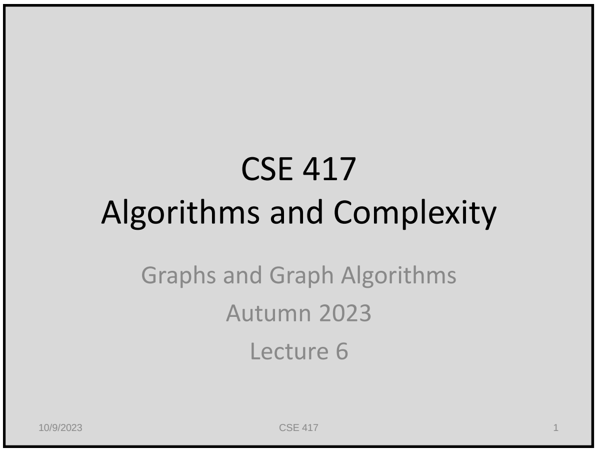

CSE 417 Algorithms and Complexity

Graphs and Graph Algorithms

Autumn 2023

Lecture 6

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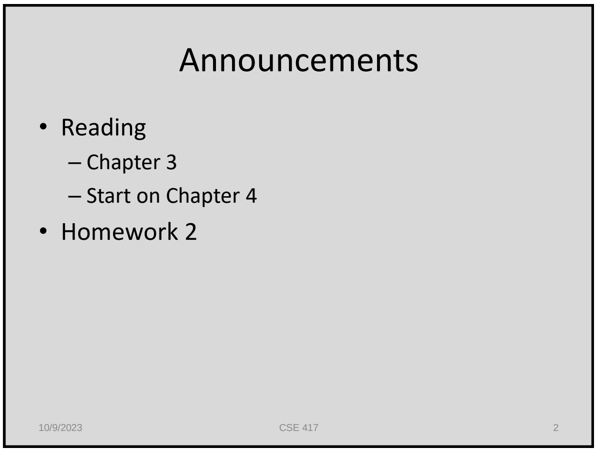

Announcements

- Reading
  - Chapter 3
  - Start on Chapter 4
- Homework 2

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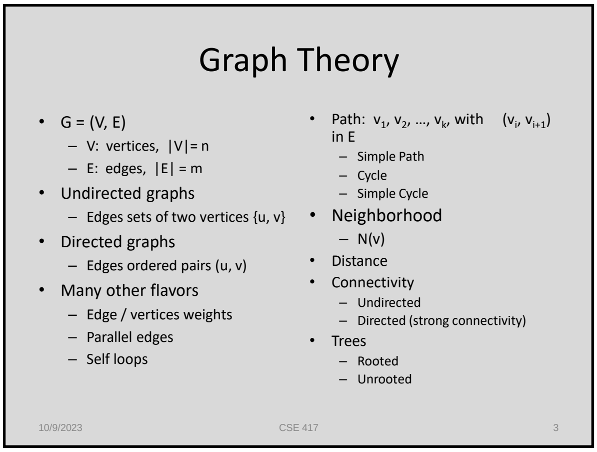

Graph Theory

- G = (V, E)
  - V: vertices, |V| = n
  - E: edges, |E| = m
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops
- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Neighborhood
  - N(v)
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

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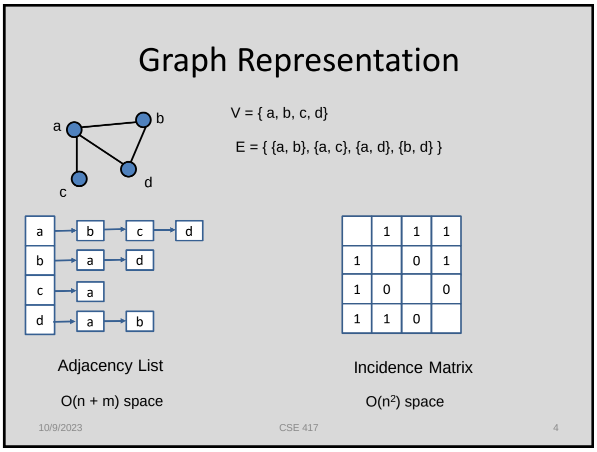

