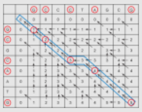
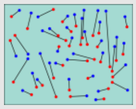
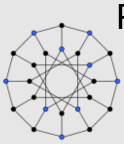
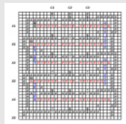




# Five Problems

CSE 417

Richard Anderson

Winter 2023, Lecture 3

## Announcements

- Course website:
  [//courses.cs.washington.edu/courses/cse417/23wi/](//courses.cs.washington.edu/courses/cse417/23wi/)
- Homework Due Friday
- Office Hours:

Richard Anderson, Tuesday, 2:30-3:30 PM, Thursday, 4:00-5:00 PM
Nickolay Perezhogin, Thursday, 10:00 AM - 12:00 PM
Artin Tajdini, Wednesday, 2:30 PM - 4:30 PM
Tom Zhaoyang Tian,Wednesday, 12:30 PM - 1:30 PM Friday, 4:30 PM - 5:30 PM
Michael Wen, Monday, 11:30 AM - 12:30 PM Thursday, 1:30 PM - 2:30 PM
Albert Weng, Tuesday, 10:30 AM - 11:30 AM Friday, 9:30 AM -10:30 AM
Yilin Zhang, Monday, 3:30 PM - 5:30 PM

## Theory of Algorithms

- What is expertise?
- How do experts differ from novices?

## Introduction of five problems

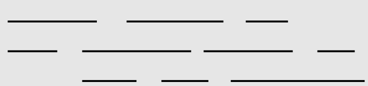
- Show the types of problems we will be considering in the class
- Examples of important types of problems
- Similar looking problems with very different characteristics
- Problems
  - Scheduling
  - Weighted Scheduling
  - Bipartite Matching
  - Maximum Independent Set
  - Competitive Facility Location

## What is a problem?

- Instance
- Solution
- Constraints on solution
- Measure of value

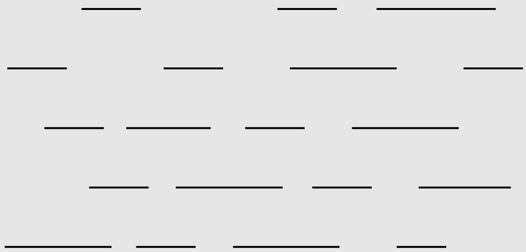
## Problem: Scheduling

- Suppose that you own a banquet hall
- You have a series of requests for use of the hall: $(s_1, f_1), (s_2, f_2), \ldots$



- Find a set of requests as large as possible with no overlap

## What is the largest solution?

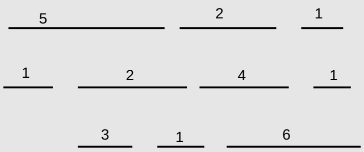


## Greedy Algorithm

- Test elements one at a time if they can be members of the solution
- If an element is not ruled out by earlier choices, add it to the solution
- Many possible choices for ordering (length, start time, end time)
- For this problem, considering the jobs by increasing end time works

## Suppose we add values?

- $(s_i, f_i, v_i)$, start time, finish time, payment
- Maximize value of elements in the solution




## Greedy Algorithms

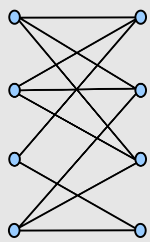
- Earliest finish time
- Maximum value
- Give counter examples to show these algorithms don't find the maximum value solution

## Dynamic Programming

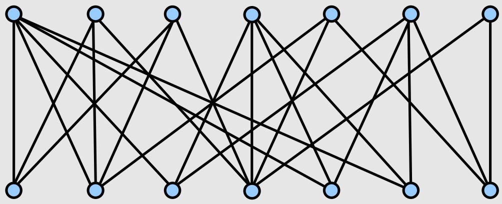
- Requests $R_1, R_2, R_3, \ldots$
- Assume requests are in increasing order of finish time $(f_1 < f_2 < f_3 \ldots)$
- $Opt_i$ is the maximum value solution of $\{R_1, R_2, \ldots, R_i\}$ containing $R_i$
- $Opt_i = \text{Max}\{ j \mid f_j < s_i \}[Opt_j + v_i]$

