CSE 417 Algorithms

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Lecture 5

Announcements

Worst Case Runtime Function

- Problem P: Given instance I compute a solution S
- A is an algorithm to solve P
- T(I) is the number of steps executed by A on instance I
- T(n) is the maximum of T(I) for all instances of size n

Ignore constant factors

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight
- Express run time as $T(n) = O(f(n))$

Formalizing growth rates

- T(n) is O(f(n)) [T : $Z^+ \rightarrow R^+$]
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for n > n_0, T(n) < c f(n)

- T(n) is O(f(n)) will be written as:
  T(n) = O(f(n))
  - Be careful with this notation

Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let c =

Let n_0 =

$T(n)$ is $O(f(n))$ if there exist c, n_0, such that for $n > n_0$,
$T(n) < c\, f(n)$

Order the following functions in increasing order by their growth rate

a) $n \log^4 n$
b) $2n^2 + 10n$
c) $2^{n/100}$
d) $1000n + \log^8 n$
e) n^{100}
f) 3^n
g) $1000 \log^{10} n$
h) $n^{1/2}$

Lower bounds

- T(n) is $\Omega(f(n))$
  - T(n) is at least a constant multiple of f(n)
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of Ω vary

- T(n) is $\Theta(f(n))$ if T(n) is O(f(n)) and T(n) is $\Omega(f(n))$

Useful Theorems

- If lim (f(n) / g(n)) = c for c > 0 then f(n) = Θ(g(n))

- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))

- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

Ordering growth rates

- For $b > 1$ and $x > 0$
  - $\log^b n$ is $O(n^x)$

- For $r > 1$ and $d > 0$
  - n^d is $O(r^n)$

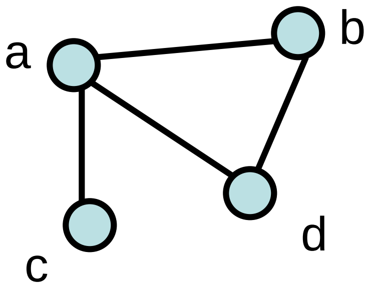
Graph Theory

- G = (V, E)
  - V – vertices
  - E – edges
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops

Definitions

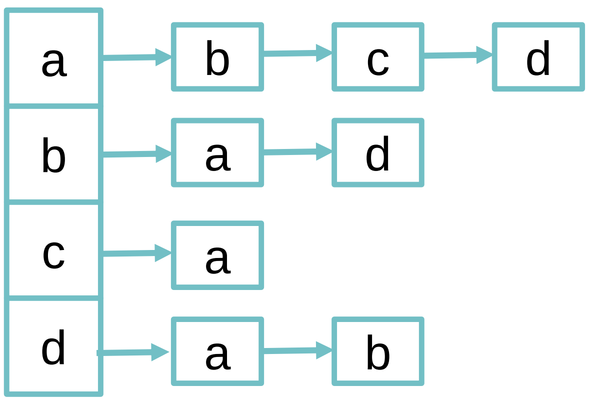
- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Neighborhood
  - N(v)
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

