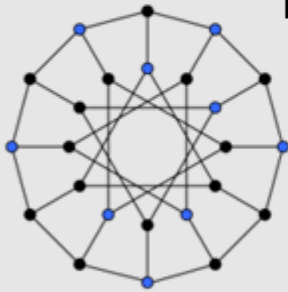
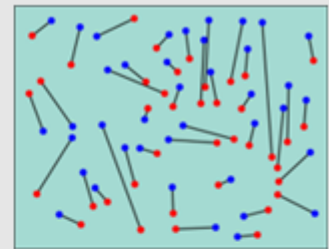
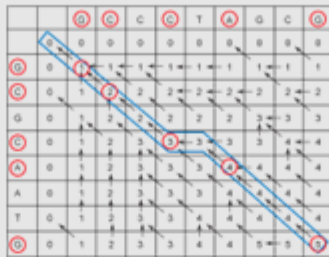


Lecture03

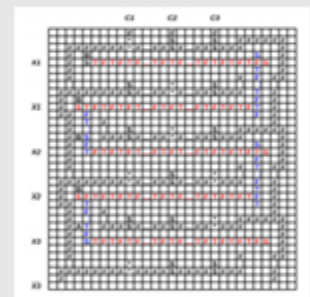


Five Problems

CSE 417

Richard Anderson

Autumn 2023, Lecture 3



Announcements

- Course website:

[//courses.cs.washington.edu/courses/cse417/23au/](https://courses.cs.washington.edu/courses/cse417/23au/)

- Homework Due Friday
- Office Hours:

Richard Anderson, Monday 2-3 pm, Thursday, 4-5 pm

Megh Bhalerao, Friday, 4:30-6:30 pm

Tiernan Kennedy, Wednesday, 9-10 am, Friday, 9-10 am

Yigao Li, Tuesday, 11:30am-12:30 pm, Friday, 1-2 pm

Kaiyuan Liu, Tuesday, 3-4 pm, Thursday, 2-3 pm

Sravani Nanduri, Monday, 4:30-5:30 pm, Friday, 11:30am-12:30pm

Albert Weng, Monday, 3:30-4:30 pm, Wednesday, 3:30-4:30 pm

Theory of Algorithms

- What is expertise?
- How do experts differ from novices?

Introduction of five problems

- Show the types of problems we will be considering in the class
- Examples of important types of problems
- Similar looking problems with very different characteristics
- Problems
 - Scheduling
 - Weighted Scheduling
 - Bipartite Matching
 - Maximum Independent Set
 - Competitive Facility Location

$$\begin{array}{ccc} \underline{3} & \underline{2} & \underline{6} \\ \underline{7} & \underline{4} & \end{array}$$

NP

Minimum Spanning Tree

What is a problem?

