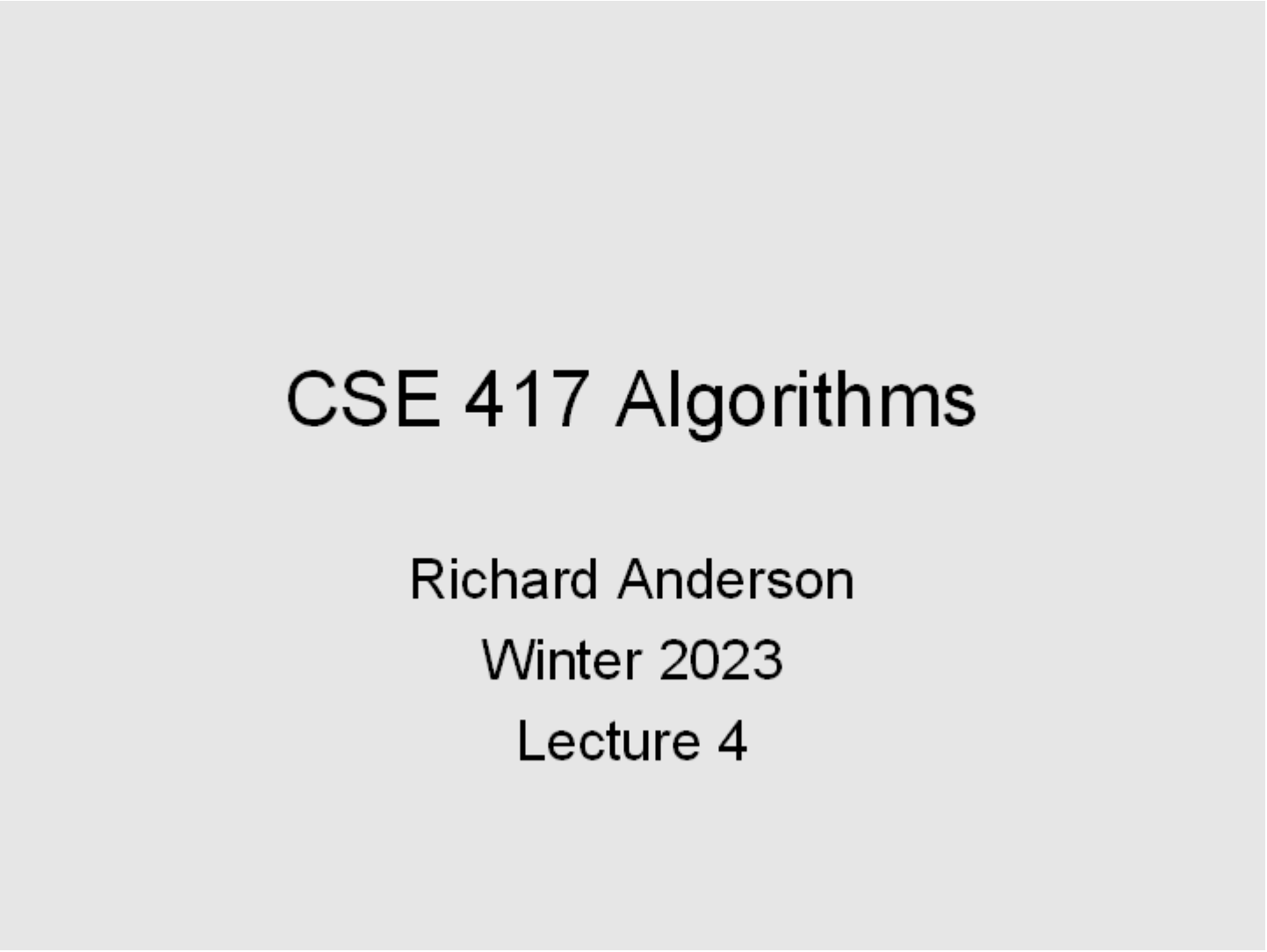
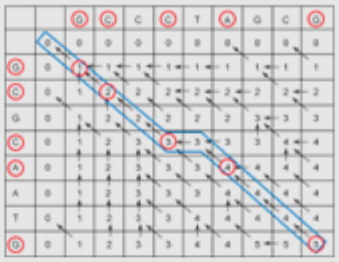
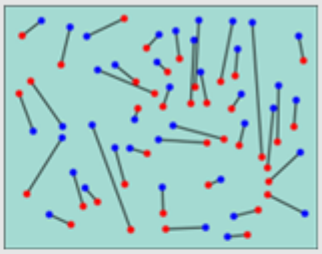
# Lecture04