Graph Representation



V = { a, b, c, d}

E = { {a, b}, {a, c}, {a, d}, {b, d} }



Adjacency List

	1	1	1
1		0	1
1	0		0
1	1	0	

Incidence Matrix

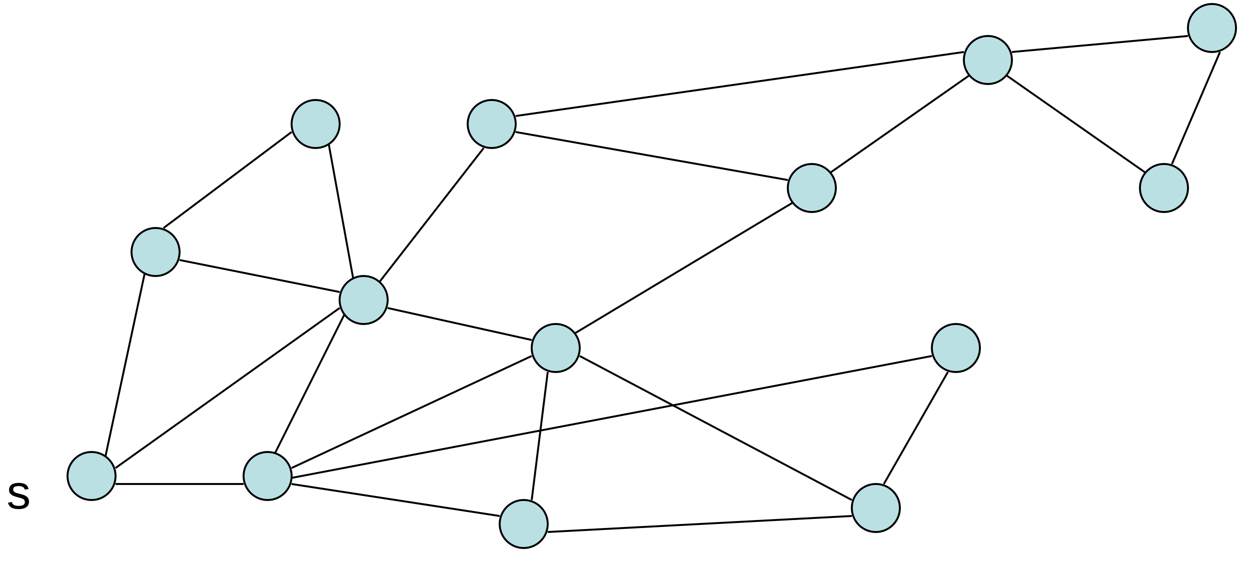
Graph search

- Find a path from s to t

```
S = {s}
while S is not empty
    u = Select(S)
    visit u
    foreach v in N(u)
        if v is unvisited
            Add(S, v)
            Pred[v] = u
        if (v = t) then path found
```

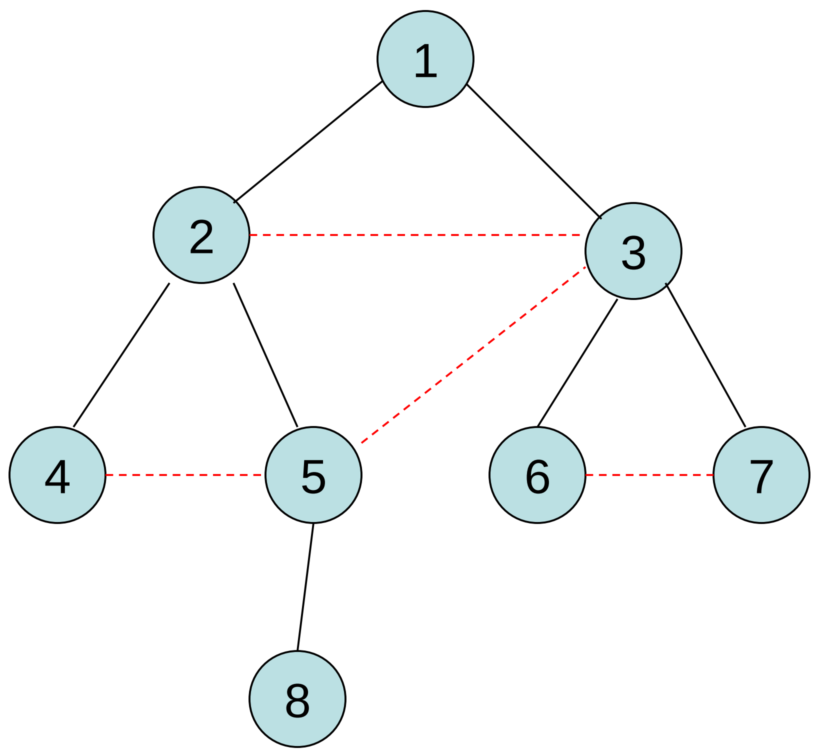
Breadth first search

- Explore vertices in layers
  - s in layer 1
  - Neighbors of s in layer 2
  - Neighbors of layer 2 in layer 3 . . .