- Instance
- Solution
- Constraints on solution
- Measure of value

Graph

Set of edges

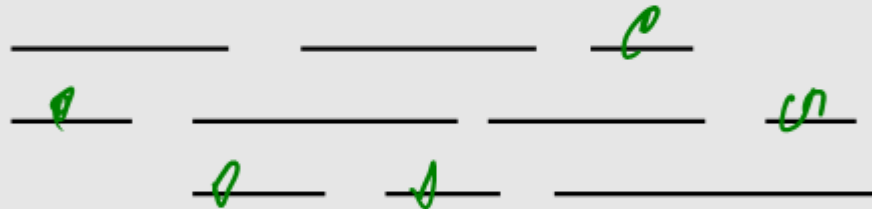
Spanning Tree

Min Edge Costs



Problem: Scheduling

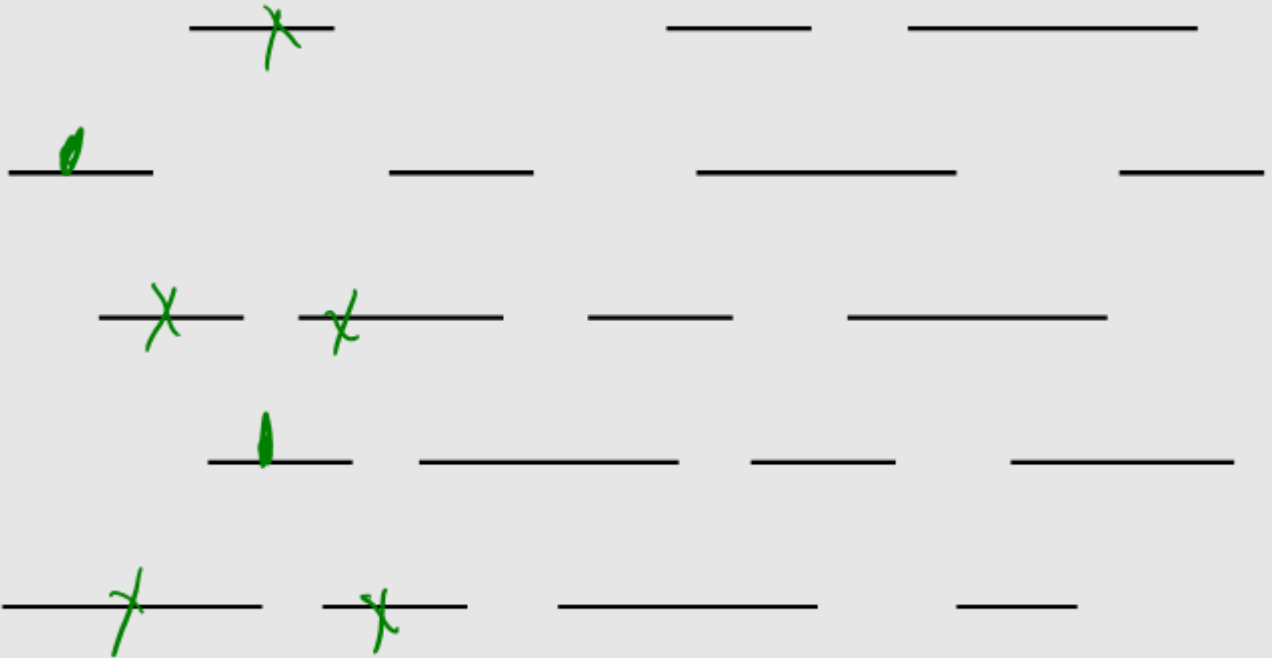
- Suppose that you own a banquet hall
- You have a series of requests for use of the hall:
 $(s_1, f_1), (s_2, f_2), \dots$



- Find a set of requests as large as possible with no overlap

sort by finish time

What is the largest solution?

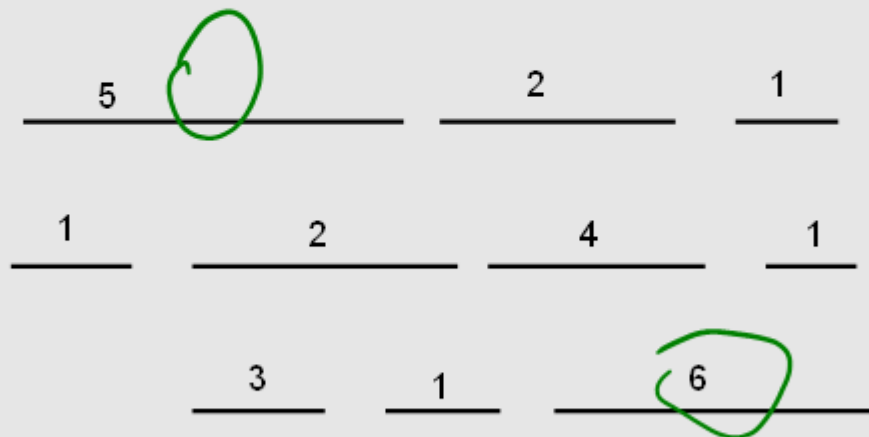


Greedy Algorithm

- Test elements one at a time if they can be members of the solution
- If an element is not ruled out by earlier choices, add it to the solution
- Many possible choices for ordering (length, start time, end time)
- For this problem, considering the jobs by increasing end time works

Suppose we add values?

- (s_i, f_i, v_i) , start time, finish time, payment
- Maximize value of elements in the solution



Greedy Algorithms

- Earliest finish time



A handwritten diagram consisting of a horizontal line with a vertical tick mark at its left end. Below the line, the number '10' is written, and a horizontal line is drawn underneath the '10'.

- Maximum value



A handwritten diagram showing two horizontal bars. Each bar has a '2' written below it. A horizontal line connects the two bars, and a '3' is written below this line.

