University of Washington March 9, 2020
Department of Computer Science and Engineering
CSE 417, Winter 2020

Homework 10, 5:00 pm Wednesday, March 18, 2020

Problem 1 (10 points):

Answer the following questions with "yes", "no", or "unknown, as this would resolve the P vs. NP question." Give a brief explanation of your answer.

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

a) Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

Problem 2 (10 points):

Suppose that you have an $O(n^3)$ time algorithm for the Hamiltonian Circuit Problem. Prove that $P = NP$.

Problem 3 (10 points):

(Kleinberg-Tardos, Page 507, Problem 7). Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define *4-Dimensional Matching* as follows. Given sets W, X, Y, and Z, each of size n, and a collection C of ordered 4-tuples of the form (w_i, x_j, y_k, z_l), do there exist n 4-tuples from C so that no two have an element in common?
Prove that 4-Dimensional Matching is NP-Complete.

Problem 4 (10 Points):

(Kleinberg-Tardos, Page 513, Problem 17). You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The *Zero-Weight-Cycle Problem* is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that the Zero-Weight-Cycle problem is NP-Complete. (Hint: Hamiltonian PATH)