# CSE 418 Algorithms

Lecture 18, Winter 2020

Dynamic Programming

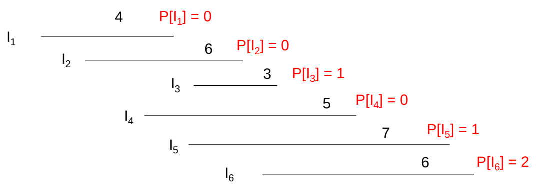
# Announcements

- Reading:
  - 6.1-6.2, Weighted Interval Scheduling
  - 6.3 Segmented Least Squares
  - 6.4 Knapsack and Subset Sum

Intervals sorted by end time

# Dynamic Programming

- Weighted Interval Scheduling
- Given a collection of intervals $I_1,\ldots,I_n$ with weights $w_1,\ldots,w_n$, choose a maximum weight set of non-overlapping intervals




Intervals sorted by end time

# Optimality Condition

- Opt[ j ] is the maximum weight independent set of intervals $I_1, I_2, \ldots, I_j$
- Opt[ j ] = max( Opt[ j – 1], $w_j$ + Opt[ p[ j ] ])
  - Where p[ j ] is the index of the last interval which finishes before $I_j$ starts

# Iterative Algorithm

```
int[] M = new int[n+1];
char[] R = new char[n+1];

M[0] = 0;
for (int j = 1; j < n+1; j++){
        v1 = M[j-1];
        v2 = W[j] + M[P[j]];
        if (v1 > v3) {
                M[j] = v1;
                R[j] = 'A';
        }
        else {
                M[j] = v2;
                R[j] = 'B';
        }
}
```

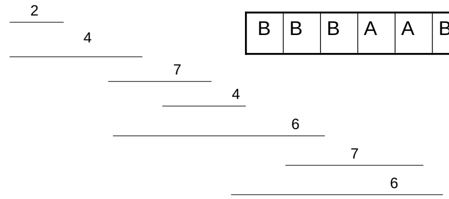
# Computing the solution

Opt[ j ] = max (Opt[ j – 1], $w_j$ + Opt[ p[ j ] ])

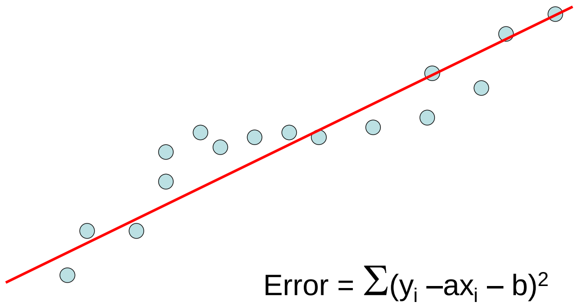
Record which case is used in Opt computation

