $f_i \geq d_i$

Example



Determine the minimum lateness