CSE 417
Algorithms and Complexity

Autumn 2023

Lecture 10 – Greedy Algorithms III

Announcements

- Today's lecture
  - Kleinberg-Tardos, 4.3, 4.4
- Friday
  - Kleinberg-Tardos, 4.4, 4.5

- Text book has lots of details on some of the proofs that I cover quickly

Greedy Algorithms

- Solve problems with the simplest possible algorithm
- Today's problems (Sections 4.3, 4.4)
  - Another homework scheduling task
  - Optimal Caching
- Start Dijkstra's shortest paths algorithm

Scheduling Theory

- Tasks
  - Execution time, value, release time, deadline
- Processors
  - Single processor, multiple processors
- Objective Function – many options, e.g.
  - Maximize tasks completed
  - Minimize number of processors to complete all tasks
  - Minimize the maximum lateness
  - Maximize value of tasks completed by deadline

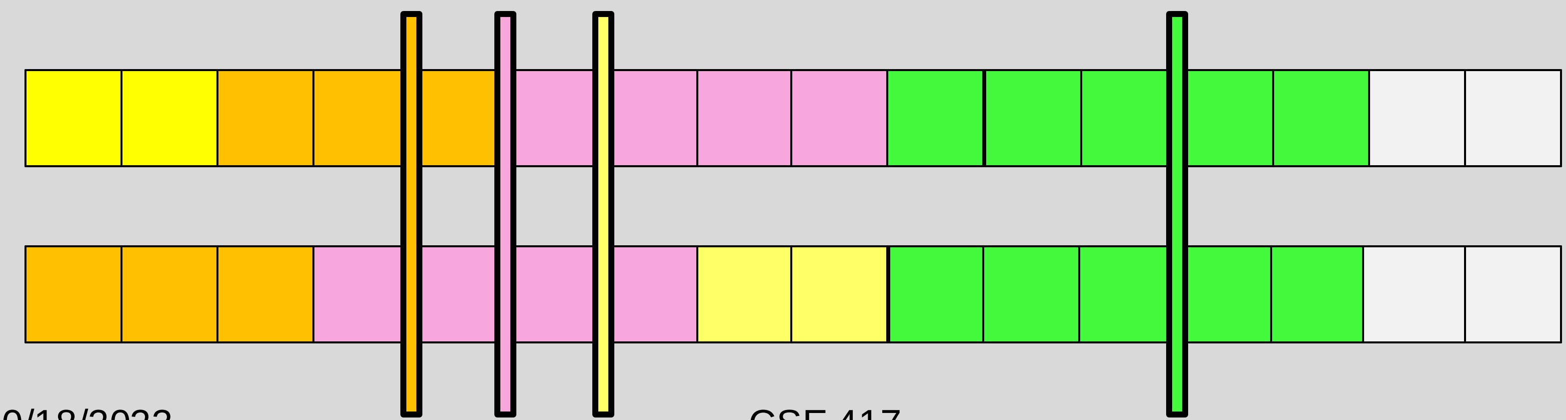
Homework Scheduling

- Each task has a length t_i and a deadline d_i
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

- Goal minimize maximum lateness
  - Lateness: $L_i = f_i - d_i$ if $f_i \geq d_i$

Result: Earliest Deadline First is Optimal for Min Max Lateness

	Time	Deadline	Lateness A_1	Lateness A_2
a_1	2	6	0	3
a_2	3	4	1	0
a_3	4	5	4	2
a_4	5	12	2	2



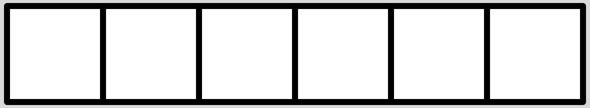
Another version of HW scheduling

- Assign values to HW units
- Maximize value completed by deadlines

- Simplifying assumptions
  - All Homework items take one unit of time
  - All items available at time 0
  - Each item has an integer deadline
  - Each item has a value
  - Maximize value of items completed before their deadlines

Example

Task	**Value**	**Deadline**
T_1	2	2
T_2	3	2
T_3	4	4
T_4	4	4
T_5	5	4
T_6	1	6
T_7	1	6
T_8	6	6



What is the maximum value of tasks you can complete by their deadlines?

