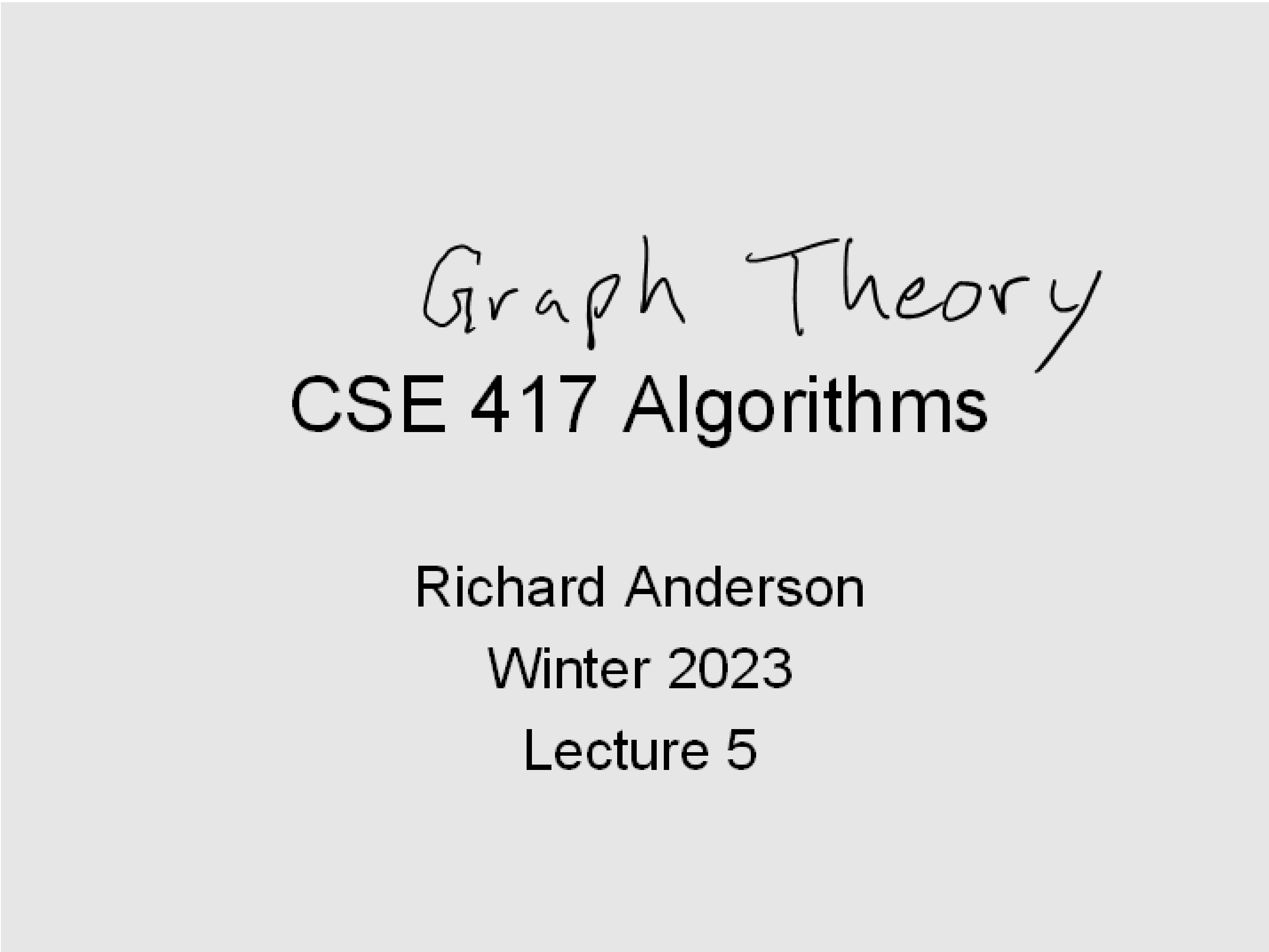
Lecture05


Graph Theory
CSE 417 Algorithms
Richard Anderson
Winter 2023
Lecture 5

Announcements

- HW 1 Due tonight on Gradescope, turn in open until Sunday, 11:59 pm
- HW 2 Available
- No class on Monday, January 16

Worst Case Runtime Function

- Problem P: Given instance I compute a solution S
- A is an algorithm to solve P
- T(I) is the number of steps executed by A on instance I
- T(n) is the maximum of T(I) for all instances of size n

Ignore constant factors

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight
- Express run time as $T(n) = O(f(n))$

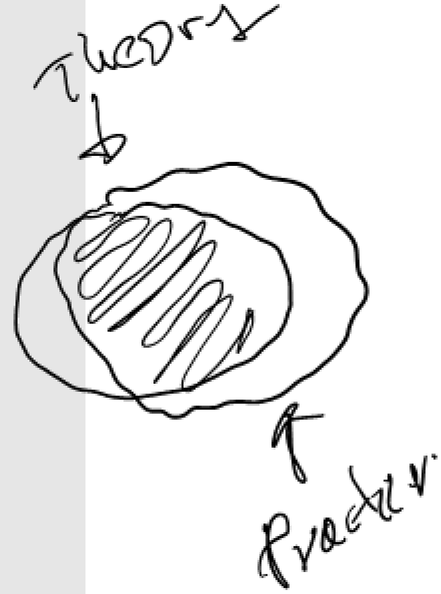
Formalizing growth rates

- T(n) is O(f(n)) [$T : Z^+ \rightarrow R^+$]
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$
- T(n) is Ω(f(n))
  - T(n) is at least a constant multiple of f(n)
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- T(n) is Θ(f(n)) if T(n) is O(f(n)) and T(n) is Ω(f(n))

