University of Washington
Department of Computer Science and Engineering
CSE 417, Winter 2020

February 12, 2020

Homework 6, Due February 19, 2020

Turn-in instructions: Electronic submission using the CSE 417 canvas site. Each numbered problem is to be turned in as a separate PDF.

On all problems provide justification of your answers. For the algorithms problems, provide an clear explanation of why your algorithm solves the problem, as well as a justification of the run time. Since this assignment is from the Divide and Conquer section, express your algorithms in a recursive manner.

Problem 1 (10 points):

Suppose you are working in the quality control of a factory that produces quarters for the US government and your job is to make sure that all quarters have exactly the same weight. You are given 2^k quarters for $k \geq 2$ and you know that at most one of them can be defective. A defective quarter will weight higher or lower than normal. You are given a scale with two trays: Each time you can put a set S of quarters in the left and a set T in the right (for disjoints sets S, T). The scale will show if S is heavier than T, or T is heavier than S, or they have exactly the same weight. Design an algorithm to find the defective quarter (if it exists) by using the scale only $O(k)$ many times. (Note that your algorithm will run by a human not a computer.) Justify your algorithm is correct.

Problem 2 (10 points):

Suppose A is an array of n integers that is a strictly decreasing sequence, followed by a strictly increase sequence such as $[12, 9, 8, 6, 3, 4, 7, 9, 11]$. Give an $O(\log n)$ algorithm to find the minimum element of the array. Justify your algorithm is correct.

Problem 3 (10 points):

Let A and B be two sorted arrays of integers, each of length n. Show how you can find the median of the combined set of elements in $O(\log n)$ comparisons. (As in the Median algorithm discussed in lecture, you will need to solve the Select the k-th largest problem.) Justify your algorithm is correct.

Programming Problem 4 (10 points):

Implement the randomized selection algorithm discussed in class. (The algorithm is also discussed in the text, starting at page 727.) The algorithm is given an array of n integers, and an integer k and returns the k-th largest element of the array. For this problem, submit your code.

Programming Problem 5 (10 points):

It can be shown that the expected number of comparisons to find the median on n integers using the randomized algorithm is cn. Experimentally determine the value of c. You will want run the program for a range of values of n, and for each value of n you should run a number of times to get an average. Submit a plot of your results and your estimate for the value of c.

You will need to instrument your code to count the number of comparisons. By a comparison, we refer to comparisons between elements in the array, which happens when the problem is being split into subproblems. Do not count the comparisons that are used to test when you are finished with a for loop.