Graph Representation

V = { a, b, c, d}

E = { {a, b}, {a, c}, {a, d}, {b, d} }

a → b → c → d
b → a → d
c → a
d → a → b

Adjacency List

O(n + m) space

	1	1	1
1		0	1
1	0		0
1	1	0	

Incidence Matrix

O(n^2) space

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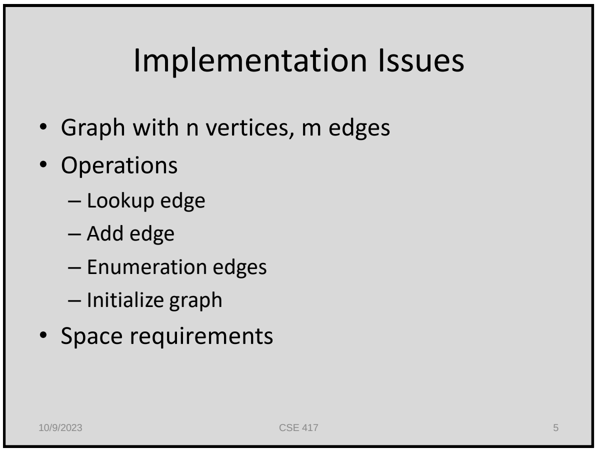

Implementation Issues

- Graph with n vertices, m edges
- Operations
  - Lookup edge
  - Add edge
  - Enumeration edges
  - Initialize graph
- Space requirements

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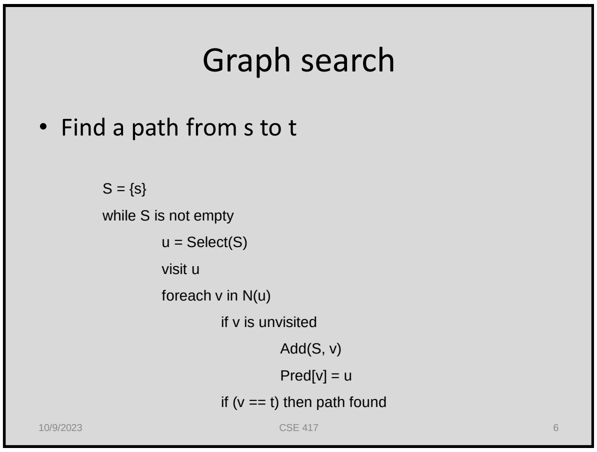

Graph search

- Find a path from s to t

```
S = {s}
while S is not empty
    u = Select(S)
    visit u
    foreach v in N(u)
        if v is unvisited
            Add(S, v)
            Pred[v] = u
        if (v == t) then path found
```

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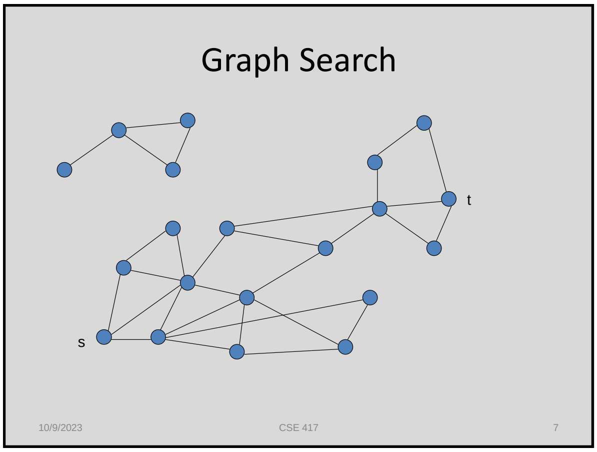

Graph Search

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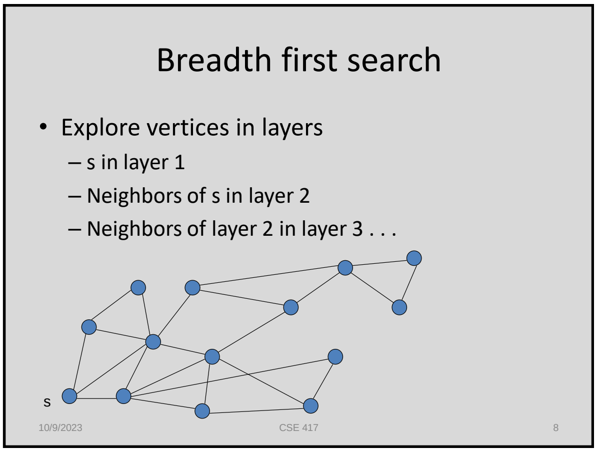

Breadth first search

- Explore vertices in layers
  - s in layer 1
  - Neighbors of s in layer 2
  - Neighbors of layer 2 in layer 3 . . .