What do you do first?

Problem transformation

- Convert to an equivalent problem with release times and a uniform deadline

- If D is the latest deadline, set r'_i as $D-d_i$ and d'_i as D

Greedy Algorithm

- Starting from t = 0, schedule the highest value available task

```
S = ∅;
for i = 0 to D – 1
     Add tasks with release time i to S;
     Remove highest value task t from S;
     Schedule task t at i;
```

Correctness argument

- Show that the item at t = 1 is scheduled correctly
  - The argument can be repeated for t=2, 3, . . .
  - Or the argument can be put in the framework of mathematical induction

First item scheduled is correct

- Let t be the task scheduled at $i = 1$, then there exists an optimal schedule with t at $i = 1$

- Suppose Opt = $\{a_1, a_2, a_3, \ldots\}$ is an optimal schedule:
  - Case 1: $t = a_1$
  - Case 2: $t \notin$ Opt
  - Case 3: $t \neq a_1$ and $t \in$ Opt

Interpretation

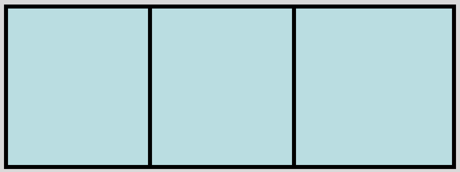
- The transformation was done so that we could think about the first item to schedule, as opposed to the last item to schedule
- In the original problem with deadlines, this is asking “what task do I do last”
  – So this is a procrastination based approach!

Optimal Caching

- Memory Hierarchy
  - Fast Memory (RAM)
  - Slow Memory (DISK)
  - Move big blocks of data from DISK to RAM for processing

- Caching problem:
  - Maintain collection of items in local memory
  - Minimize number of items fetched

Caching example



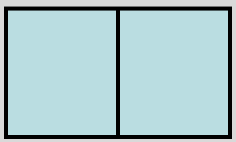
A, B, C, D, A, E, B, A, D, A, C, B, D, A

Optimal Caching

- If you know the sequence of requests, what is the optimal replacement pattern?
- Note – it is rare to know what the requests are in advance – but we still might want to do this:
  - Some specific applications, the sequence is known
    - Register allocation in code generation
  - Competitive analysis, compare performance on an online algorithm with an optimal offline algorithm