Efficient Algorithms

$O(n^4)$

- Polynomial Time (P): Class of all problems that can be solved with algorithms that have polynomial runtime functions
- Polynomial Time has been a very successful tool for theoretical computer science
- Problems in Polynomial Time often have practical solutions

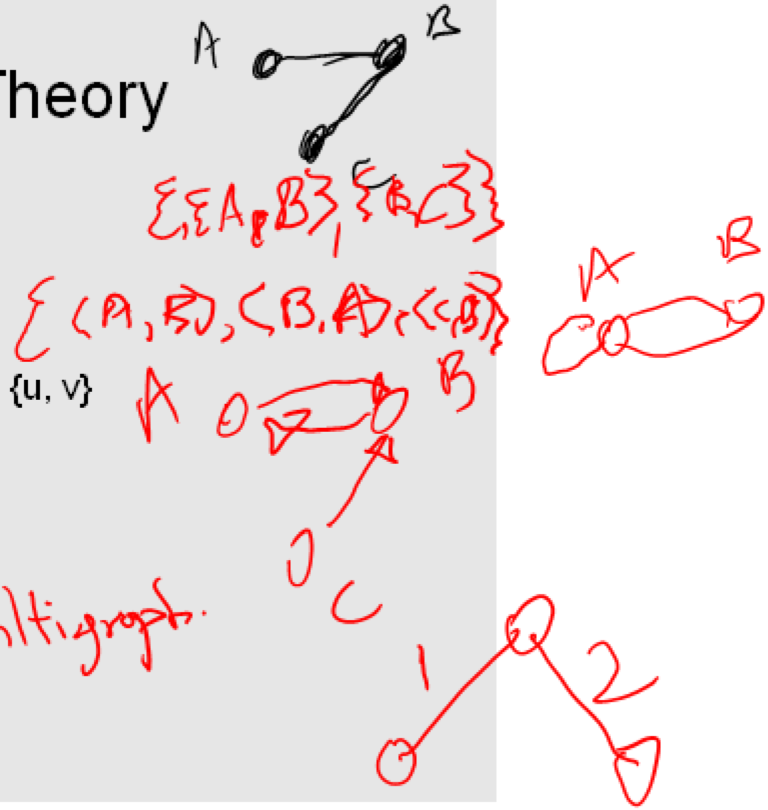


$|V| = n$

$|E| = m$

Graph Theory

- G = (V, E)
  - V – vertices
  - E – edges
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges – Multigraph.
  - Self loops



Examples

FaceBook Graph = (V, E)

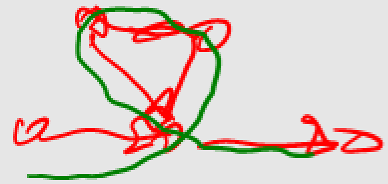
Profiles

FB Friend

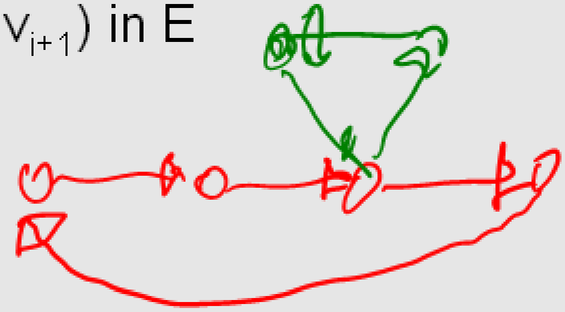
$|V| \approx 2B$

Currency Conversion Graph

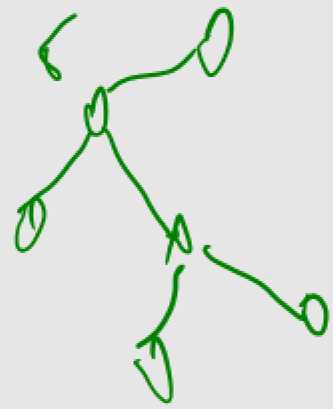
Definitions



- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle

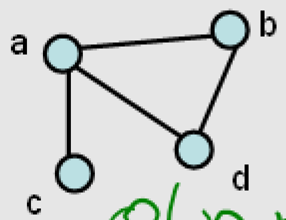


- **Neighborhood**
  - N(v) $N^+(v)$ $N^-(v)$
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted



$|V| = n \quad |E| = m$

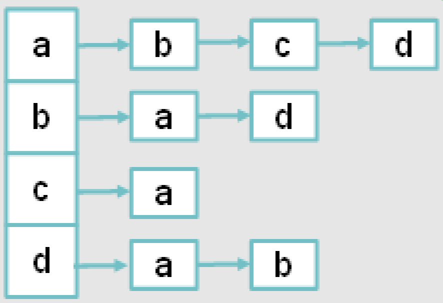
Graph Representation



V = { a, b, c, d}

E = { {a, b}, {a, c}, {a, d}, {b, d} }

$O(n+m)$



Adjacency List

n^2 space

	1	1	1
1		0	1
1	0		0
1	1	0	

Incidence Matrix

Implementation Issues

- Graph with n vertices, m edges
- Operations
  - Lookup edge
  - Add edge
  - Enumeration edges
  - Initialize graph
- Space requirements