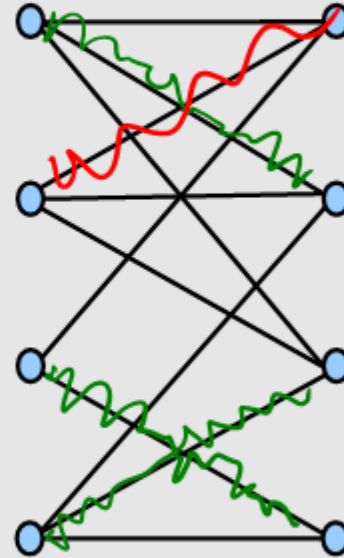
- Give counter examples to show these algorithms don't find the maximum value solution

Dynamic Programming

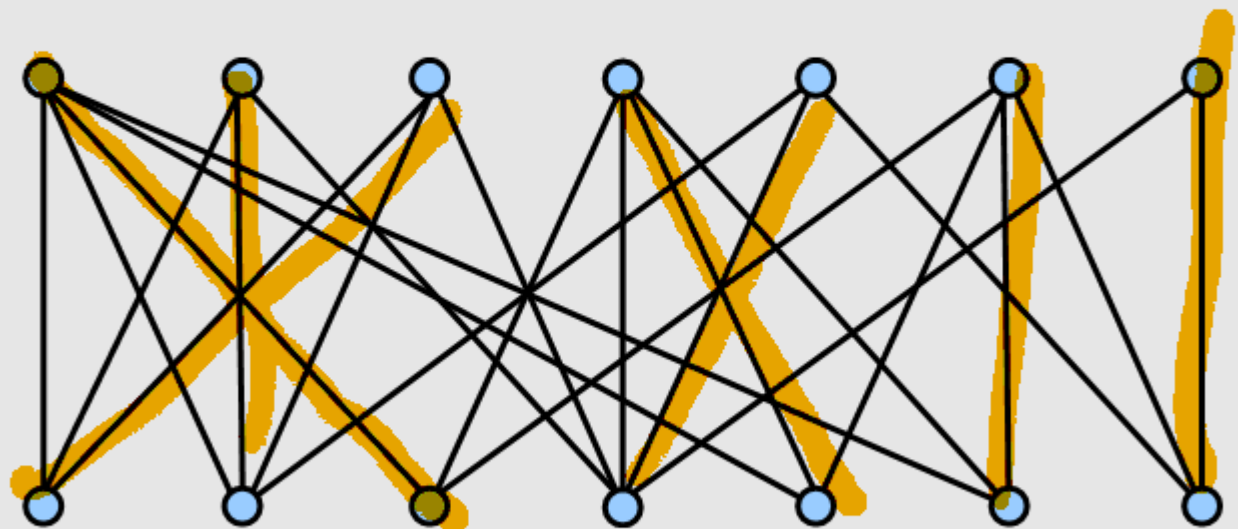
- Requests R_1, R_2, R_3, \dots
- Assume requests are in increasing order of finish time ($f_1 < f_2 < f_3 \dots$)
- Opt_i is the maximum value solution of $\{R_1, R_2, \dots, R_i\}$ containing R_i
- $\text{Opt}_i = \text{Max}\{j \mid f_j < s_i\}[\text{Opt}_j + v_i]$

Matching (Combinatorial Optimization)

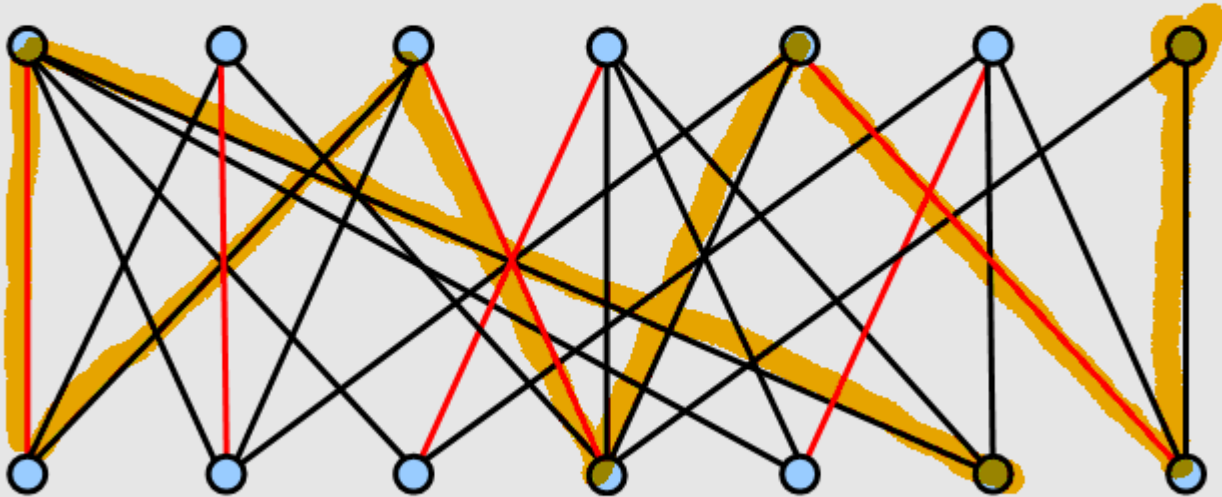
- Given a bipartite graph $G=(U,V,E)$, find a subset of the edges M of maximum size with no common endpoints.
- Application:
 - U : Professors
 - V : Courses
 - (u,v) in E if Prof. u can teach course v