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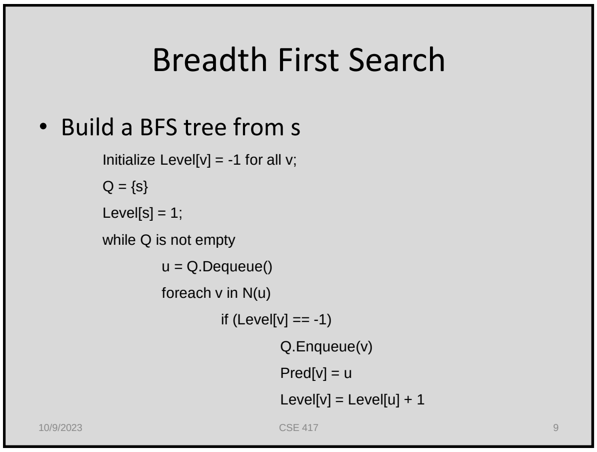

Breadth First Search

- Build a BFS tree from s

```
Initialize Level[v] = -1 for all v;
Q = {s}
Level[s] = 1;
while Q is not empty
    u = Q.Dequeue()
    foreach v in N(u)
        if (Level[v] == -1)
            Q.Enqueue(v)
            Pred[v] = u
            Level[v] = Level[u] + 1
```

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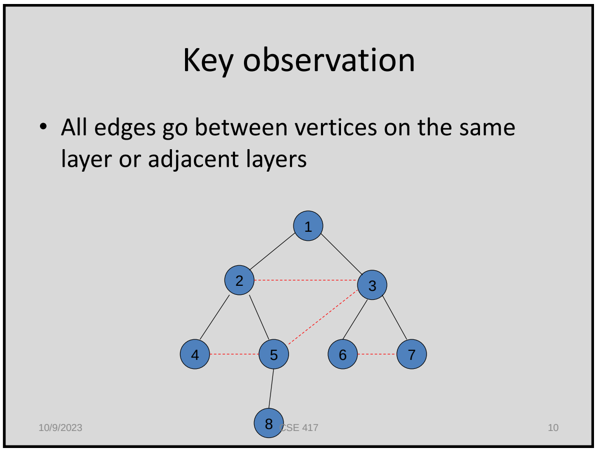

Key observation

- All edges go between vertices on the same layer or adjacent layers

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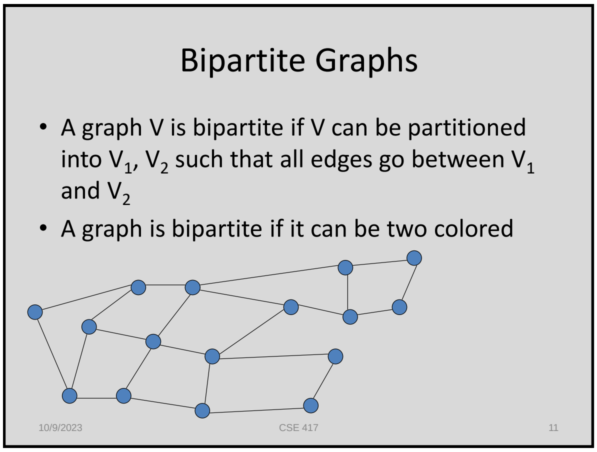

Bipartite Graphs

- A graph V is bipartite if V can be partitioned into V_1, V_2 such that all edges go between V_1 and V_2
- A graph is bipartite if it can be two colored

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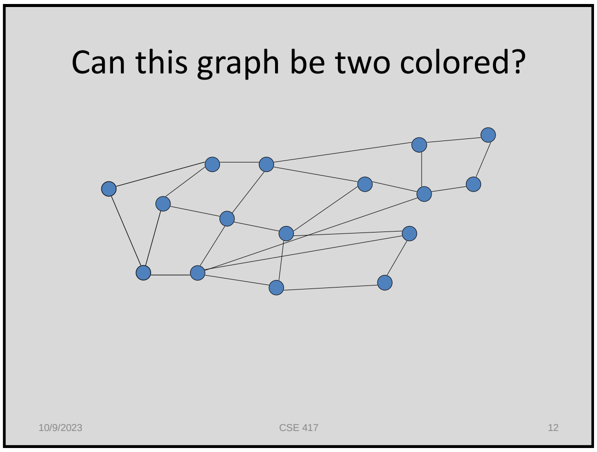

Can this graph be two colored?