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 4 | 9 | 9 | 9 | 16 | 16 |

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | A | A | B | A |

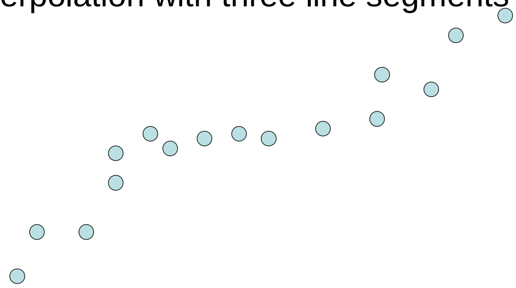


## Optimal linear interpolation

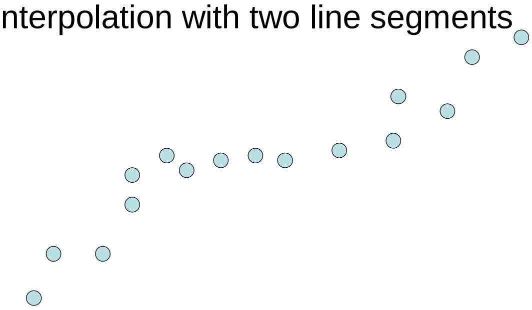


$\text{Error} = \sum (y_i - ax_i - b)^2$

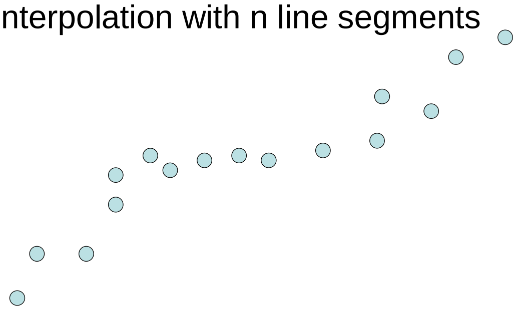
## What is the optimal linear interpolation with three line segments



## What is the optimal linear interpolation with two line segments

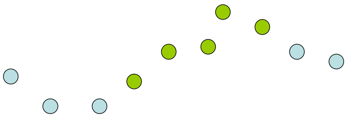


## What is the optimal linear interpolation with n line segments



## Notation

- Points $p_1, p_2, \ldots, p_n$ ordered by x-coordinate ($p_i = (x_i, y_i)$)
- $E_{i,j}$ is the least squares error for the optimal line interpolating $p_i, \ldots p_j$



## Optimal interpolation with two segments

- Give an equation for the optimal interpolation of $p_1,\ldots,p_n$ with two line segments

- $E_{i,j}$ is the least squares error for the optimal line interpolating $p_i, \ldots p_j$

## Optimal interpolation with k segments

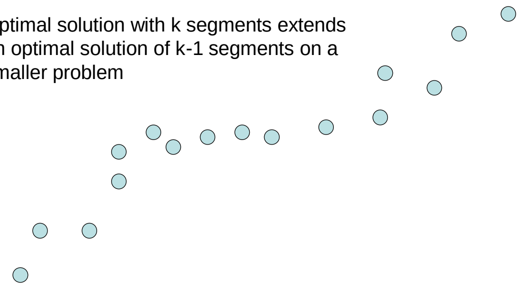
- Optimal segmentation with three segments
  - $Min_{i,j}\{E_{1,i} + E_{i,j} + E_{j,n}\}$
  - $O(n^2)$ combinations considered
- Generalization to k segments leads to considering $O(n^{k-1})$ combinations

## $Opt_k[\ j\ ]$ : Minimum error approximating $p_1 \ldots p_j$ with k segments

How do you express $Opt_k[\ j\ ]$ in terms of $Opt_{k-1}[1], \ldots, Opt_{k-1}[\ j\ ]$?

## Optimal sub-solution property

Optimal solution with k segments extends an optimal solution of k-1 segments on a smaller problem



## Optimal multi-segment interpolation

Compute Opt[ k, j ] for 0 < k < j < n

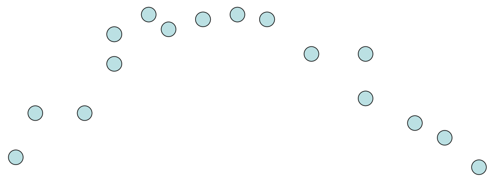
```
for j = 1 to n
   Opt[1, j] = E1,j;

for k = 2 to n-1
   for j = 2 to n
      t = E1,j
      for i = 1 to j-1
         t = min(t, Opt[k-1, i] + Ei,j)
      Opt[k, j] = t
```

## Determining the solution

- When Opt[k,j] is computed, record the value of i that minimized the sum
- Store this value in a auxiliary array
- Use to reconstruct solution

## Variable number of segments

- Segments not specified in advance
- Penalty function associated with segments
- Cost = Interpolation error + C x #Segments



## Penalty cost measure

- Opt[ j ] = min($E_{1,j}$, $\min_i$(Opt[ i ] + $E_{i,j}$ + P))