Find a maximum matching

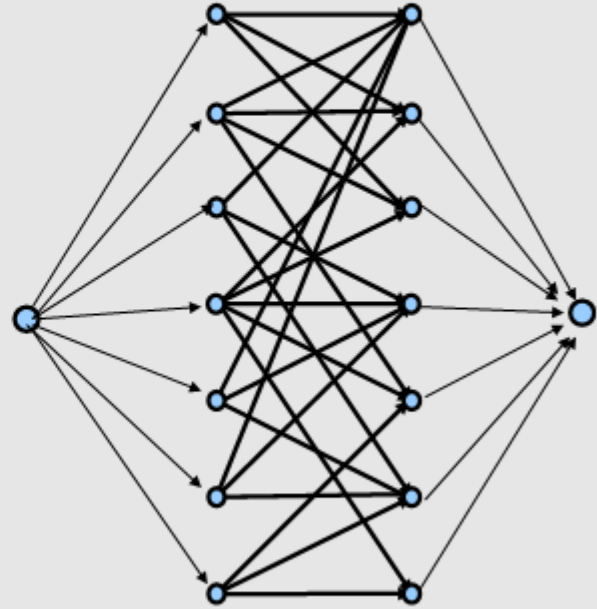


Augmenting Path Algorithm



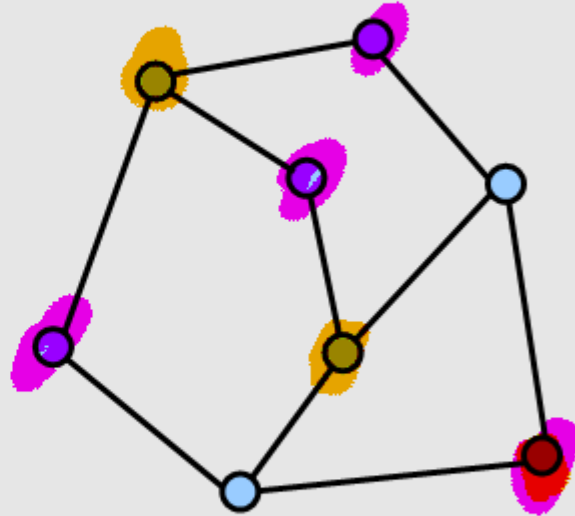
Reduction to network flow

- More general problem
- Send flow from source to sink
- Flow subject to capacities at edges
- Flow conserved at vertices
- Can solve matching as a flow problem