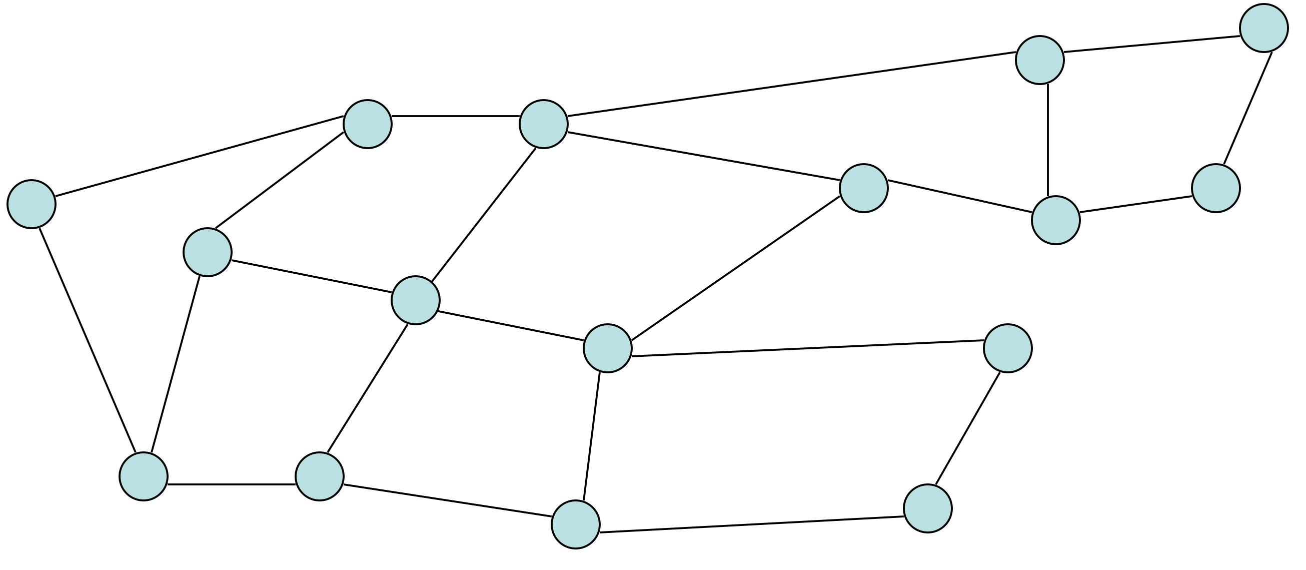
Key observation

- All edges go between vertices on the same layer or adjacent layers

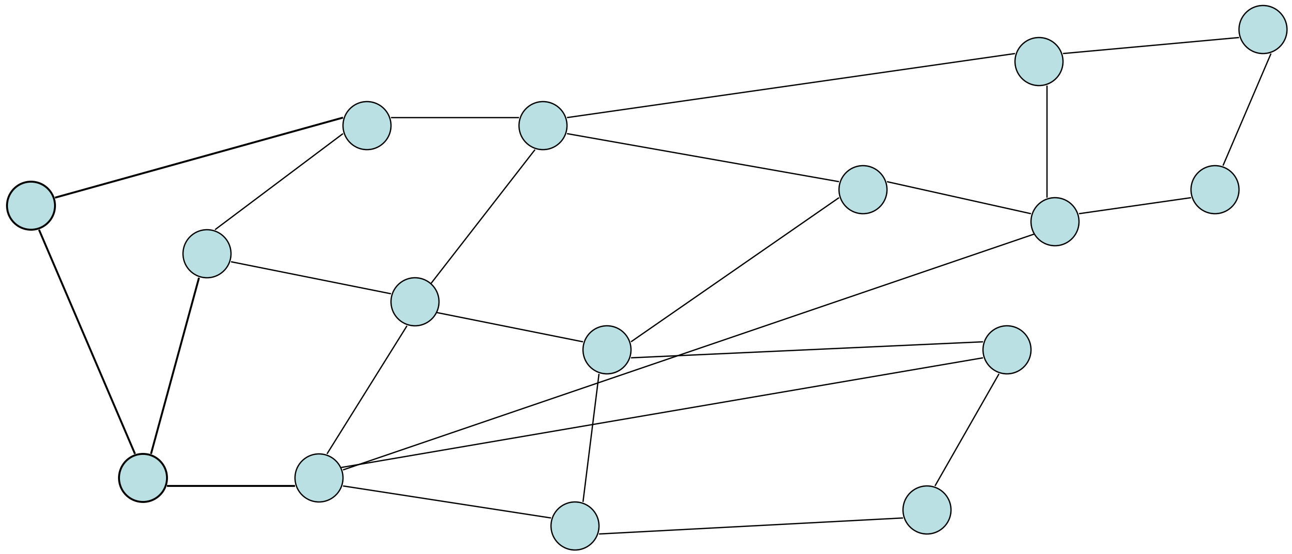


Bipartite Graphs

- A graph V is bipartite if V can be partitioned into V_1, V_2 such that all edges go between V_1 and V_2
- A graph is bipartite if it can be two colored



Can this graph be two colored?



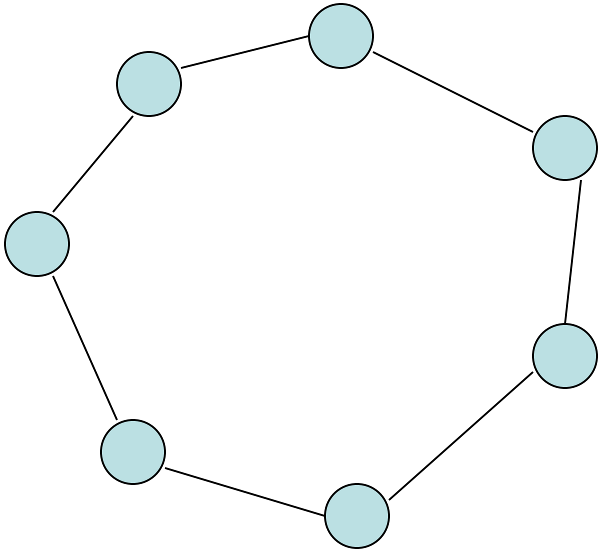
Algorithm

- Run BFS
- Color odd layers red, even layers blue
- If no edges between the same layer, the graph is bipartite
- If edge between two vertices of the same layer, then there is an odd cycle, and the graph is not bipartite

Theorem: A graph is bipartite if and only if it has no odd cycles

Lemma 1

- If a graph contains an odd cycle, it is not bipartite



Lemma 2

- If a BFS tree has an *intra-level edge*, then the graph has an odd length cycle

