CSE 417
Algorithms and Complexity

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Lecture 13, Winter 2020
Recurrences

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Announcements

- Midterm, Monday, February 10, 2020
  - Coverage through KT 5.2
  - Sample midterm questions will be posted
  - 50 minutes
  - Closed book
  - No notes
  - No calculators or electronic devices

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Divide and Conquer

- Recurrences, Sections 5.1 and 5.2
- Algorithms
  - Fast Matrix Multiplication
  - Counting Inversions (5.3)
  - Closest Pair (5.4)
  - Multiplication (5.5)

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Divide and Conquer

```
Array Mergesort(Array a){
        n = a.Length;
        if (n <= 1)
                return a;
        b = Mergesort(a[0 .. n/2]);
        c = Mergesort(a[n/2+1 .. n-1]);
        return Merge(b, c);
}
```

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Algorithm Analysis

- Cost of Merge
- Cost of Mergesort

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$T(n) = 2T(n/2) + cn; T(1) = c;$

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Recurrence Analysis

- Solution methods
  - Unrolling recurrence
  - Guess and verify
  - Plugging in to a "Master Theorem"

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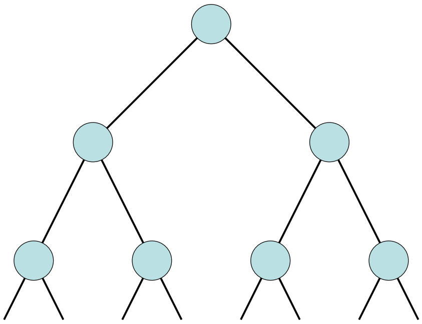
Unrolling the recurrence



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T(n) = 2T(n/2) + n; T(1) = 1;

Substitution

Prove $T(n) \le n(\log_2 n + 1)$ for $n \ge 1$

Induction:
Base Case:

Induction Hypothesis:

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A better mergesort (?)

- Divide into 3 subarrays and recursively sort
- Apply 3-way merge

What is the recurrence?

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Unroll recurrence for $T(n) = 3T(n/3) + dn$

11

$T(n) = aT(n/b) + f(n)$

12

$T(n) = T(n/2) + cn$

Where does this recurrence arise?

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Solving the recurrence exactly

14

$T(n) = 4T(n/2) + n$

15

$T(n) = 2T(n/2) + n^2$

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$T(n) = 2T(n/2) + n^{1/2}$

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Recurrences

- Three basic behaviors
  - Dominated by initial case
  - Dominated by base case
  - All cases equal – we care about the depth

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