Farthest in the future algorithm

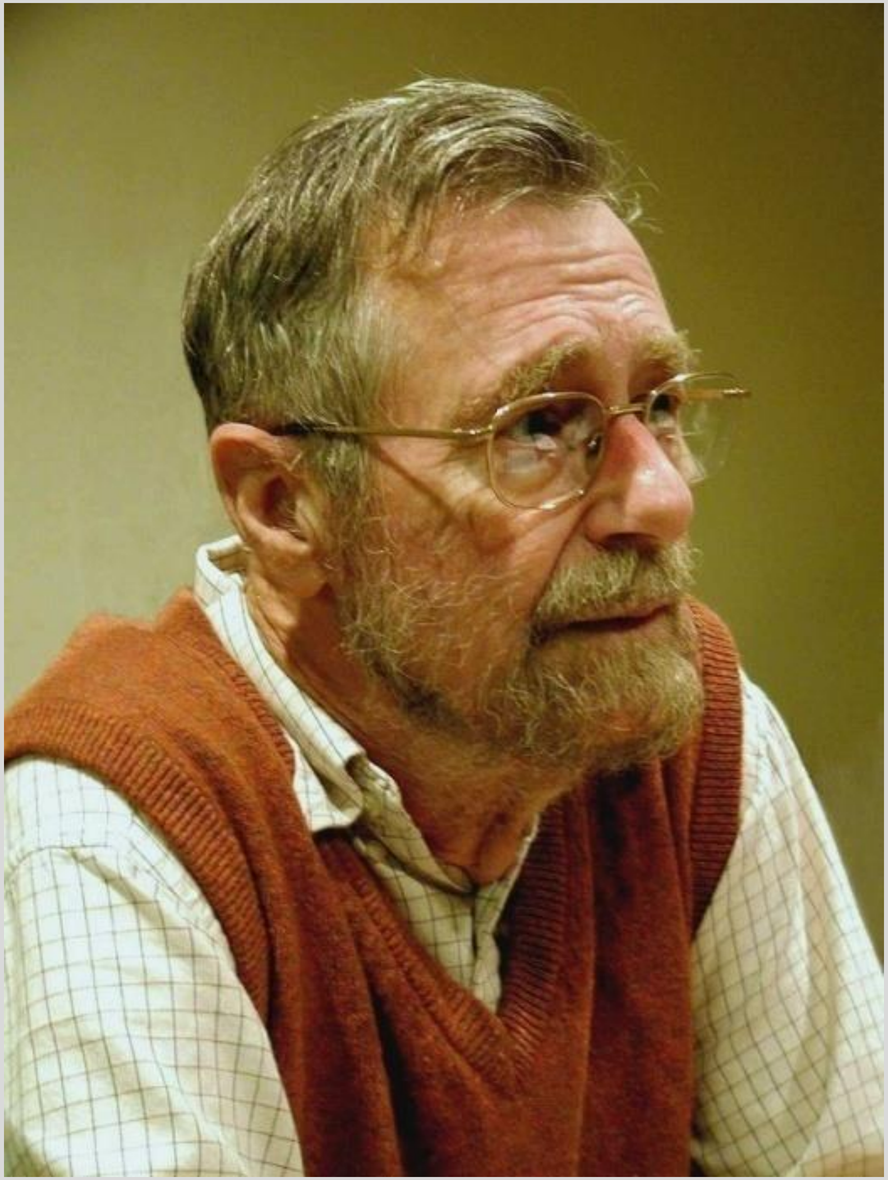
- Discard element used farthest in the future



A, B, C, A, C, D, C, B, C, A, D

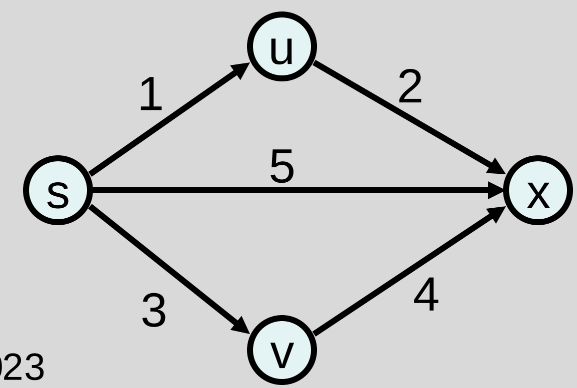
Correctness Proof

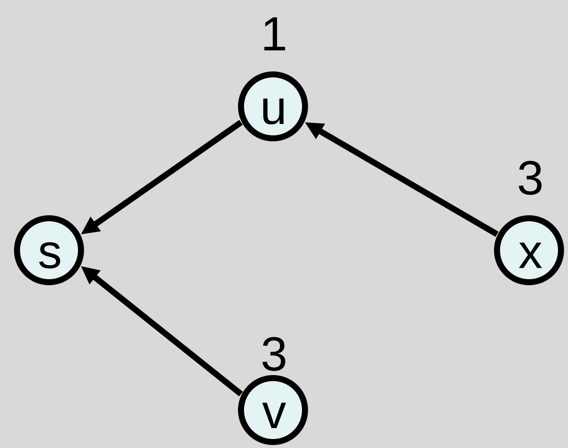
- Sketch
- Start with Optimal Solution O
- Convert to Farthest in the Future Solution F-F
- Look at the first place where they differ
- Convert O to evict F-F element
  - There are some technicalities here to ensure the caches have the same configuration . . .