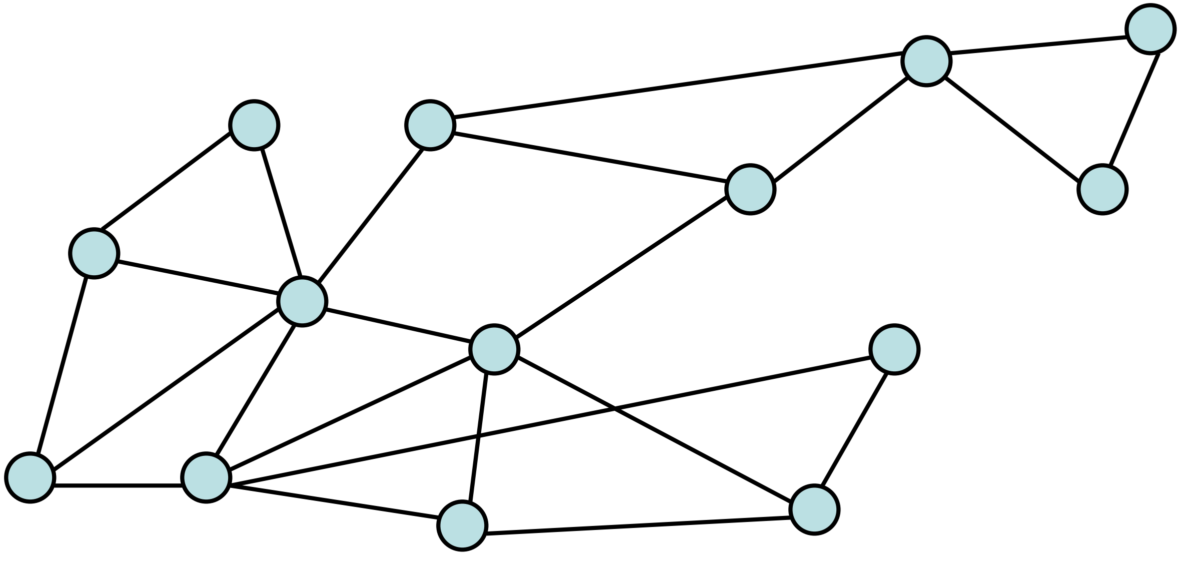
Intra-level edge: both end points are in the same level

Lemma 3

- If a graph has no odd length cycles, then it is bipartite

Graph Search

- Data structure for next vertex to visit determines search order



Graph search

Breadth First Search

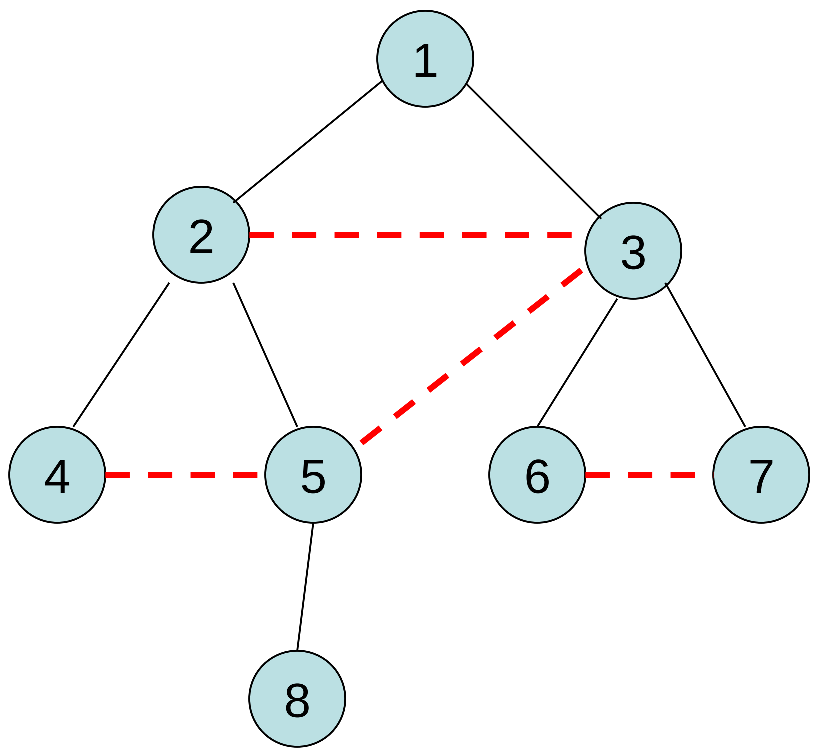
```
S = {s}
while S is not empty
    u = Dequeue(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Enqueue(S, v)
```

Depth First Search

```
S = {s}
while S is not empty
    u = Pop(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Push(S, v)
```

Breadth First Search

- All edges go between vertices on the same layer or adjacent layers



Depth First Search

- Each edge goes between vertices on the same branch
- No cross edges