Lecture04

CSE 417 Algorithms

Richard Anderson

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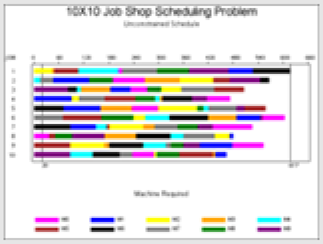
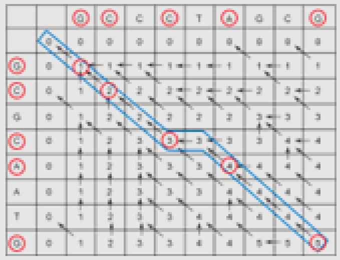
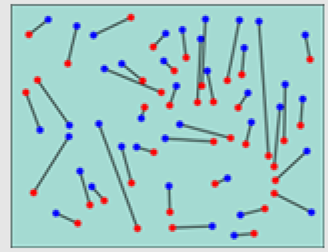
Lecture 4

Announcements

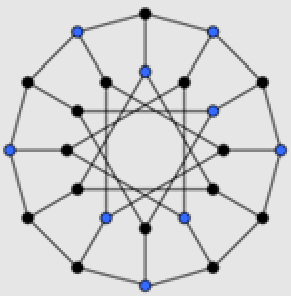
- Reading
  - Chapter 2.1, 2.2
  - Chapter 3 (Mostly review)
  - Start on Chapter 4
- Homework Guidelines
  - Submit homework with Gradescope
  - Describing an algorithm
    - Clarity is most important
    - Pseudocode generally preferable to just English
      - But sometimes both methods combined work best
  - Prove that your algorithm works
    - A proof is a "convincing argument"
  - Give the run time for your algorithm
    - Justify that the algorithm satisfies the runtime bound
  - You may lose points for style
  - Homework assignments will (probably) be worth the same amount

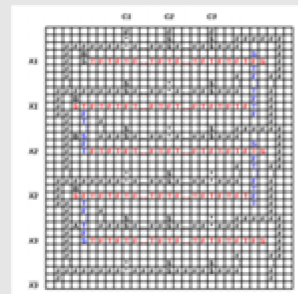






Five Problems





Scheduling
Weighted Scheduling
Bipartite Matching
Maximum Independent Set
Competitive Facility Location

Summary – Five Problems

- Scheduling
- Weighted Scheduling
- Combinatorial Optimization
- Maximum Independent Set
- Competitive Scheduling

What does it mean for an algorithm to be efficient?

Fast in Practice

Definitions of efficiency

- Fast in practice
- Qualitatively better worst case performance than a brute force algorithm

$T(n) = f(n)$

Polynomial time efficiency

$T(n) = 3n^3 + 2n^2$

- An algorithm is efficient if it has a polynomial run time
- Run time as a function of problem size
  - Run time: count number of instructions executed on an underlying model of computation
  - T(n): maximum run time for all problems of size at most n

n^{27}

Polynomial Time

- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)

Why Polynomial Time?

- Generally, polynomial time seems to capture the algorithms which are efficient in practice
- The class of polynomial time algorithms has many good, mathematical properties

Polynomial vs. Exponential Complexity

- Suppose you have an algorithm which takes n! steps on a problem of size n
- If the algorithm takes one second for a problem of size 10, estimate the run time for the following problems sizes:

12	14	16	18	20
2 min	6 hrs	2 mo	50 yrs	20K yr

$42n^2$ $O(n^2)$

Ignoring constant factors

- Express run time as O(f(n))
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award

$O(n^3)$ $O(n^{2.87})$ $O(n^{2.77})$ $n^{2.76}$

Why ignore constant factors?

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight

Why emphasize growth rates?

- The algorithm with the lower growth rate will be faster for all but a finite number of cases
- Performance is most important for larger problem size
- As memory prices continue to fall, bigger problem sizes become feasible
- Improving growth rate often requires new techniques

Formalizing growth rates

- T(n) is O(f(n)) $[T : Z^+ \rightarrow R^+]$
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$

- T(n) is O(f(n)) will be written as:
  T(n) = O(f(n)) $T(n) \in O(f(n))$
  - Be careful with this notation

Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let c = 6 suppose $n > 5$

Let n_0 = 5

$3n^2 + 5n + 20 < 3n^2 + n^2 + n^2$
$= 5n^2 < 6n^2$

T(n) is O(f(n)) if there exist c, n_0, such that for $n > n_0$,
$T(n) < c\, f(n)$

Order the following functions in increasing order by their growth rate

a) $n \log^4 n$

b) $2n^2 + 10n$

c) $2^{n/100}$

d) $1000n + \log^8 n$

e) n^{100}

f) 3^n

g) $1000 \log^{10} n$

h) $n^{1/2}$

Lower bounds

n^2 is $O(n^3)$

- $T(n)$ is $\Omega(f(n))$
  - $T(n)$ is at least a constant multiple of $f(n)$
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of Ω vary

$\Theta(T(n))$

- $T(n)$ is $\Theta(f(n))$ if $T(n)$ is $O(f(n))$ and $T(n)$ is $\Omega(f(n))$

$T(n) = \Theta(f(n))$

Useful Theorems

- If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = c for c > 0 then f(n) = Θ(g(n))
- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))
- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

Ordering growth rates

- For b > 1 and x > 0
  - $\log^b n$ is $O(n^x)$
- For r > 1 and d > 0
  - n^d is $O(r^n)$