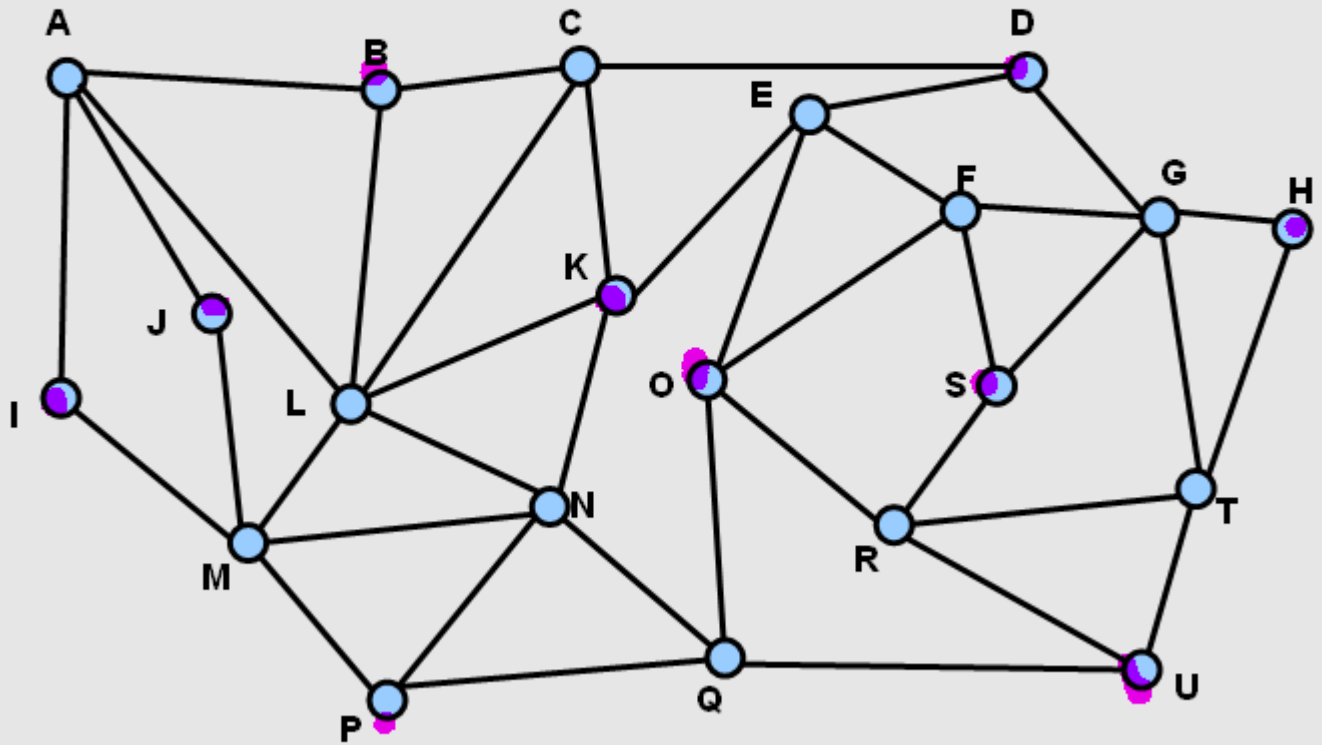


Maximum Independent Set

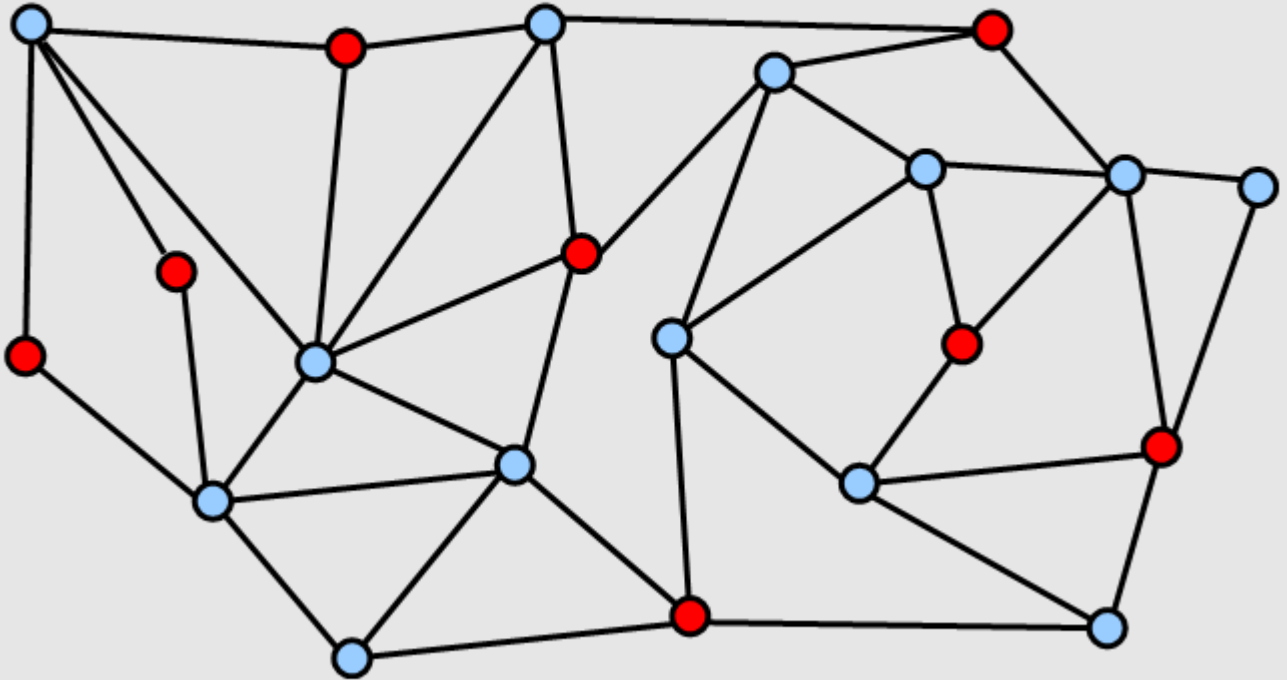
- Given an undirected graph $G=(V,E)$, find a set I of vertices such that there are no edges between vertices of I
- Find a set I as large as possible