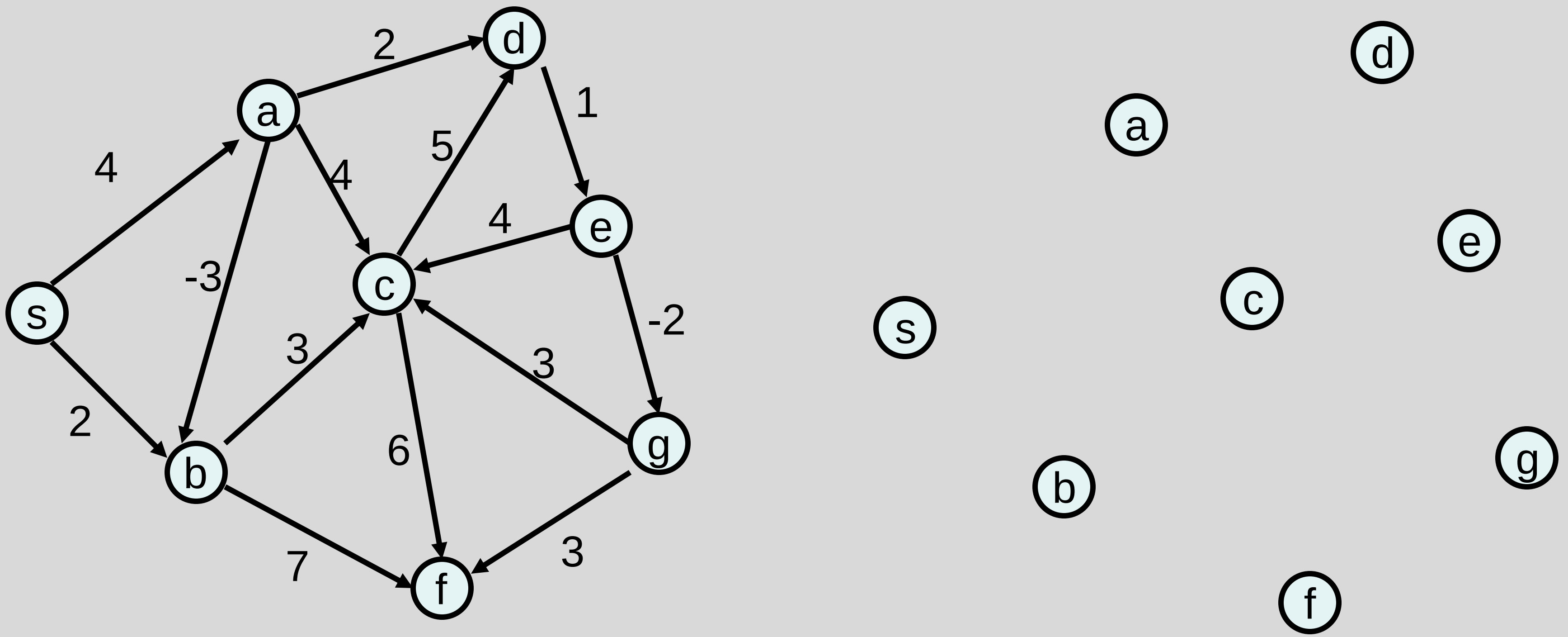
Single Source Shortest Path Problem

- Given a graph and a start vertex s
  - Determine distance of every vertex from s
  - Identify shortest paths to each vertex
    - Express concisely as a “shortest paths tree”
    - Each vertex has a pointer to a predecessor on shortest path