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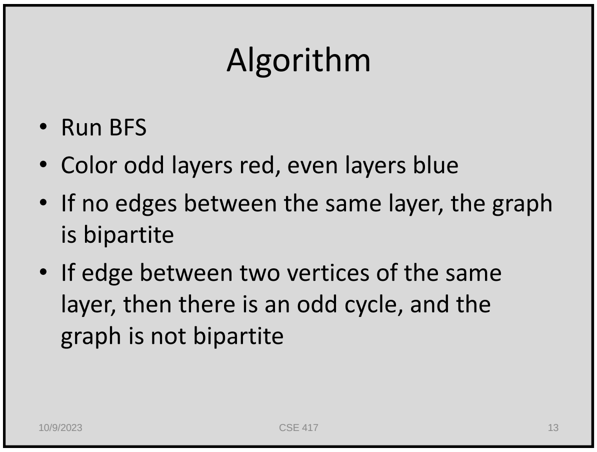

Algorithm

- Run BFS
- Color odd layers red, even layers blue
- If no edges between the same layer, the graph is bipartite
- If edge between two vertices of the same layer, then there is an odd cycle, and the graph is not bipartite

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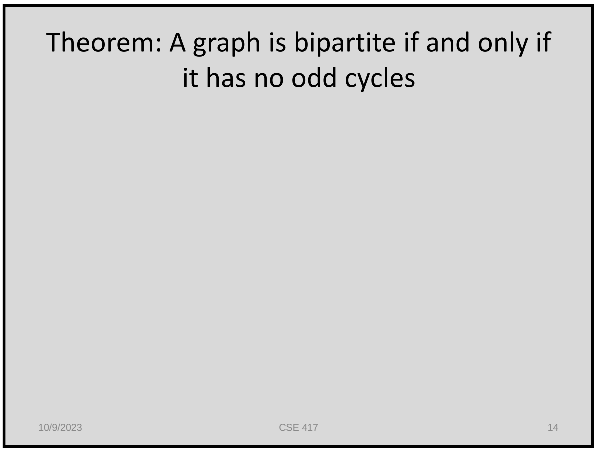

Theorem: A graph is bipartite if and only if it has no odd cycles

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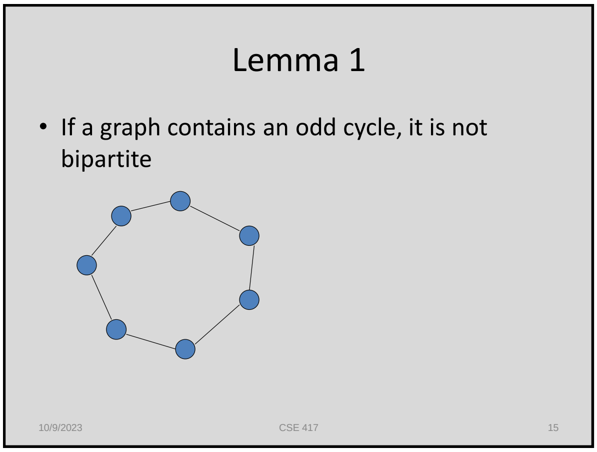

Lemma 1

- If a graph contains an odd cycle, it is not bipartite

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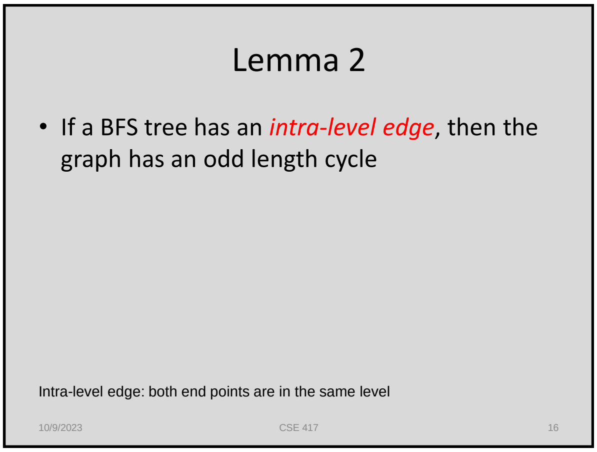

Lemma 2

- If a BFS tree has an *intra-level edge*, then the graph has an odd length cycle