Find a Maximum Independent Set



Verification: Prove the graph has an independent set of size 8



Key characteristic

- Hard to find a solution
- Easy to verify a solution once you have one
- Other problems like this
 - Hamiltonian circuit
 - Clique
 - Subset sum
 - Graph coloring

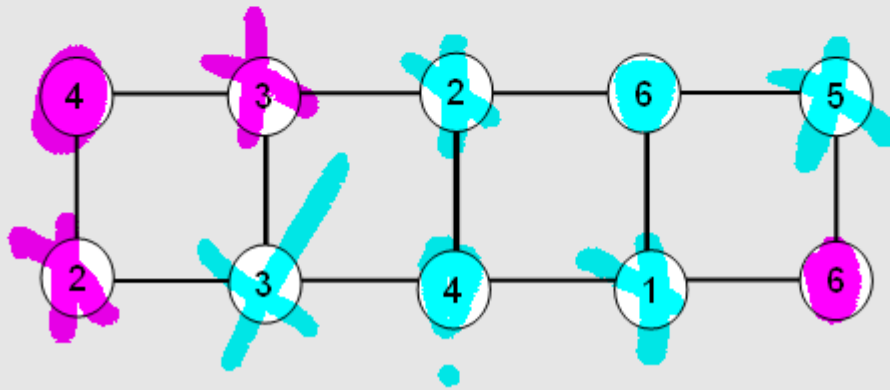


NP-Completeness

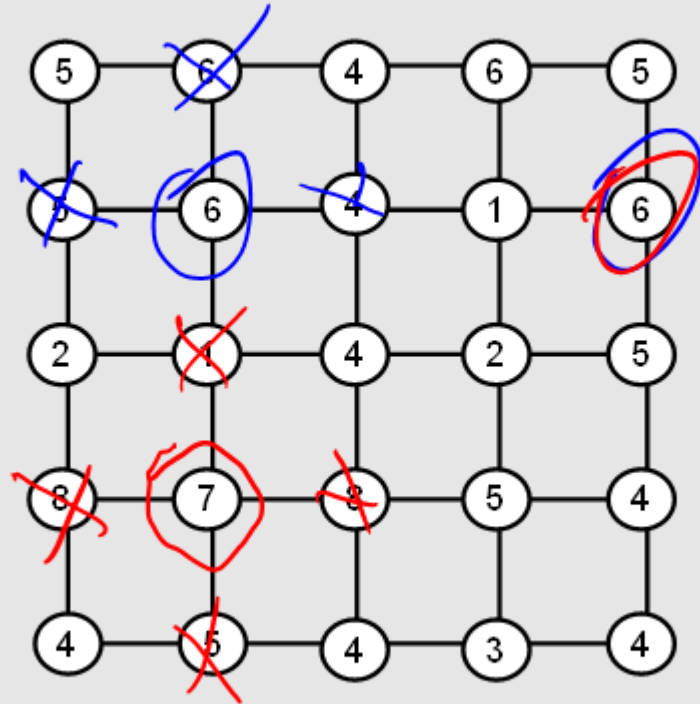
- Theory of Hard Problems
- A large number of problems are known to be equivalent
- Very elegant theory

Are there even harder problems?

- Simple game:
 - Players alternate selecting nodes in a graph
 - Score points associated with node
 - Remove nodes neighbors
 - When neither can move, player with most points wins



4 6
10 10



7
6

0

Competitive Facility Location

- **Choose location for a facility**
 - Value associated with placement
 - Restriction on placing facilities too close together
- **Competitive**
 - Different companies place facilities
 - E.g., KFC and McDonald's

Complexity theory

- These problems are P-Space complete instead of NP-Complete
 - Appear to be much harder
 - No obvious certificate
 - G has a Maximum Independent Set of size 10
 - Player 1 wins by at least 10 points

Summary

- Scheduling
- Weighted Scheduling
- Bipartite Matching
- Maximum Independent Set
- Competitive Scheduling