## Matching (Combinatorial Optimization)

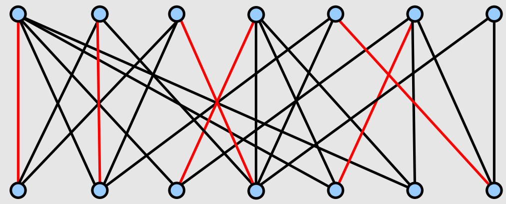
- Given a bipartite graph G=(U,V,E), find a subset of the edges M of maximum size with no common endpoints.
- Application:
  - U: Professors
  - V: Courses
  - (u,v) in E if Prof. u can teach course v



## Find a maximum matching

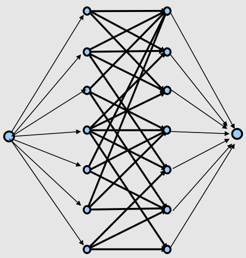


## Augmenting Path Algorithm



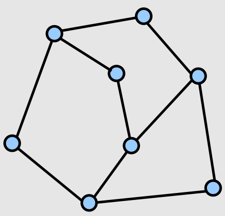
## Reduction to network flow

- More general problem
- Send flow from source to sink
- Flow subject to capacities at edges
- Flow conserved at vertices
- Can solve matching as a flow problem

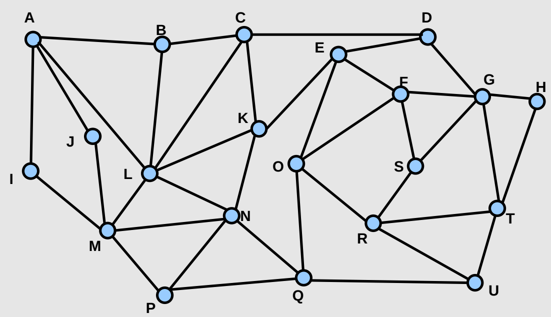


## Maximum Independent Set

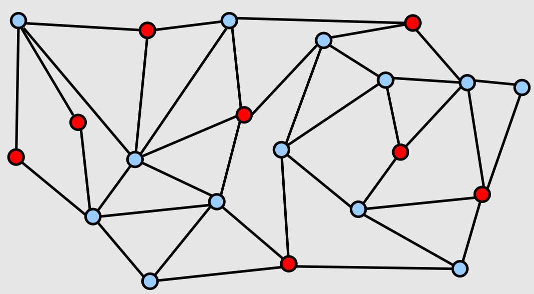
- Given an undirected graph G=(V,E), find a set I of vertices such that there are no edges between vertices of I
- Find a set I as large as possible



## Find a Maximum Independent Set




## Verification: Prove the graph has an independent set of size 8



## Key characteristic

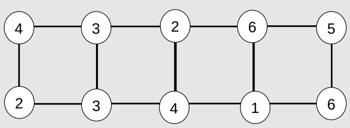
- Hard to find a solution
- Easy to verify a solution once you have one
- Other problems like this
  – Hamiltonian circuit
  – Clique
  – Subset sum
  – Graph coloring

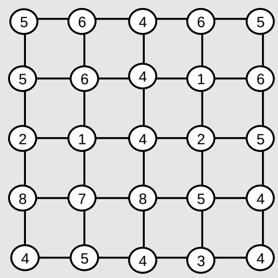
## NP-Completeness

- Theory of Hard Problems
- A large number of problems are known to be equivalent
- Very elegant theory

## Are there even harder problems?

- Simple game:
  – Players alternate selecting nodes in a graph
    - Score points associated with node
    - Remove nodes neighbors
  – When neither can move, player with most points wins




## Competitive Facility Location

- Choose location for a facility
  – Value associated with placement
  – Restriction on placing facilities too close together
- Competitive
  – Different companies place facilities
    - E.g., KFC and McDonald’s

## Complexity theory

- These problems are P-Space complete instead of NP-Complete
  – Appear to be much harder
  – No obvious certificate
    - G has a Maximum Independent Set of size 10
    - Player 1 wins by at least 10 points

## Summary

- Scheduling
- Weighted Scheduling
- Bipartite Matching
- Maximum Independent Set
- Competitive Scheduling