Intra-level edge: both end points are in the same level

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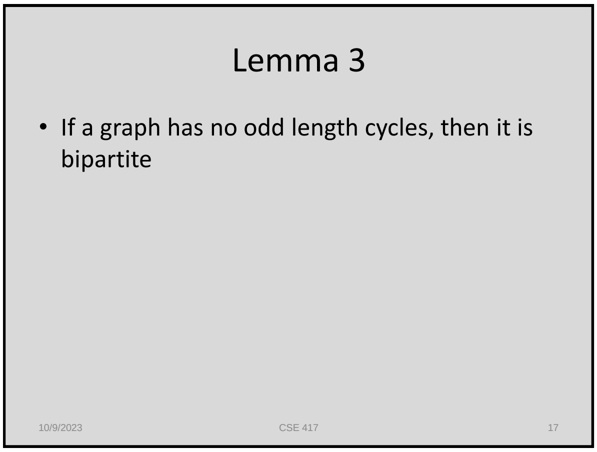

Lemma 3

- If a graph has no odd length cycles, then it is bipartite

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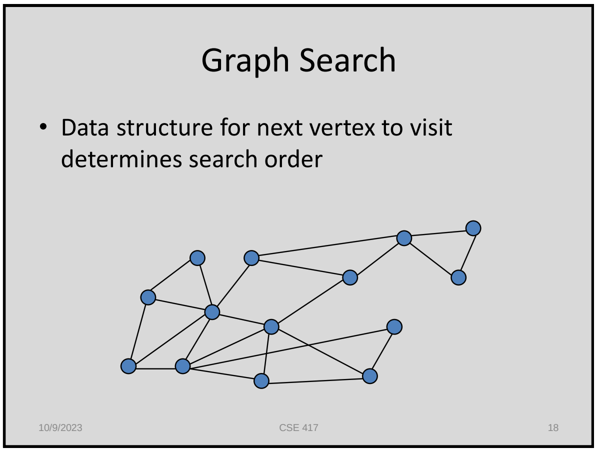

Graph Search

- Data structure for next vertex to visit determines search order

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Graph search

Breadth First Search

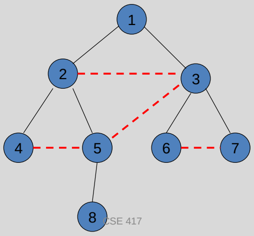
```
S = {s}
while S is not empty
    u = Dequeue(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Enqueue(S, v)
```

Depth First Search

```
S = {s}
while S is not empty
    u = Pop(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Push(S, v)
```

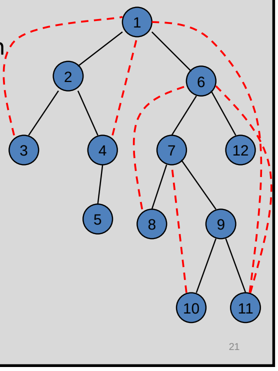
Breadth First Search

- All edges go between vertices on the same layer or adjacent layers



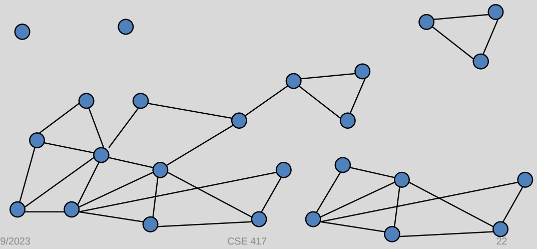
Depth First Search

- Each edge goes between vertices on the same branch
- No cross edges



Connected Components

- Undirected Graphs

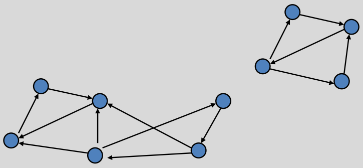


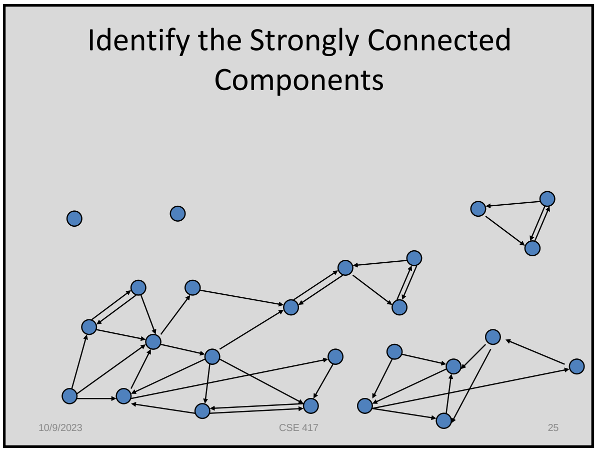
Computing Connected Components in O(n+m) time

- A search algorithm from a vertex v can find all vertices in v's component
- While there is an unvisited vertex v, search from v to find a new component

Directed Graphs

- A Strongly Connected Component is a subset of the vertices with paths between every pair of vertices.





Identify the Strongly Connected Components

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