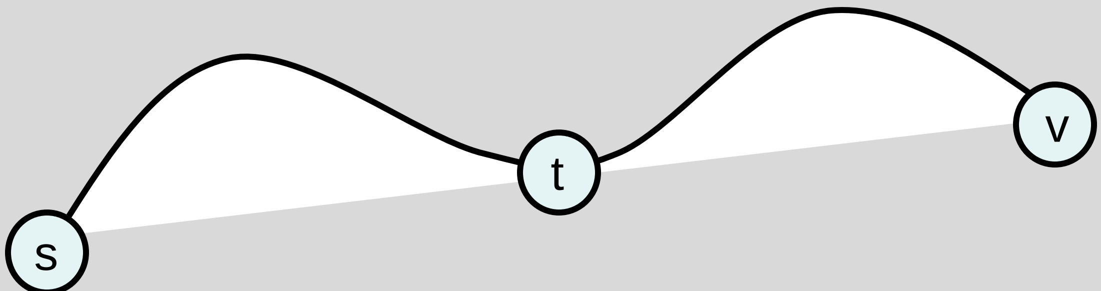


Construct Shortest Path Tree from s



Warmup

- If P is a shortest path from s to v, and if t is on the path P, the segment from s to t is a shortest path between s and t



- WHY?

Assume all edges have non-negative cost

Dijkstra's Algorithm

S = { }; d[s] = 0; d[v] = infinity for v != s

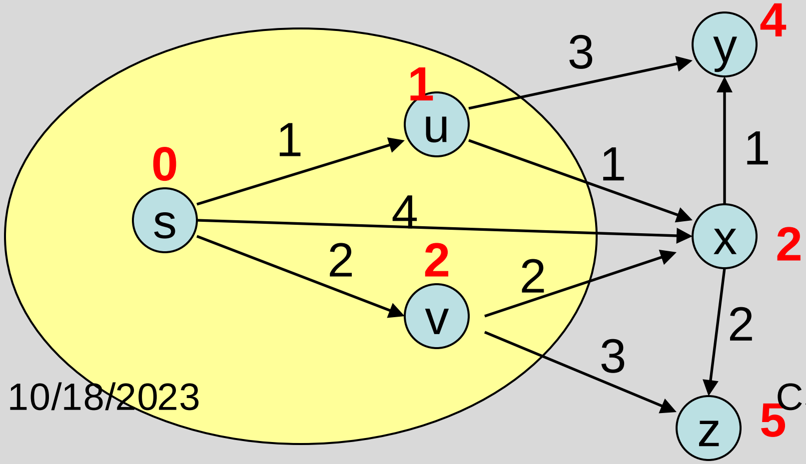
While S != V

Choose v in V-S with minimum d[v]

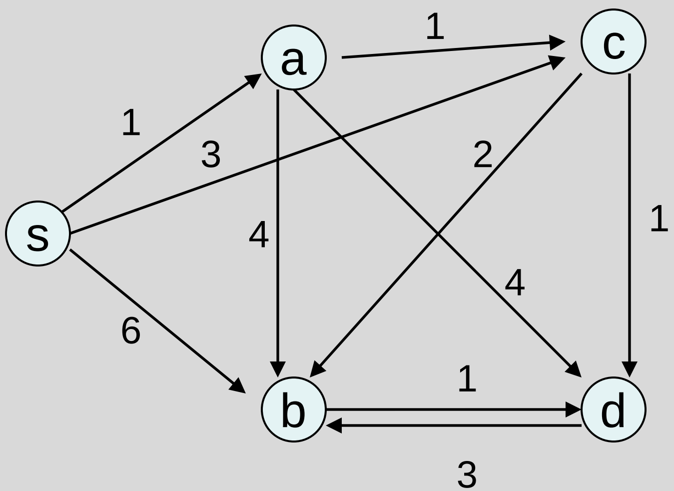
Add v to S

For each w in the neighborhood of v

d[w] = min(d[w], d[v] + c(v, w))



Simulate Dijkstra's algorithm (starting from s) on the graph



Round	Vertex Added	s	a	b	c	d
1						
2						
3						
4						
5						