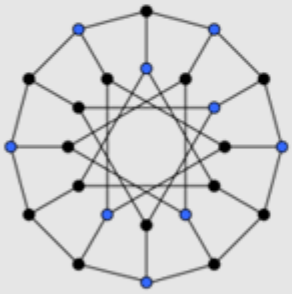


# Announcements

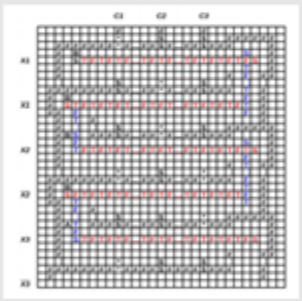
- Reading
  - Chapter 2.1, 2.2
  - Chapter 3 (Mostly review)
  - Start on Chapter 4
- Homework Guidelines
  - Submit homework with Gradescope
  - Describing an algorithm
    - Clarity is most important
    - Pseudocode generally preferable to just English
      - But sometimes both methods combined work best
  - Prove that your algorithm works
    - A proof is a "convincing argument"
  - Give the run time for your algorithm
    - Justify that the algorithm satisfies the runtime bound
  - You may lose points for style
  - Homework assignments will (probably) be worth the same amount





# Five Problems



Scheduling
Weighted Scheduling
Bipartite Matching
Maximum Independent Set
Competitive Facility Location

# Summary – Five Problems

- Scheduling – greedy
- Weighted Scheduling – dynamic programming
- Combinatorial Optimization
- Maximum Independent Set – NP
- Competitive Scheduling – PSpace

# What does it mean for an algorithm to be efficient?

- Space & Time
- multiple resources

# Definitions of efficiency

- Fast in practice
- Qualitatively better worst case performance than a brute force algorithm

# Polynomial time efficiency

- An algorithm is efficient if it has a polynomial run time
- Run time as a function of problem size
  - Run time: count number of instructions executed on an underlying model of computation
  - T(n): maximum run time for all problems of size at most n

Average

# Polynomial Time

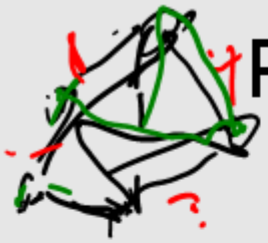
- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)

# Why Polynomial Time?

- Generally, polynomial time seems to capture the algorithms which are efficient in practice

- The class of polynomial time algorithms has many good, mathematical properties

- set polynomials is closed under natural operations



# Polynomial vs. Exponential Complexity

- Suppose you have an algorithm which takes n! steps on a problem of size n
- If the algorithm takes one second for a problem of size 10, estimate the run time for the following problems sizes:

| 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|
| 2 min | 6 hrs | 2 wks | 50 y | 20K y |

# Ignoring constant factors

- Express run time as O(f(n))
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award

# Why ignore constant factors?

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight

# Why emphasize growth rates?

- The algorithm with the lower growth rate will be faster for all but a finite number of cases
- Performance is most important for larger problem size
- As memory prices continue to fall, bigger problem sizes become feasible
- Improving growth rate often requires new techniques

# Formalizing growth rates

- T(n) is O(f(n)) $[T : Z^+ \rightarrow R^+]$
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, $n_0$, such that for $n > n_0$, $T(n) < c\, f(n)$
- T(n) is O(f(n)) will be written as:
  T(n) = O(f(n))
  - Be careful with this notation

$T(n) \in O(f(n))$

# Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let c = 6

Let $n_0$ = 5

$n > 5$

$3n^2 + 5n + 20 < 3n^2 + n^2 + n^2$

$= 5n^2 < 6n^2$

T(n) is O(f(n)) if there exist c, $n_0$, such that for $n > n_0$,
$T(n) < c\ f(n)$

# Order the following functions in increasing order by their growth rate

a) $n \log^4 n$

b) $2n^2 + 10n$

c) $2^{n/100}$

d) $1000n + \log^8 n$

e) $n^{100}$

f) $3^n$

g) $1000 \log^{10} n$

h) $n^{1/2}$

# Lower bounds

- T(n) is $\Omega(f(n))$
  - T(n) is at least a constant multiple of f(n)
  - There exists an $n_0$, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of $\Omega$ vary

$3n^2$ is $\Theta(n^2)$

- T(n) is $\Theta(f(n))$ if T(n) is O(f(n)) and T(n) is $\Omega(f(n))$

# Useful Theorems

- If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = c for c > 0 then f(n) = $\Theta$(g(n))

- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))

- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

# Ordering growth rates

- For $b > 1$ and $x > 0$
  - $\log^b n$ is $O(n^x)$
- For $r > 1$ and $d > 0$
  - $n^d$ is $O(r^n)$